Symplectic Codes and Quantum Capacity of Noisy Channels

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Contents

Goal

To provide evidence that quantum error-correcting codes (QECCs) [or standard algebraic QECCs = symplectic codes = stabilizer codes] work reliably at positive rates in the presence of quantum noise, and to determine the highest such rate, i.e., the quantum capacity, of the channel in the spirit of Shannon.

Towards this goal, this talk gives lower bounds on the capacity. Especially, I give the conditional capacity of the depolarizing channel on symplectic (stabilizer) codes.

Talk is mainly based on MH (2002, quant-ph/0207113)

Remarks on Quantum Channel Coding

There are two major settings:

1. sending classical messages over noisy quantum channels
   - classical capacity
     Coding theorem exists: Holevo (1998), Schumacher and Westmoreland (1997); additivity problem, King
2. protecting quantum states from quantum noise
   = sending entanglement over noisy quantum channels
   (Gives insight into realization of quantum computers)
   - quantum capacity
     Shor (this workshop!)

History

1. Shor (1995) posed the problem of determining quantum capacity
2. Schumacher and Nielsen (1996), coherent information
3. Bennett *et al.* (1996) gave a lower bound on capacity of general binary quantum discrete memoryless channels (QDMCs);
   argued with an entanglement purification protocol
4. Shor and Smolin (1996) improved this for very noisy channels
5. Preskill (1998) gave a lower bound $1 - H(P)$ for Pauli channel with probabilities $P(s,t)$ of occurrence of $\sigma_x^s \sigma_z^t = X^s Z^t,\ s,t \in \{0,1\}$;
   used standard QECCs (symplectic or stabilizer codes)
6. MH (*IEEE IT*, 2002) extended Preskill's lower bound to general QDMCs;
   used symplectic codes;
   not smaller than those previously known except Shor and Smolin's

Quantum Channels

- $\mathsf{L}(\mathsf{H}') =$ the set of all linear operators on a Hilbert space H'
- Completely positive (CP) map $\mathcal{M} : \mathsf{L}(\mathsf{H}') \to \mathsf{L}(\mathsf{H}')$ has form $\mathcal{M}(\rho) = \sum_i M_i \rho M_i^\dagger$, where $M_i \in \mathsf{L}(\mathsf{H}')$.

  Notation: $\mathcal{M} \sim \{M_i\} \iff \mathcal{M}$ is specified by $\{M_i\}$ in this way
- A quantum discrete memoryless channel (QDMC) is a trace-preserving CP map (TPCP map) $\mathcal{A} : \mathsf{L}(\mathsf{H}) \to \mathsf{L}(\mathsf{H})$; supposed to act as $\mathcal{A}^{\otimes n}(\rho)$ on $\rho \in \mathsf{L}(\mathsf{H}^{\otimes n})$
- Assumption: $\dim \mathsf{H} = d$ is a prime number

Quantum Capacity

- A *(quantum) code* $=$ a pair $(\mathcal{C}_n, \mathcal{R}_n)$ consisting of a subspace $\mathcal{C}_n \subseteq \mathsf{H}^{\otimes n}$ and a TPCP linear map $\mathcal{R}_n : \mathsf{L}(\mathsf{H}^{\otimes n}) \to \mathsf{L}(\mathsf{H}^{\otimes n})$ ($\mathcal{R}_n$: a recovery operator)
- Rate of code $(\mathcal{C}_n, \mathcal{R}_n) = \dfrac{\log_d \dim \mathcal{C}_n}{n}$
- Fidelity (minimum fidelity)

$$F(\mathcal{C}_n, \mathcal{A}) = \min_{|\psi\rangle \in \mathcal{C}_n} \langle \psi | \mathcal{R}_n \circ \mathcal{A}^{\otimes n}(|\psi\rangle\langle\psi|) | \psi \rangle$$

- A number $R \geq 0$ is said to be achievable on $\mathcal{A}$ if there exists a sequence of codes $\{(\mathcal{C}_n, \mathcal{R}_n)\}$ of rate not less than R such that $\lim_n F(\mathcal{C}_n, \mathcal{A}) = 1$
- $\mathsf{Q}(\mathcal{A})$ $=$ quantum capacity of $\mathcal{A}$ $= \sup\{R \mid R \text{ is achievable on } \mathcal{A}\}$

Coherent Information

For a density operator $\rho \in \mathsf{L}(\mathsf{H}')$ and a TPCP map $\mathcal{A}' : \mathsf{L}(\mathsf{H}') \to \mathsf{L}(\mathsf{H}')$, the coherent information $I_{\mathrm{c}}(\rho, \mathcal{A}')$ is defined by

$$I_{\mathrm{c}}(\rho, \mathcal{A}') = S\big(\mathcal{A}'(\rho)\big) - S\big([\mathrm{Id} \otimes \mathcal{A}'](|\Psi\rangle\langle\Psi|)\big),$$

where $S(\sigma)$ denotes the von Neumann entropy of σ, and $|\Psi\rangle \in \mathsf{H}'' \otimes \mathsf{H}'$ is a purification of ρ.

Upper bounds on $\mathsf{Q}(\mathcal{A})$:

- $\mathsf{Q}(\mathcal{A}) \leq \lim\limits_{n\to\infty} \max\limits_{\rho} \dfrac{I_{\mathrm{c}}(\rho, \mathcal{A}^{\otimes n})}{n}$ (Barnum *et al.*, 2000)
  where the maximum is over all states on $\mathsf{H}^{\otimes n}$.
- $\mathsf{Q}(\mathcal{A}) \leq \lim\limits_{n\to\infty} \max\limits_{\mathcal{C}} \dfrac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n})}{n}$,
  where $\Pi_{\mathcal{C}}$ is the projection onto $\mathcal{C}$ divided by $\dim \mathcal{C}$, and the maximum is over all subspaces of $\mathsf{H}^{\otimes n}$.

These two bounds are the same.

Weyl's Unitary Basis of $\mathsf{L}(\mathsf{H})$

- $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$: an arbitrary basis of H. ω : a primitive d-th root of unity. $\{0, \ldots, d-1\} = \mathbb{Z}/d\mathbb{Z} = \mathsf{F}$; $\mathcal{X} = \mathsf{F}^2$
- Following H. Weyl (1928), define unitary operators $X, Z \in \mathsf{L}(\mathsf{H})$ by

$$X|j\rangle = |j-1\rangle, \quad Z|j\rangle = \omega^j |j\rangle, \quad j \in \mathsf{F}$$

  Note: $\{|i\rangle\} \overset{\text{Fourier T.}}{\longrightarrow} \{|i\rangle'\}$, then $Z|i\rangle' = |j-1\rangle'$
- The $\mathsf{N} = \{N_{(s,t)} = X^s Z^t\}_{(s,t)\in\mathcal{X}}$ is an orthonormal basis of $\mathsf{L}(\mathsf{H})$ (w.r.t. inner product $\langle A, B\rangle = d^{-1}\mathrm{Tr}A^\dagger B$).
- We put $\mathsf{N}_n = \{N_x\}_{x\in\mathcal{X}^n}$, where $N_{(x_1,\ldots,x_n)} = N_{x_1} \otimes \ldots \otimes N_{x_n}$

N-channel

An N-channel is a memoryless channel $\mathcal{A} \sim \{\sqrt{P(u)}N_u\}_{u\in\mathcal{X}}$, where P is a probability distribution on $\mathcal{X}$. Also known as a Pauli channel.
Example: $P(u)$ is uniform except $u = (0,0) \Longrightarrow$ depolarizing channel

Lower Bound on the Quantum Capacity

Theorem 1 *For any* N*-channel, we have*

$$\mathsf{Q}(\mathcal{A}) \geq \lim_{n\to\infty} \max_{\mathcal{C}\in \mathsf{S}_n(\mathsf{N})} \frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n})}{n},$$

where $\Pi_{\mathcal{C}}$ *is the unit-trace operator proportional to the projection onto* $\mathcal{C}$ *and* $\mathsf{S}_n(\mathsf{N})$ *is the set of all symplectic (stabilizer) codes designed with* N_n.

Remark. A symplectic code is a simultaneous eigenspace of a set of commuting operators $\in \mathsf{N}_n$.

Cf. Upper bound

$$\mathsf{Q}(\mathcal{A}) \leq \lim_{n\to\infty} \max_{\mathcal{C}} \frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n})}{n},$$

where the maximum is taken over all subspaces of $\mathsf{H}^{\otimes n}$.

Remark 1. Conditional Capacity

- Imagine only a certain class T_n of subspaces of $\mathsf{H}^{\otimes n}$ are available as quantum codes. In this situation, we consider the 'conditional capacity' $\mathsf{Q}(\mathcal{A}|\{\mathsf{T}_n\})$ of a channel $\mathcal{A}$.
- Follows an upper bound on the conditional capacity

$$\mathsf{Q}(\mathcal{A}|\{\mathsf{T}_n\}) \leq \lim_{n\to\infty} \max_{\mathcal{C}\in\mathsf{T}_n} \frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n})}{n}$$

  for a general channel $\mathcal{A}$.

- When S_n is the set of all symplectic codes, the bound in Theorem1 is the conditional capacity $\mathsf{Q}(\mathcal{A}|\{\mathsf{S}_n\})$ of the depolarizing channel $\mathcal{A}$:

$$\mathsf{Q}(\mathcal{A}|\{\mathsf{S}_n\}) = \lim_{n\to\infty} \max_{\mathcal{C}\in\mathsf{S}_n} \frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n})}{n}.$$

Remark 2. Superadditivity of Coherent Information

The lower bound in Theorem 1 is the supremum of b_n/n, where

$$b_n = \sup_{\mathcal{C} \in \mathsf{S}_n(\mathsf{N})} I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n}), \quad n = 1, 2, \ldots. \tag{1}$$

Does $b_n/n > b_1$ hold for some n or not? Shor and Smolin numerically demonstrated that $b_n/n > b_1$ for very noisy 2-dimensional depolarizing channels. Recall that $\lim_n b'_n/n$, where

$$b'_n = \sup_{\mathcal{C}\text{: subspace}} I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \mathcal{A}^{\otimes n}), \tag{2}$$

is an upper bound on the usual (unconditional) capacity $\mathsf{Q}(\mathcal{A})$. For the erasure channel, $\lim_n b'_n/n = b'_1$, which is indeed the capacity.

Remark 3. Coset Arrays and Probability Arrays

- The lower bound is the supremum of

$$\frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}_L}, \mathcal{A}^{\otimes n})}{n} = \frac{k - H_{\mathrm{cond}}(P_L)}{n}$$

  over all choices for L, where $H_{\mathrm{cond}}(P_L)$ denotes the conditional entropy of P_L to be specified later, the L is an $[[n, k]]$ code, which encodes k qudits into n qudits, P_L is determined from L and P ($\mathcal{A} \sim \{\sqrt{P(u)}N_u\}$).

- L: the $[[1, 1]]$ code $\Longrightarrow$ we recover the known lower bound $1 - H(P)$
- L: the $[[n, 1]]$ repetition code $\Longrightarrow$ we recover the Shor-Smolin bound

What is $H_{\mathrm{cond}}(P_L)$?

Symplectic (Stabilizer) Codes

- Symplectic inner product: For $x = (u_1, v_1, \ldots, u_n, v_n) \in \mathsf{F}^{2n}$, $y = (u'_1, v'_1, \ldots, u'_n, v'_n) \in \mathsf{F}^{2n}$, $(x, y)_{\mathrm{sp}} = \sum_{i=1}^{n} u_i v'_i - v_i u'_i$
- $N_x N_y = \omega^{(x,y)_{\mathrm{sp}}} N_y N_x$
- A subspace $L \subseteq \mathsf{F}^{2n}$ is self-orthogonal $\leftrightarrow L \subseteq L^{\perp}$, where $L^{\perp} = \{y \in \mathsf{F}^{2n} \mid \forall x \in L, (x, y)_{\mathrm{sp}} = 0\}$
- Symplectic codes: Once a self-orthogonal code $L \subseteq \mathsf{F}^{2n}$ with $\dim L = n - k$ is obtained, we get $S = d^{n-k}$ subspaces $\mathcal{C}_L^{(0)}, \ldots, \mathcal{C}_L^{(S-1)} \subseteq \mathsf{H}^{\otimes n}$ with $\dim \mathcal{C}_L^{(i)} = d^k$.
  We can use any $\mathcal{C}_L^{(i)}$ as a quantum symplectic code

Coset Array of L

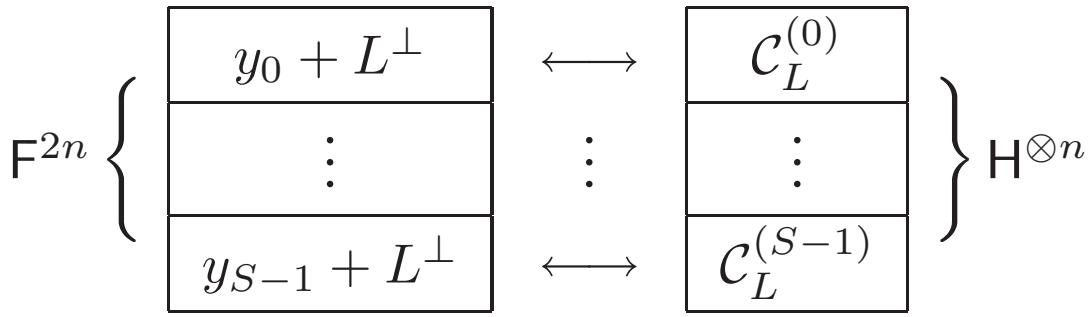
$$\begin{array}{cccc} y_0 + x_0 + L & y_0 + x_1 + L & \cdots & y_0 + x_{K-1} + L \\ y_1 + x_0 + L & y_1 + x_1 + L & \cdots & y_1 + x_{K-1} + L \\ \vdots & \vdots & & \vdots \\ y_{S-1} + x_0 + L & y_{S-1} + x_1 + L & \cdots & y_{S-1} + x_{K-1} + L, \end{array}$$

where $K = d^{2k}$, $S = d^{n-k}$, $\{x_i\}$ is a transversal of the cosets of L in $L^{\perp}$, and $\{y_i\}$ is that of the cosets of $L^{\perp}$ in F^{2n}.

Each row form a coset of $L^{\perp}$ in F^{2n}.

Cf. Standard array (of $L^{\perp}$) in coding theory

Tracing Errors Using Coset Arrays



Assume an error N_z, $z \in \mathsf{F}^{2n}$, occurs on code $\mathcal{C}_L^{(0)}$. Decompose z into

$$z = w + y_i + x_j, \quad w \in L.$$

Then, $$N_z = \alpha N_{x_j} N_{y_i} N_w, \quad \alpha \in \mathbb{C}.$$

- N_w does nothing, (hence, $\{N_w\}_{w \in L}$: stabilizer)
- N_{y_i} moves any state in $\mathcal{C}_L^{(0)}$ to $\mathcal{C}_L^{(i)}$, (syndrome i can be measured)
- N_{x_j} stirs states in $\mathcal{C}_L^{(i)}$, and its action is that of Pauli matrices (Weyl's unitaries) for encoded (logical) qudits

Decoding Symplectic Codes

- error x = error N_x, $\quad \mathsf{F}^{2n} \ni x \overset{1:1}{\longleftrightarrow} N_x \in \mathsf{N}_n$
- Design of a decoder = to choose a set J_0 of coset representatives of cosets of $L^\perp$ in F^{2n}.
  For any such set J_0, any $\mathcal{C}_L^{(i)}$ is J-correcting, where
  $J = J_0 + L = \{w + v \mid w \in L, v \in J_0\}$

E.g., J is union of $\boxed{\cdots}$:

$$\begin{array}{cccc}
y_0 + x_0 + L & \boxed{y_0 + x_1 + L} & \cdots & y_0 + x_{K-1} + L \\
\boxed{y_1 + x_0 + L} & y_1 + x_1 + L & \cdots & y_1 + x_{K-1} + L \\
\vdots & \vdots & & \vdots \\
y_{S-1} + x_0 + L & y_{S-1} + x_1 + L & \cdots & \boxed{y_{S-1} + x_{K-1} + L}
\end{array}$$

Probability Array of L $(d = 2)$

$$\begin{array}{rrcr}
P_L(0_{n-k}, 0_{2k}) & P_L(0_{n-k}, 0\ldots01) & \cdots & P_L(0_{n-k}, 11\ldots1) \\
P_L(0\ldots01, 0_{2k}) & P_L(0\ldots01, 0\ldots01) & \cdots & P_L(0\ldots01, 11\ldots1) \\
\vdots & \vdots & & \vdots \\
P_L(11\ldots1, 0_{2k}) & P_L(11\ldots1, 0\ldots01) & \cdots & P_L(11\ldots1, 11\ldots1)
\end{array}$$

$P_L(s, \widetilde{u})$ is the probability of errors (vectors) in the corresponding coset: $P_L(s, \widetilde{u}) = \sum_{x \in \mathrm{coset}(s, \widetilde{u})} P^n(x)$. Row index s is syndrome.

- The conditional entropy $H_{\mathrm{cond}}(P_L)$ appearing the lower bound $[k - H_{\mathrm{cond}}(P_L)]/n$ is $H(X|Y)$ where (Y, X) is drawn according to P_L.
- Interpretation: the less average entropy of row is, the better code L is.
- Clearly, $P^n(J_0) \leq P^n(J)$. Evaluating $P^n(J_0)$ results in the old bound $1 - H(P)$, Bennett *et al.* ('96), Preskill ('98), MH (*IEEE IT*, 2002).

Ideas for Proof of Theorem 1

- Concatenated code $\mathsf{cat}(L, L_{\rm out})$ (two-stage coding)

  L: inner $[[n, k]]$ code, $L_{\rm out}$: outer code. Both are self-orthogonal

- Theorem was proved with a random coding argument.
  Namely, I proved $\forall L$, $\forall \mathcal{A} \sim \{\sqrt{P(u)}N_u\}_{u\in\mathcal{X}}$,

$$\frac{1}{|\mathsf{E}|}\sum_{L_{\rm out}\in\mathsf{E}} F(\mathcal{C}_{\mathsf{cat}(L,L_{\rm out})}, \mathcal{A}) \geq 1 - \exp_d[-mG(R,P,L) + o(m)]$$

  where the ensemble E consists of all self-orthogonal codes over F with fixed size, and $\mathcal{C}_L$ denotes a symplectic code associated with L.

  Cf. Shor and Smolin restricted L to (concat. of) repetition codes.

- $G(R, P, L) > 0$ for $R < I_{\rm c}(\Pi_{\mathcal{C}_L}, \mathcal{A}^{\otimes n})/n$.

- I used the method of types from information theory.

Ideas for Proof of Theorem 1 (Continued)

- Encoding with concatenated codes

	block 1	$\cdots$	block ν
Encode by L:	k qubits in n	$\cdots$	k qubits in n

$\kappa < k\nu$ qubits are encoded by $[[\kappa, k\nu]]$ outer code L_{out}

- Decoding

	block 1	$\cdots$	block ν
	k qubits in n	$\cdots$	k qubits in n
measure syndromes	$\downarrow s_1$	$\cdots$	$\downarrow s_\nu$
	$\mathcal{V}_{s_1}(k \text{ qubits})$	$\cdots$	$\mathcal{V}_{s_\nu}(k \text{ qubits})$

Varying channel $\mathcal{V}_{s_1} \otimes \cdots \otimes \mathcal{V}_{s_\nu}$ for outer code

Conditioning of $H_{\text{cond}}(P_L)$ in $[k - H_{\text{cond}}(P_L)]/n$ is on syndrome s_i

Remark 4. The Case of General Quantum Discrete Memoryless Channels

For a channel $\mathcal{A} \sim \{A_u\}_{u\in\mathcal{X}}$, expand each A_u in terms of basis N as $A_u = \sum_{v\in\mathcal{X}} a_{uv} N_v$, $u \in \mathcal{X}$. Define a probability distribution $\widehat{P}_{\mathcal{A}}$ by

$$\widehat{P}_{\mathcal{A}}(v) = \sum_{u\in\mathcal{X}} |a_{uv}|^2, \quad v \in \mathcal{X}.$$

Then, we have

$$\mathsf{Q}(\mathcal{A}) \geq \lim_{n\to\infty} \max_{\mathcal{C}\in\mathsf{S}_n(\mathsf{N})} \frac{I_{\mathrm{c}}(\Pi_{\mathcal{C}}, \widehat{\mathcal{A}}^{\otimes n})}{n}$$

where $\widehat{\mathcal{A}} \sim \{\sqrt{\widehat{P}_{\mathcal{A}}(u)} N_u\}_{u\in\mathcal{X}}$.

Proof. Roughly speaking, $F(\mathcal{C}, \mathcal{A}) \geq F(\mathcal{C}, \widehat{\mathcal{A}})$ for any symplectic (stabilizer) code $\mathcal{C}$ owing to the next lemma.

Reduction to Classical Coding Problem

Recall a self-orthogonal L and a set J of coset representatives of $L^{\perp}$ gives symplectic codes $\mathcal{C}_L^{(i)}$, $i = 0, \ldots, S-1$.

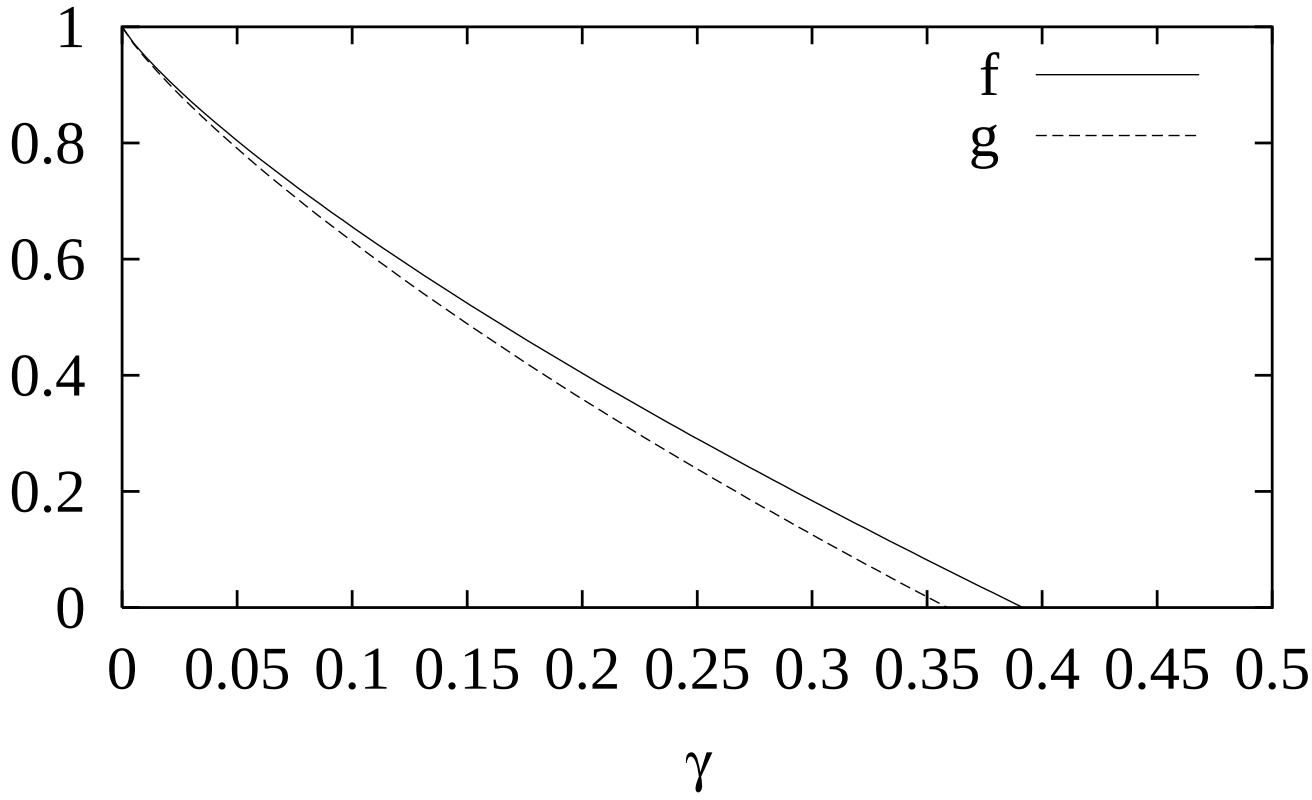
Lemma (MH, *IEEE IT*, '02, quant-ph/0112103; based on Preskill, '98). For any such L, any such choice of J, and any memoryless channel $\mathcal{A}$, we have

$$\frac{1}{S}\sum_{i=0}^{S-1}[1 - F_{\mathrm{a}}(\mathcal{C}_L^{(i)})] \leq \sum_{x \notin J} \widehat{P}_{\mathcal{A}}^n(x),$$

where $\widehat{P}_{\mathcal{A}}^n(x) = \widehat{P}_{\mathcal{A}}\big((u_1, v_1)\big) \cdots \widehat{P}_{\mathcal{A}}\big((u_n, v_n)\big)$ for $x = (u_1, v_1, \ldots, u_n, v_n) \in \mathsf{F}^{2n}$.

Remark. To prove the lower bound for general QDMCs, I used random coding methods twice: $\mathsf{E} = \{\text{self-orthogonal } L \subseteq \mathsf{F}^{2n}\}$, and $\{\mathcal{C}_L^{(0)}, \ldots, \mathcal{C}_L^{(S-1)} \subseteq \mathsf{H}^{\otimes n}\}$.

Next gives example of b_1 while bound $\sup_n b_n/n$ is hard to evaluate



Lower bounds on the capacity
of the amplitude-damping channel

f: MH, *IEEE IT*, 2002
g: Bennett *et al.*, PRA, 1996

$$\sim \left\{ \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix} \right\}.$$

Remark 5. Error Exponent

Let $F^{\star}_{m,\kappa}(\mathcal{A}^{\otimes m})$ denote the highest fidelity of quantum $[[m,\kappa]]$ codes used on a QDMC $\mathcal{A}$. MH (quant-ph/0207113) has actually shown

$$1 - F^{\star}_{m,Rm}(\mathcal{A}^{\otimes m}) \leq \exp_d[-m \sup_{L:\ \text{self-orthogonal}} G(R, \widehat{P}_{\mathcal{A}}, L) + o(m)],$$

i.e., that error exponent $\sup_{L:\ \text{self-orthogonal}} G(R, \widehat{P}_{\mathcal{A}}, L)$ is attainable.

Remark. "Error exponent E is attainable" means
$1 - F^{\star}_{m,Rm}(\mathcal{A}^{\otimes m}) \leq \exp_d[-mE + o(m)]$.

• Research problem: Determine the largest attainable error exponent (reliability function).

The lower bound in the theorem follows from

$$R < \frac{k - H_{\text{cond}}(P_L)}{n} \Longrightarrow G(R, P, L) > 0.$$

Simple Attainable Error Exponent

When L is the $[[1,1]]$ code, $G(R,P,L)$ becomes

$$E(R,P) = \min_Q \{D(Q||P) + \max\{1 - H(Q) - R, 0\}\},$$

$D(P||Q) = \sum_{x\in\mathcal{X}} P(x)\log_d \frac{P(x)}{Q(x)}$, $H(Q) = -\sum_{x\in\mathcal{X}} Q(x)\log_d Q(x)$,
the minimization is over all probability distributions Q on
$\mathcal{X} = \{0,1,\ldots,d-1\}^2$.

Thus $E(R,\widehat{P}_{\mathcal{A}})$ is an attainable exponent for $\mathcal{A}$
(MH, *IEEE IT*, 2002, quant-ph/0112103).

The lower bound $1 - H(\widehat{P}_{\mathcal{A}})$ on the capacity follows from

$$R < 1 - H(P) \Longrightarrow E(R,P) > 0.$$

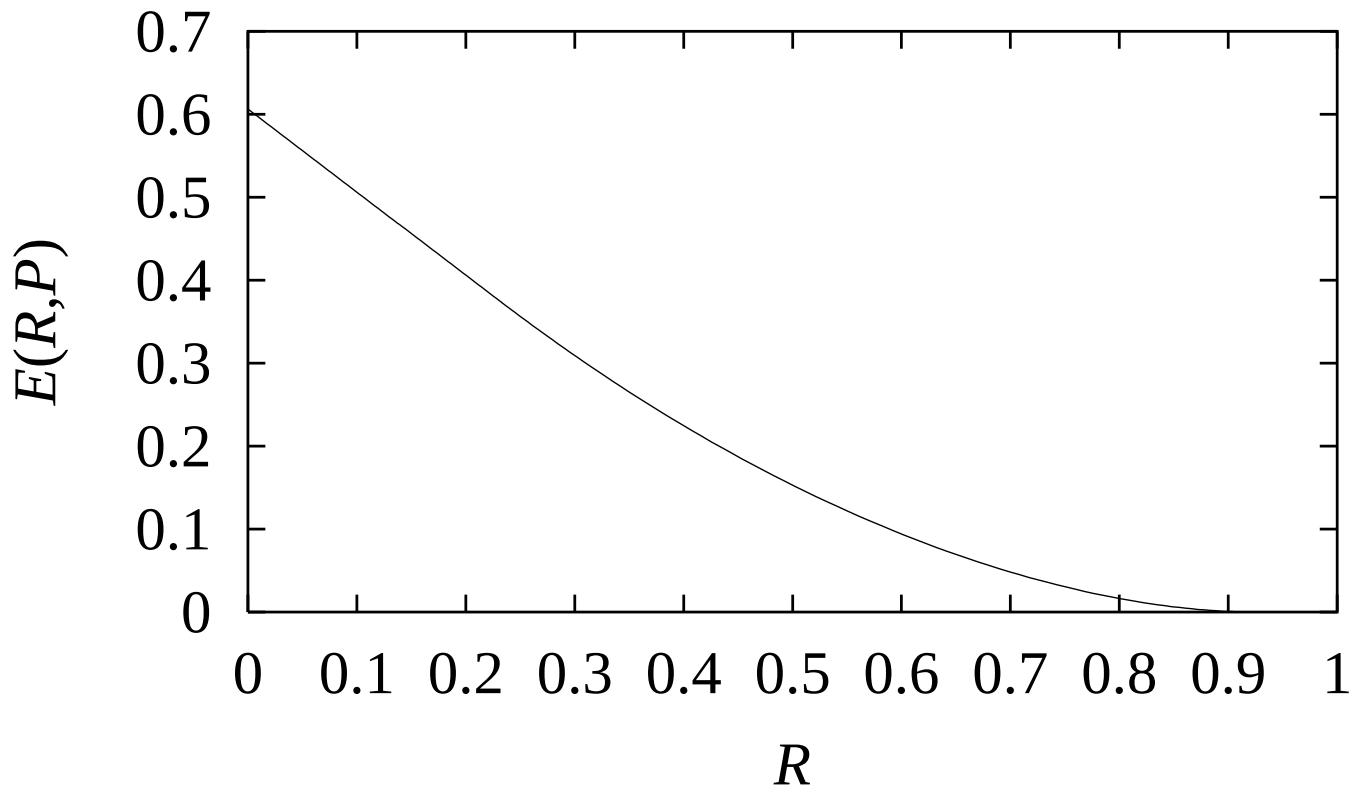
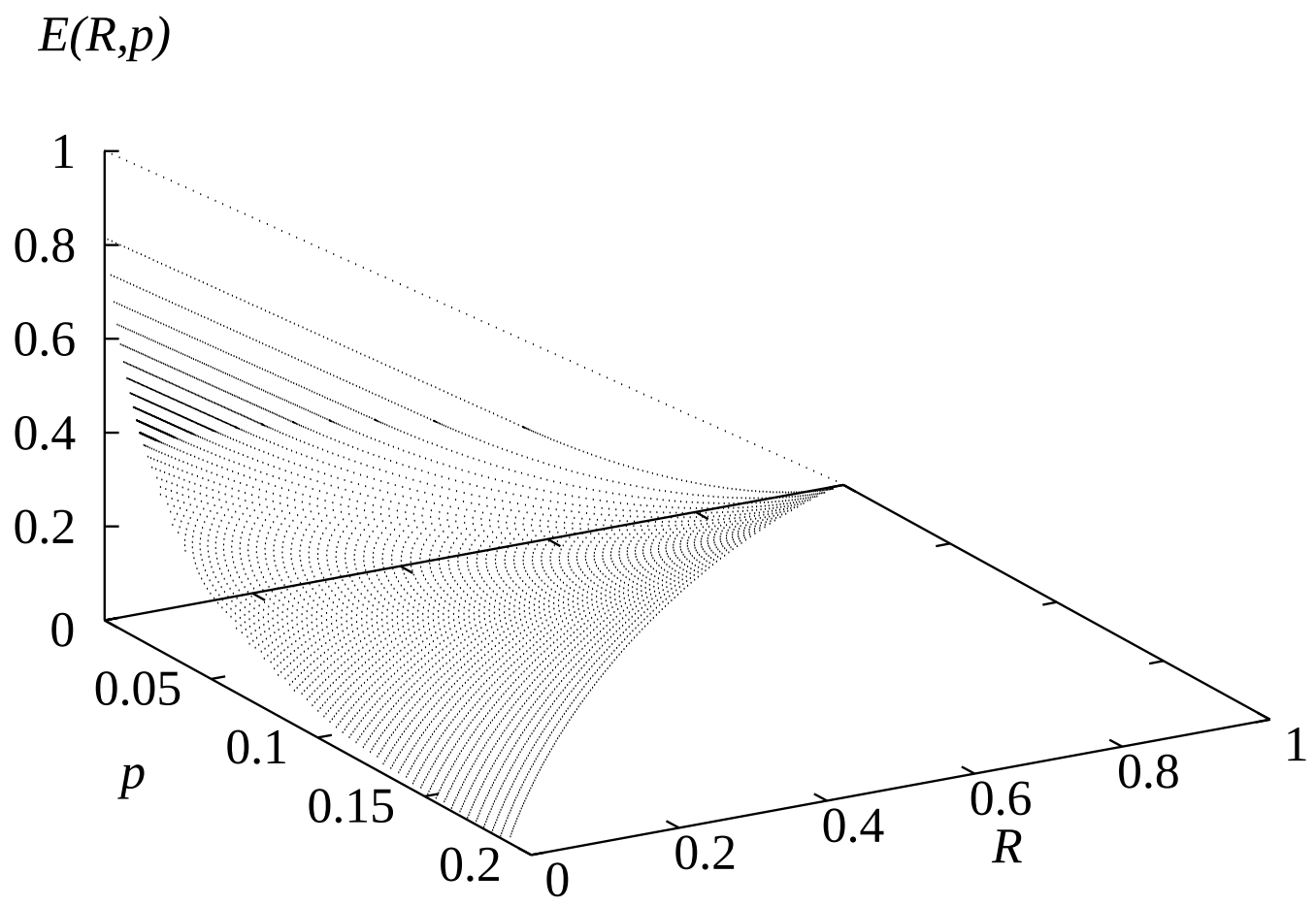


Figure 1: The function $E(R, P)$ in the case where $d = 2$ and $P((0, 0)) = 1 - p$, $P(u) = p/3$ for $u \neq (0, 0)$, $u \in \mathcal{X} = \{0, 1\}^2$, with $p = 0.0075$, which applies to the depolarizing channel.


E(R,p)
1
0.8
0.6
0.4
0.2
0
0.05
0.1
0.15
0.2
p
0
0.2
0.4
0.6
0.8
1
R

Conclusion

This talk presented a lower bound on the quantum capacity which can be achieved with symplectic codes and has a close relation to the known upper bound written with coherent information.

Talk was mainly based on quant-ph/0207113