ENVIRONMENT-ASSISTED INVARIANCE, IGNORANCE & INFORMATION IN QUANTUM PHYSICS*

[BORN'S RULE* FROM CAUSALITY]

$$|\Psi\rangle = \sum_{i=1}^{N} \psi_i |s_i\rangle \Longrightarrow p_i = |\psi_i|^2$$

PROBABILITIES OF $|s_i\rangle$s

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* MAX BORN, ZEITSCHRIFT FÜR PHYSIK, 37 pp. 863-867 (1926)

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* WHZ, "DECOHERENCE, EINSELECTION, & THE QUANTUM ORIGINS OF THE CLASSICAL" quant-ph/0105127, RMP (Apr. 2003)

+ A.M. GLEASON, J. MATH. MECH. 6 (1957), p. 885

"STRONG SUPERPOSIT'N PRINCIPLE + REASONABLE CONTINUITY ARGUMENTS"

$$\Longrightarrow \langle A \rangle = \mathrm{Tr}\, \rho A$$

If one translates this result into terms of particles, only one interpretation is possible. $\Phi_{n_\tau m}(\alpha, \beta, \gamma)$ gives the probability* for the electron, arriving from the z-direction, to be thrown out into the direction designated by the angles α, β, γ, with the phase change δ. Here its energy τ has increased by one quantum $h\nu_{nm}^0$ at the cost of the energy of the atom (collision of the first kind for $W_n^0 < W_m^0$, $h\nu_{nm}^0 < 0$; collision of the second kind $W_n^0 > W_m^0$, $h\nu_{nm}^0 > 0$).

Schrödinger's quantum mechanics therefore gives quite a definite answer to the question of the effect of the collision; but there is no question of any causal description. One gets no answer to the question, "what is the state after the collision," but only to the question, "how probable is a specified outcome of the collision" (where naturally the quantum mechanical energy relation must be fulfilled).

Here the whole problem of determinism comes up. From the standpoint of our quantum mechanics there is no quantity which in any individual case causally fixes the consequence of the collision; but also experimentally we have so far no reason to believe that there are some inner properties of the atom which condition a definite outcome for the collision. Ought we to hope later to discover such properties (like phases or the internal atomic motions) and determine them in individual cases? Or ought we to believe that the agreement of theory and experiment—as to the impossibility of prescribing conditions for a causal evolution—is a pre-established harmony founded on the nonexistence of such conditions? I myself am inclined to give up determinism in the world of atoms. But that is a philosophical question for which physical arguments alone are not decisive.

In practical terms indeterminism is present for experimental as well as for theoretical physicists. The "yield function" Φ so much investigated by experimentalists is now also sharply defined theoretically. One can determine it from the potential energy of interaction, $V(x, y, z; q_k)$. However, the calculations required

* Addition in proof: More careful consideration shows that the probability is proportional to the square of the quantity $\Phi_{n_\tau m}$.

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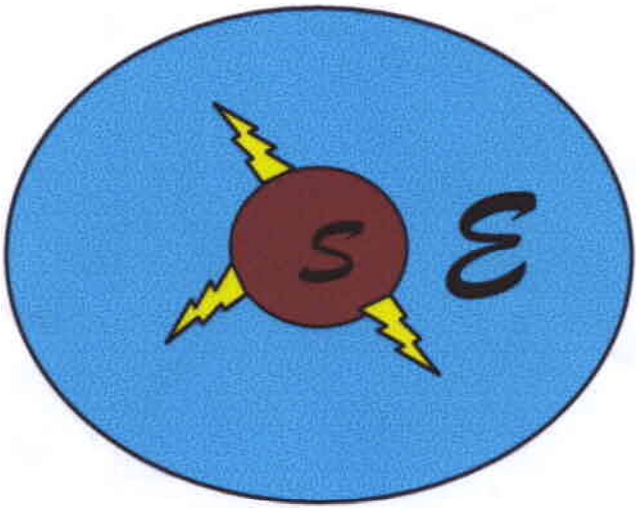
EINSELECTION, POINTER BASIS, AND DECOHERENCE



$$|\Phi_{\mathcal{SE}}(0)\rangle = |\psi_{\mathcal{S}}\rangle \otimes |\varepsilon_0\rangle = \left(\sum_i \alpha_i |\sigma_i\rangle\right) \otimes |\varepsilon_0\rangle$$

$$\xrightarrow[\text{Entanglement}]{\text{Interaction}} \sum_i \alpha_i |\sigma_i\rangle \otimes |\varepsilon_i\rangle = |\Phi_{\mathcal{SE}}(t)\rangle$$

REDUCED DENSITY MATRIX

$$\rho_{\mathcal{S}}(t) = Tr_{\mathcal{E}} |\Phi_{\mathcal{SE}}(t)\rangle\langle\Phi_{\mathcal{SE}}(t)| = \sum_i |\alpha_i|^2 |\sigma_i\rangle\langle\sigma_i|$$

EINSELECTION* leads to POINTER STATES

(same states appear on the diagonal of $\rho_{\mathcal{S}}(t)$ for times long compared to the decoherence time)

***Environment InducedSuperSELECTION**

DECOHERENCE & EINSELECTION

THESIS: QUANTUM THEORY CAN EXPLAIN EMERGENCE OF "THE CLASSICAL".
PRINCIPLE OF SUPERPOSITION LOSES VALIDITY IN "OPEN" SYSTEMS (I.E. SYSTEMS INTERACTING WITH THEIR ENVIRONMENTS)

DECOHERENCE RESTRICTS STABLE STATES (STATES THAT "EXIST") TO "EXCEPTIONAL"...

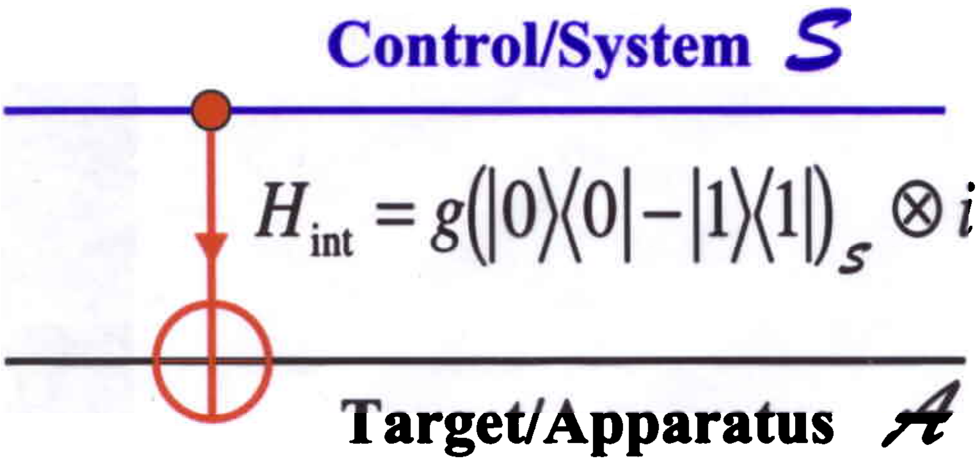
... POINTER STATES THAT EXIST OR PREDICTABLY EVOLVE IN SPITE OF THE IMMERSION OF THE SYSTEM IN THE ENVIRONM'NT

PREDICTABILITY SIEVE CAN BE USED TO "SIFT" THE HILBERT SPACE OF THE OPEN SYSTEM IN SERCH FOR THESE POINTER STATES

EINSELECTION (ENVIRONMENT INDUCED SUPERSELECTION) IS THE MECHANISM OF SELECTION OF THESE PREFERRED POINTER STATES

- FOR MACROSCOPIC SYSTEMS DECOHERENCE & EINSELECTION CAN BE VERY EFFECTIVE ("NO SCHRÖDINGER CATS")
- EINSELECTION INTRODUCES AN EFFECTIVE BORDER BETWEEN QUANTUM & CLASSICAL, MAKING A POINT OF VIEW SIMILAR TO BOHR'S C.I. POSSIBLE (BUT JUSTIFYING IT QUITE DIFFERENTLY.)

Measurements and Controlled Not (C-NOT)



$$H_{\text{int}} = g\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right)_{\mathcal{S}} \otimes i\left(|0\rangle\langle 1| - |1\rangle\langle 0|\right)_{\mathcal{A}}$$

Initially

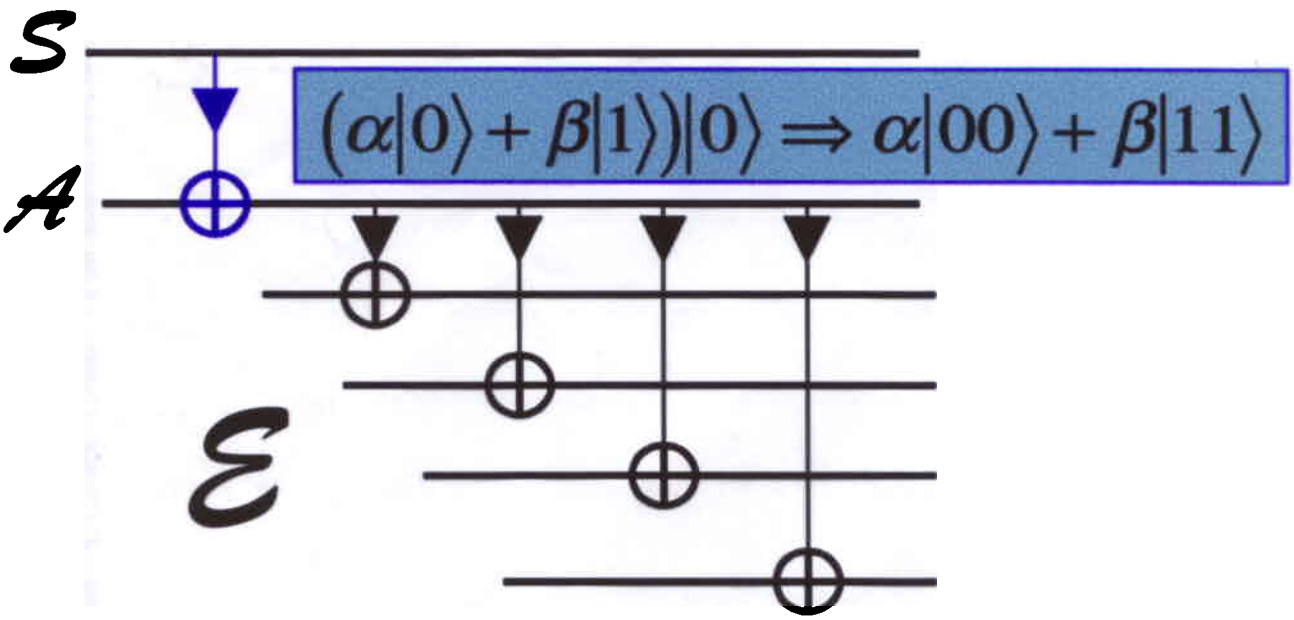
$$|c\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$|t\rangle = |0\rangle$$

After a time $2\pi\hbar/g$

$$|c\rangle|t\rangle \Rightarrow \alpha|0\rangle_c|0\rangle_t + \beta|1\rangle_c|1\rangle_t$$

Quantum Entanglement

DECOHERENCE AS A MEASUREMENT BY THE ENVIRONMENT

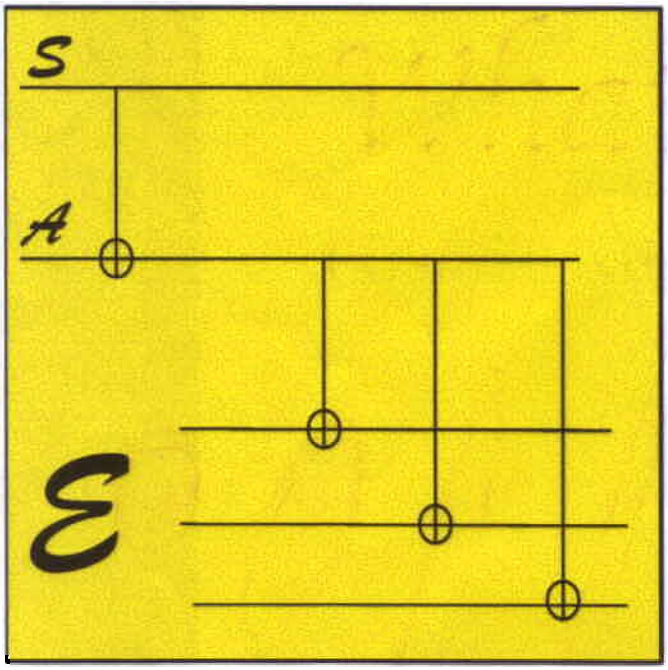


In presence of decoherence, classical correlations remain,but entanglement disappears.

$$|\alpha|^2|00\rangle\langle 00|+\alpha\beta^*|00\rangle\langle 11| +\alpha^*\beta|11\rangle\langle 00|+|\beta|^2|11\rangle\langle 11| \longrightarrow |\alpha|^2|00\rangle\langle 00| +|\beta|^2|11\rangle\langle 11|$$

IMPLICATIONS OF DECOHERENCE AND EINSELECTION

1. MEASUREMENTS



$$\left(\sum_k \alpha_k |s_k\rangle\right)|A_0\rangle|\varepsilon_0\rangle$$

$$\Downarrow$$

$$\left(\sum_k \alpha_k |s_k\rangle|A_k\rangle\right)|\varepsilon_0\rangle$$

$$\Downarrow$$

$$\sum_k \alpha_k |s_k\rangle|A_k\rangle|\varepsilon_k\rangle = |\Phi_{SAE}\rangle$$

$$\rho_{SA} = Tr_{\mathcal{E}}|\Phi_{SAE}\rangle\langle\Phi_{SAE}| \cong \sum_k |\alpha_k|^2 |s_k\rangle\langle s_k||A_k\rangle\langle A_k|$$

2. DYNAMICS

- States in Hilbert space “censored”, restricted to localized quantum approximations of points
- Classical equations of motion

$$i\hbar\frac{d\rho_{S\mathcal{E}}}{dt} = [H,\rho_{S\mathcal{E}}] \Rightarrow \dot{W}_{S}(x,p) \cong \{H_S, W_S\}_P$$

POINTER STATES
(their emergence and properties)

•Pointer states are measured by, but do not entangle with the environment $\mathcal{E}$

•Correlations with pointer states are preserved in spite of $\mathcal{E}$

$$\left(\sum_i \alpha_i |\sigma_i\rangle \otimes |s_i\rangle\right) \otimes |\varepsilon_0\rangle \Rightarrow \sum_i \alpha_i |\sigma_i\rangle \otimes |s_i\rangle \otimes |\varepsilon_i\rangle$$

•Pointer states are the most predictable over times long compared to decoherence time

Simple example of pointer states: When the Hamiltonian of interaction dominates then the pointer observable satisfies

$$[H_{\mathcal{SE}}, \Lambda] = 0$$

and is a constant of motion. Pointer states are its eigenstates.

PREDICTABILITY SIEVE

States in the Hilbert space of the open system evolve from pure into mixed under the influence of both the self Hamiltonian and the interaction Hamiltonian. They can be sorted according to predictability.

$$|\psi\rangle \Rightarrow \rho_\psi(t) \;\ldots\ldots\; H(\rho_\psi(t)) = Tr\rho_\psi^2(t)$$

$$|\varphi\rangle \Rightarrow \rho_\varphi(t) \;\ldots\ldots\; H(\rho_\varphi(t)) = Tr\rho_\varphi^2(t)$$

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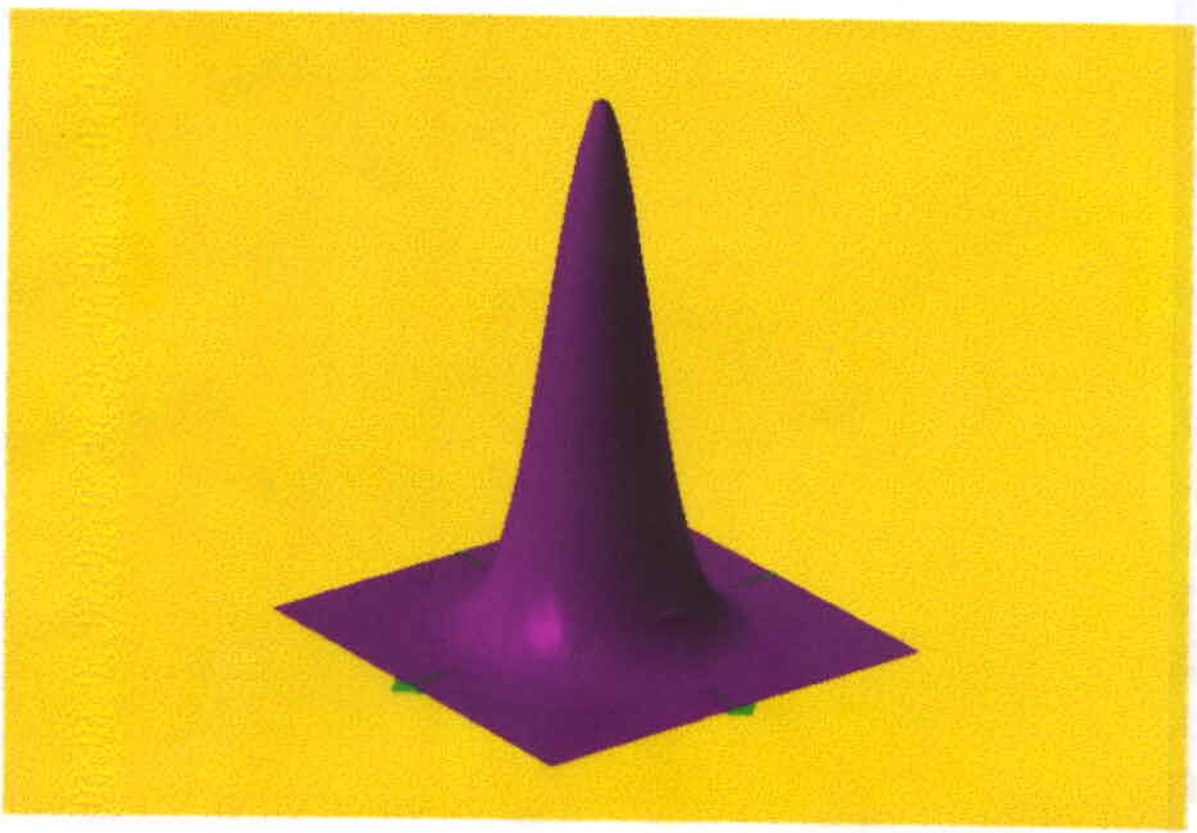
$$|\xi\rangle \Rightarrow \rho_\xi(t) \;\ldots\ldots\; H(\rho_\xi(t)) = Tr\rho_\xi^2(t)$$



PROBABILITIES FROM QUANTUM THEORY VIA DECOHERENCE

APPROACHES:

(i) "RELATIVE FREQUENCY"
(ii) "MEASURE OF CERTAINTY"
(iii) "EQUAL LIKELIHOOD"

A SIMPLE EXAMPLE:

$$\rho_S \propto \sum_{k=1}^{N} |k\rangle\langle k|$$

$$\Rightarrow p_k = 1/N \quad \forall_k$$

$$\Rightarrow P_{k_1 \vee k_2 \vee \ldots \vee k_n} = n/N$$

WARNING!!

- CANT USE "REDUCED DENSITY MATRIX" TO DERIVE $|\psi|^2 \sim p$!!

(BORN'S RULE USED TO JUSTIFY IT!!)

ENVARIANCE

(ENVIRONMENT-ASSISTED INVARIANCE)

CONSIDER A COMPOSITE QUANTUM OBJECT CONSISTING OF $\mathcal{S}$ AND $\mathcal{E}$

SYSTEM ENVIRONMENT

WHEN THE COMBINED STATE $|\psi_{\mathcal{SE}}\rangle$ IS TRANSFORMED BY:

$$U_{\mathcal{S}} = u_{\mathcal{S}} \otimes \mathbb{1}_{\mathcal{E}}$$

BUT CAN BE "UNTRANSFORMED" BY ACTING SOLELY ON $\mathcal{E}$, I.E. $\exists\, U_{\mathcal{E}}$ →

$$U_{\mathcal{E}} = \mathbb{1}_{\mathcal{S}} \otimes u_{\mathcal{E}}$$

THEN $|\psi_{\mathcal{SE}}\rangle$ IS **ENVARIANT** W.R.T. u

- $U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle = (u_{\mathcal{S}} \otimes \mathbb{1}_{\mathcal{E}})|\psi_{\mathcal{SE}}\rangle \Rightarrow |\tilde{\psi}_{\mathcal{SE}}\rangle \neq |$
- $U_{\mathcal{E}}|\tilde{\psi}_{\mathcal{SE}}\rangle = \underline{U_{\mathcal{E}} U_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle = |\psi_{\mathcal{SE}}\rangle}$

ENVARIANCE = { ENVIRONMENT-ASSISTED INVARIANCE; ENTANGLEMENT- " " ; EXTERNALLY " " }

ENVARIANCE IS A PROPERTY OF A COMBINED STATE: $\exists\, U_{\mathcal{E}}$ THAT "UNDOES" THE EFFECT OF $U_{\mathcal{S}}$.

ENVARIANCE – SOME PROPERTIES

$$\mathcal{U}_S \mathcal{U}_\mathcal{E} |\psi_{S\mathcal{E}}\rangle = e^{i\varphi} |\psi_{S\mathcal{E}}\rangle$$

NOTE # 1: ENVARIANT $|\psi_{S\mathcal{E}}\rangle$ IS AN EIGENSTATE OF $(\mathcal{U}_S \mathcal{U}_\mathcal{E})$ WITH A UNIT (UNIMODULAR) EIGENVA

NOTE # 2: ENVARIANCE FOR DENSITY MATRICES:

$$\mathcal{U}_S \mathcal{U}_\mathcal{E} \, \rho_{S\mathcal{E}} (\mathcal{U}_\mathcal{E} \mathcal{U}_S)^\dagger = \rho_{S\mathcal{E}}$$

[REMARK: WILL NOT NEED IT – ONE CAN ALWAYS PURIFY $\rho_{S\mathcal{E}} = \mathrm{Tr}_{\mathcal{E}'} |\psi_{S\mathcal{E}\mathcal{E}'}\rangle\langle\psi_{S\mathcal{E}\mathcal{E}'}$

NOTE # 3 PRODUCT OF TRANSFORMATION THAT LEAVE $|\psi_{S\mathcal{E}}\rangle$ ENVARIANT IS ALSO ENVARIANT

PROOF:

$$\left.\begin{aligned} \mathcal{U}_S^{(1)} |\psi_{S\mathcal{E}}\rangle &= (\mathcal{U}_\mathcal{E}^{(1)})^{-1} |\psi_{S\mathcal{E}}\rangle \\ \mathcal{U}_S^{(2)} |\psi_{S\mathcal{E}}\rangle &= (\mathcal{U}_\mathcal{E}^{(2)})^{-1} |\psi_{S\mathcal{E}}\rangle \end{aligned}\right\} \Rightarrow \mathcal{U}_S^{(1)} \mathcal{U}_S^{(2)} |\psi_{S\mathcal{E}}\rangle = (\mathcal{U}_\mathcal{E}^{(1)} \mathcal{U}_\mathcal{E}^{(2)}$$

$\checkmark$

$$\Downarrow$$

$$\mathcal{U}_S^{(1)} \mathcal{U}_\mathcal{E}^{(1)} \mathcal{U}_S^{(2)} \mathcal{U}_\mathcal{E}^{(2)} |\psi_{S\mathcal{E}}\rangle = |\psi_{S\mathcal{E}}\rangle \quad \text{QED}$$

ENTANGLED STATE AS AN EXAMPLE OF ENVARIANCE

$$|\psi_{SE}\rangle = \sum_{k=1}^{N} \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

SCHMIDT DECOMPOSITION: α_k COMPLEX, $\{|s_k\rangle\}$ & $\{|\varepsilon_k\rangle\}$ ORTHONORMAL

LEMMA 1: ALL UNITARY TRANSFORMATIONS WITH EIGENSTATES $\{|s_k\rangle\}$ ARE ENVARIANT (I.E. LEAVE $|\psi_{SE}\rangle$ ENVARIANT)

PROOF:

$$U_S^{\{|s_k\rangle\}} = \sum_k e^{i\varphi_k} |s_k\rangle\langle s_k|$$

$$U_S^{\{|s_k\rangle\}} |\psi_{SE}\rangle = \sum_k \alpha_k e^{i\varphi_k} |s_k\rangle|\varepsilon_k\rangle$$

BUT $\exists$ A "COUNTERTRANSFORMATION" ACTING ON $\mathcal{E}$:

$$U_\varepsilon^{\{|\varepsilon_k\rangle\}} = \sum_k e^{-i\varphi_k + 2\pi i l_k} |\varepsilon_k\rangle\langle\varepsilon_k|$$

l_k INTEGER

THAT "UNDOES" $U_S^{\{|s_k\rangle\}}$:

$$U_S^{\{|s_k\rangle\}} U_\varepsilon^{\{|\varepsilon_k\rangle\}} |\psi_{SE}\rangle = |\psi_{SE}\rangle$$

Q.E.D.

"NO ACTION AT A DISTANCE" LEMMA

(i) PROBABILITIES ARE A LOCAL PROPERTY OF A SYSTEM ("NO ACTION..." – NAAAD ASSUMPTION)

(ii) ENTANGLEMENT HAPPENS:

$$|\psi_{\mathcal{S}\mathcal{E}}\rangle = \sum_k{}' \alpha_k |s_k\rangle|\varepsilon_k\rangle$$

SCHMIDT DECOMP'STN

<u>THESIS</u>: PROBABILITIES OF $\mathcal{S}$ <u>ALONE</u> CAN DEPEND ONLY ON $|\alpha_k|$ (ABSOLUTE VALUES, <u>NOT</u> PHASES)

<u>PROOF</u>: PHASES OF α_k CAN BE CHANGED BY ACTING ON $\mathcal{E}$ ALONE.

$\therefore$ BY NAAAD $\{|\alpha_k|, |s_k\rangle\}$ MUST BE A COMPLETE LOCAL DESCRIPTION

$$P_k = f(|\alpha_k|)$$

ENVARIANCE OF AN ENTANGLED STATE

– THE CASE OF EQUAL COEFF'S.

$$|\psi_{SE}\rangle \propto \sum_{k=1}^{N} e^{i\varphi_k} |s_k\rangle|\varepsilon_k\rangle$$

ANY ORTHONORMAL BASIS OF S IS SCHMIDT

IN PARTICULAR, IN A SUBSPACE $\mathcal{H}_S^{kl}$ CAN DEFINE HADAMARD BASIS:

$$|\pm\rangle_{k,l} = (|s_k\rangle \pm |s_l\rangle)/\sqrt{2}$$

⋮

CAN USE THIS TO GENERATE A "NEW KIND" OF ENVARIANT TRANSFORMATIONS: **"SWAPS"**

$$u_S(k \leftrightarrow l) = e^{i\varphi_{kl}} |s_k\rangle\langle s_l| + h.c.$$

FOR THE CASE OF $|\alpha_k| = |\alpha_l|$ THE EFFECT OF THE SWAP CAN BE "UNDONE" BY A "COUNTERSWAP"

$$u_\varepsilon(k \leftrightarrow l) = e^{i(\varphi_{kl} + \varphi_k - \varphi_l + 2\pi L_{kl})} |\varepsilon_k\rangle\langle\varepsilon_l| + h.c.$$

PROOF:

$$u_S(k \leftrightarrow l)\, u_\varepsilon(k \leftrightarrow l)\, |\psi_{S\varepsilon}\rangle = |\psi_{S\varepsilon}\rangle$$

[WHEN SWAP IN S IS FOLLOWED BY A(SWAP IN ε, STATE SAME IF $|\alpha_k| = |\alpha_l|$

CASE OF EQUAL $|\alpha_k|$'S (COEFF'S.)

$$|\psi_{1\mathcal{E}}\rangle \sim \sum_{k=1}^{N} e^{i\varphi_k} |\sigma_k\rangle |\mathcal{E}_k\rangle$$

BY THE "NAAAD LEMMA" THE SET OF PAIRS $\{|\alpha_k|, |\sigma_k\rangle\}$ DESCRIBES $\mathcal{S}$ COMPLETELY. BUT ALL $|\alpha_k|$ ARE EQUAL.

$$\therefore \quad P(\sigma_k) = 1/N \quad \forall_k$$

$$P(\sigma_k \vee \sigma_{k+1} \vee \dots \vee \sigma_{k+m-1}) = m/N$$

CASE OF UNEQUAL COEFFICIENTS

$$|\psi_{S\mathcal{E}}\rangle = \sqrt{\tfrac{2}{3}}\,|0\rangle|+\rangle + \sqrt{\tfrac{1}{3}}\,|2\rangle|2\rangle$$

S - TWO STATES, $\{|0\rangle, |2\rangle\}$

$\mathcal{E}$ - THREE STATES, $\{|0\rangle, |1\rangle, |2\rangle\}$; $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$\mathcal{E}'$ - TO USE NAAAD FOR BOTH S & $\mathcal{E}$, INTRODUCE $\mathcal{E}'$!

$$|\psi_{S\mathcal{E}}\rangle|\mathcal{E}'\rangle = \left(\sqrt{\tfrac{2}{3}}\,|0\rangle|+\rangle + \sqrt{\tfrac{1}{3}}\,|2\rangle|2\rangle\right)|0\rangle \rightarrow$$

"C-SHIFT"
$|k\rangle|0\rangle \rightarrow |k\rangle|k\rangle$

$$\rightarrow \cancel{\sqrt{\tfrac{2}{3}}}\,|0\rangle\left(\frac{|0\rangle|0\rangle + |1\rangle|1\rangle}{\cancel{\sqrt{2}}}\right) + \sqrt{\tfrac{1}{3}}\,|2\rangle|2\rangle|2\rangle =$$

$$= \left(\underline{|0\rangle|0\rangle}\,|0\rangle + \underline{|0\rangle|1\rangle}\,|1\rangle + \underline{|2\rangle|2\rangle}\,|2\rangle\right)/\sqrt{3}$$

- STATES $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|2\rangle|2\rangle$ HAVE EQUAL LIKELIHOODS: EACH HAS A PROBABILITY OF $1/3$

- $\exists$ TWO CASES WHERE $|0\rangle$ APPEARS FOR THE SYSTEM

BORN'S RULE!

$\therefore\ P_{|0\rangle} = \frac{2}{3}$; $P_{|2\rangle} = \frac{1}{3}$!

MORE GENERALLY;

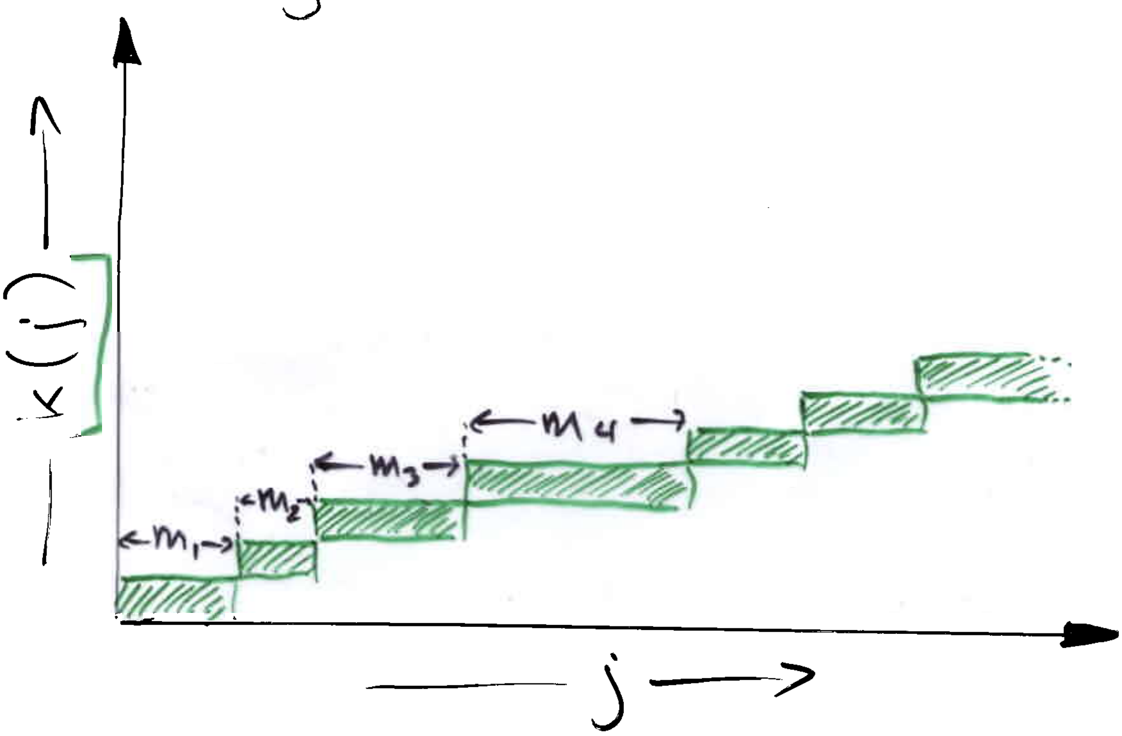
THE CASE OF COMMENSURATE PROB'S:

$$|\psi_{S\mathcal{E}}\rangle = \sum_{k=1}^{N} \underbrace{\sqrt{\frac{m_k}{M}}}_{\alpha_k} |s_k\rangle|\varepsilon_k\rangle$$

ATTACH $\mathcal{E}'$...

$$|\psi_{S\mathcal{E}}\rangle|\mathcal{E}'\rangle = \left(\sum_{k=1}^{N} \sqrt{\frac{m_k}{M}}\,|s_k\rangle\left(\sum_{j_k=1}^{m_k}|e_{j_k}\rangle\right)\Big/\sqrt{m_k}\right)|e'_0\rangle$$

$$\longrightarrow \frac{1}{\sqrt{M}}\sum_{j=1}^{M}|s_{k(j)}\rangle|e_j\rangle|e'_j\rangle$$



$$P_j = 1/M$$

$$P_k = m_k/M = |\alpha_k|^2$$

FREQUENCY INTERPRETATION

IMAGINE ENSEMBLE WITH MANY ($\mathcal{N} \gg 1$) DISTINGUISHABLE S, A & E, ALL IN THE SAME STATE:

$$|\Phi^{\mathcal{N}}_{SAE}\rangle = \bigotimes_{l=1}^{\mathcal{N}} |\psi_{SAE}\rangle_l$$

WHAT IS THE FREQUENCY OF OCURRENCE OF 0's & 1's ?

$$|\Phi^{\mathcal{N}}_{SAE}\rangle = \alpha^{\mathcal{N}}\left(|0\rangle|A_0\rangle|\varepsilon_0\rangle\right)^{\otimes\mathcal{N}} + \binom{\mathcal{N}}{1}\alpha^{\mathcal{N}-1}\beta\left(|0\rangle|A_0\rangle|\varepsilon_0\rangle\right)^{\otimes\mathcal{N}-1}\left(|1\rangle|A_1\rangle|\varepsilon_1\rangle\right)$$

$$+ \dots + \beta^{\mathcal{N}}\left(|1\rangle|A_1\rangle|\varepsilon_1\rangle\right)^{\otimes\mathcal{N}}$$

$$\alpha = \sqrt{m/M} \qquad \beta = \sqrt{(M-m)/M}$$

$$|A_0\rangle = \sum_1^m |a_k\rangle/\sqrt{m} \qquad |A_1\rangle = \sum_{m+1}^M |a_k\rangle/\sqrt{M-m}$$

$$|\Phi^{\mathcal{N}}_{SAE}\rangle = \left(\frac{1}{\sqrt{M}}\right)^{\mathcal{N}}\left(|0\rangle\left(\sum_1^m|a_k\rangle\right)|\varepsilon_0\rangle\right)^{\mathcal{N}} + \binom{\mathcal{N}}{1}\left(|0\rangle\sum_1^m\dots\right)^{\mathcal{N}-1}\dots$$

$$+ \binom{\mathcal{N}}{n}\left(|0\rangle\left(\sum_1^m|a_k\rangle\right)|\varepsilon_0\rangle\right)^{\mathcal{N}-n}\left(|1\rangle\left(\sum_{m+1}^M|a_k\rangle\right)|\varepsilon_1\rangle\right)^n$$

$$+ \dots$$

PROB. OF n 1's:

$$P_n = \binom{\mathcal{N}}{n}\left(\frac{m}{M}\right)^{\mathcal{N}-n}\left(\frac{M-m}{M}\right)^n$$

$$= \binom{\mathcal{N}}{n}|\alpha|^{2(\mathcal{N}-n)}|\beta|^{2n}$$

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SUMMARY & CONCLUSIONS*

1. NEW SYMMETRY - ENVARIANCE - OF JOINT STATES OF QUANTUM SYSTEMS

2. IN QUANTUM THEORY, PERFECT KNOWLEDGE OF THE 'WHOLE MAY IMPLY COMPLETE IGNORANCE OF A PART

3. BORN'S RULE IS A CONSEQUENCE OF ENVARIANCE (AND OF CAUSALITY - NAAAD).

4. FREQUENCY INTERPRETATION NATURALLY FOLLOWS (ALSO IN A "MANY WORLDS" CONTEXT).

5. ENVARIANCE SUPPLIES A NEW FOUNDATION FOR DECOHERENCE AND ENVIRONMENT - INDUCED SUPERSELECTION BY JUSTIFYING "REDUCED DENSITY MATRICES", "TR"...

* WHZ, RMP, IN PRESS; ALSO, IN PRE-PARAT'N...