Indistinguishablity and compressibility of quantum states

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Introduction

Distinguishability of states

Compressibility of source with distinguishable letters

Optimal compression in blind scenarios

Operations that preserve partially known quantum states

Variable length, faithful compression

Fixed length, asymptotically faithful compression

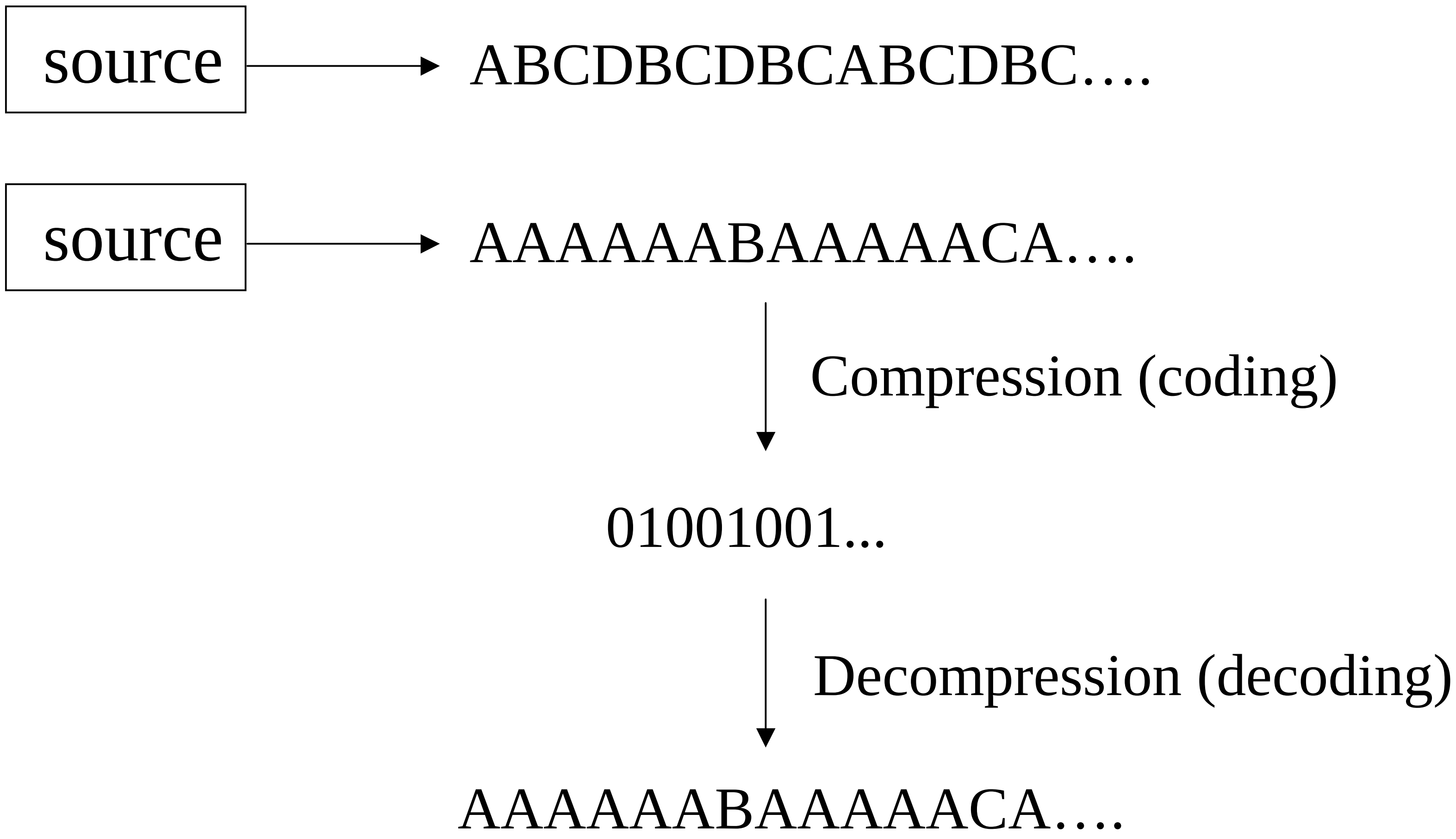
Upper bound for the information defect.

Distinguishablity and compression rates

In collaboration with Nobuyuki Imoto

A fundamental question in the information theory

Quantifying the randomness of a probabilistic source

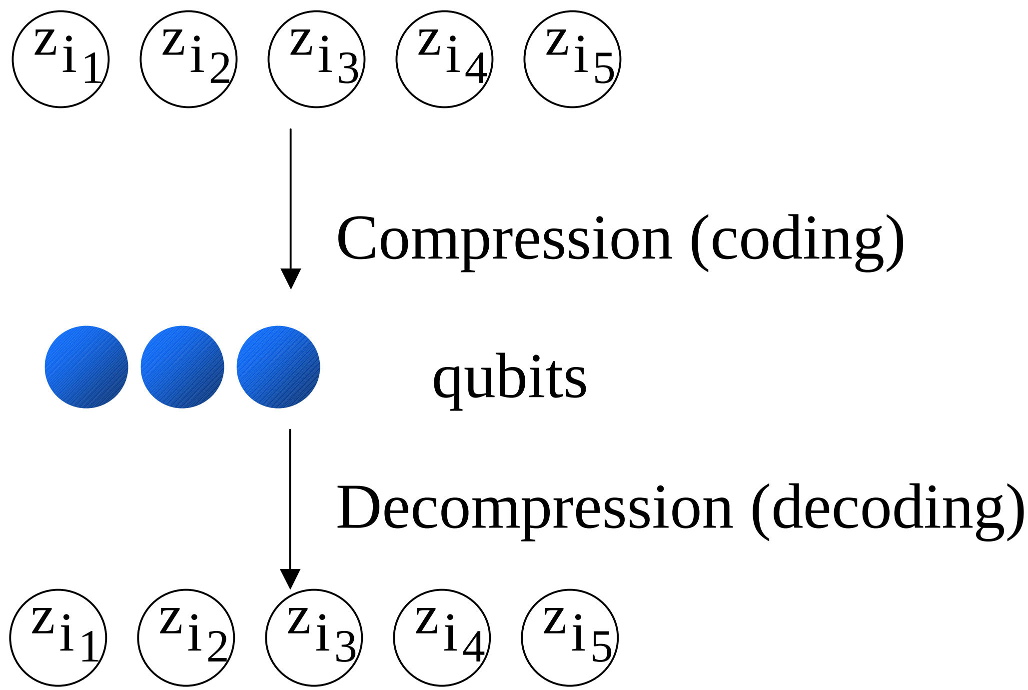


How many bits (per letter) are required to describe the sequence?

A fundamental question in quantum information theory

How many qubits (per system) are required to reproduce the state?

source $f\, p_i;\, ž_i\, g$



Source

- Quantum / Classical
- Pure / Mixed

Scenario

- Exact / Almost exact
- Side information?

How the optimal rates differ depending on scenarios?

How well can we manipulate various types of information?

source $\{p_i; i\}$ → ABCDBCDBCABCDBC….

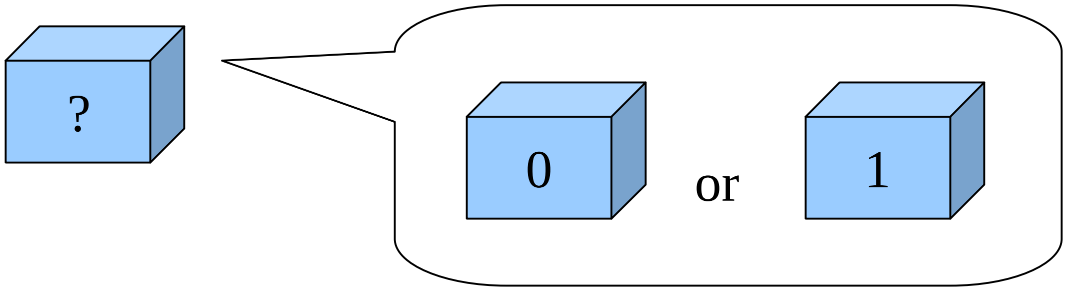
Letters are distinguishable from each other.

source $\{p_i; \check{z}_i\}$ → z_{i_1} z_{i_2} z_{i_3} z_{i_4} z_{i_5}

Letter states are <u>indistinguishable</u>.

(not completely distinguishable)

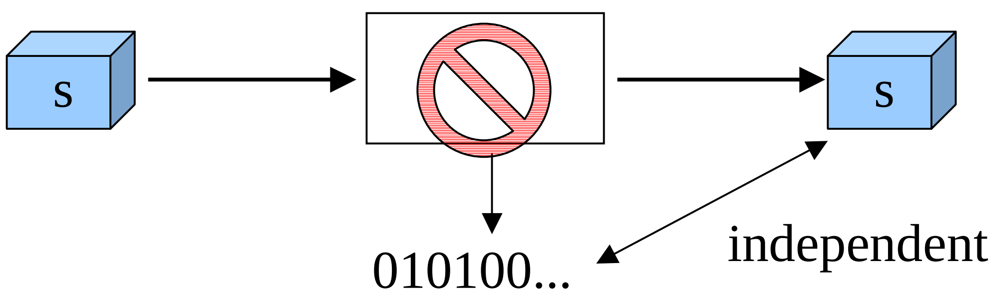
Distinguishability of the states of a physical system



The two states are indistinguishable.

There is no way to learn the identity of the state from a single sample for sure, in principle.

The following task is forbidden.

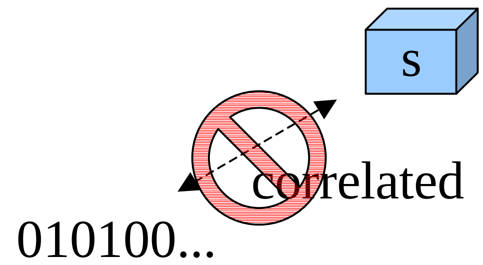


The outcome depends on s.

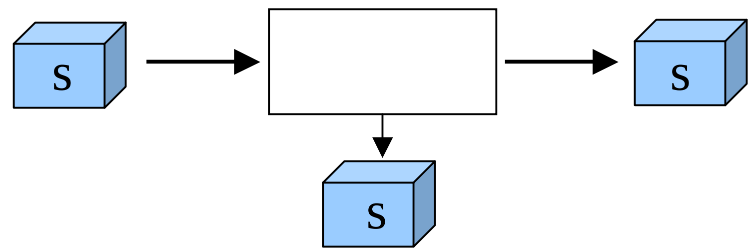
Pure source and classical source

source $\{p_s; \lambda_s\}$

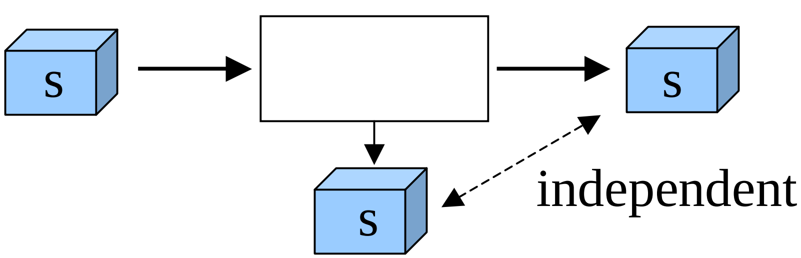
Pure: Each letter state is pure.



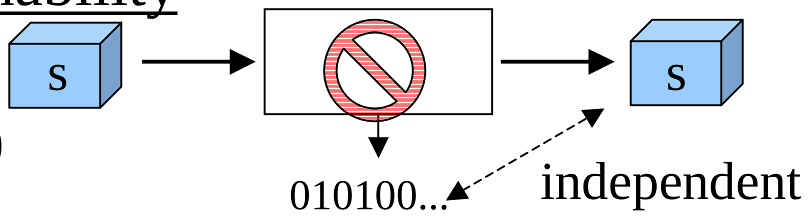
Classical: Letter states can be copied.



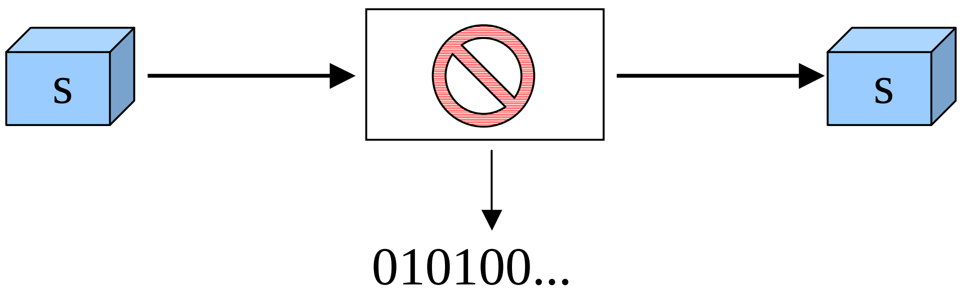
Pure & Classical → distinguishable



Restriction arising from indistinguishability

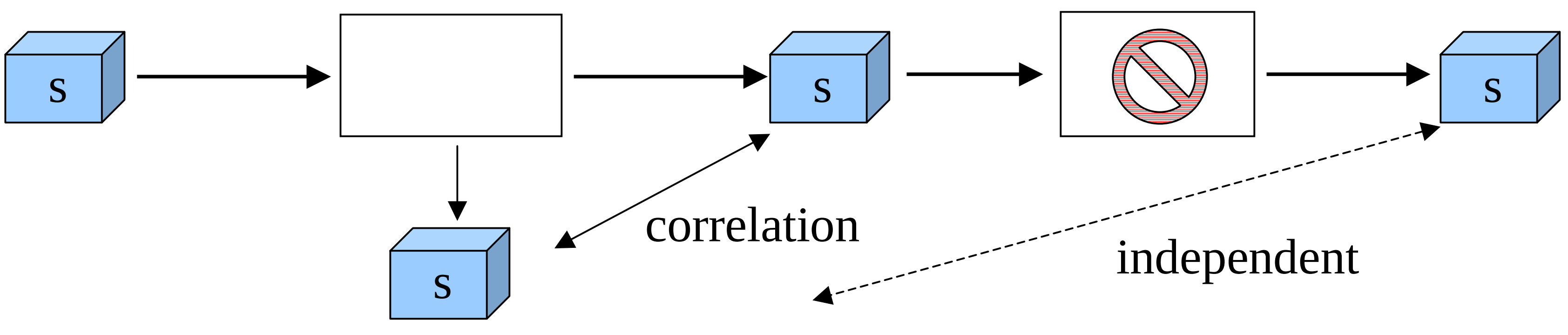


Pure & indistinguishable (pure & quantum)



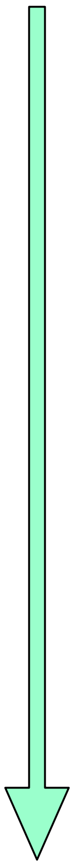
The outcome depends on s.

Classical & indistinguishable (classical & mixed)



Different scenarios of compression

Hard

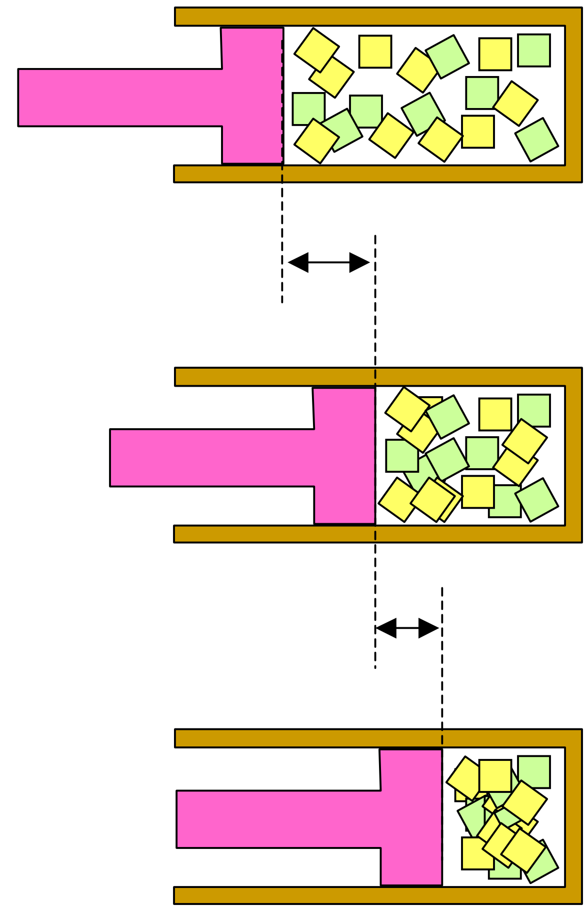


Easy

Variable-length
Faithful — Blind — $R_{opt}^{(1)}$

Fixed-length
Asymptotically Faithful — Blind — R_{opt}^{AF}

Fixed-length
Asymptotically Faithful — Visible — I_{eA}

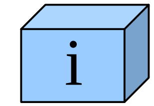


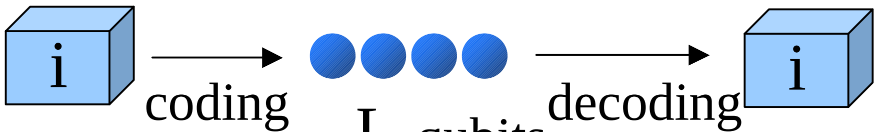
Variable-length and faithful scenario

Characterization of a source

$\{p_i; \psi_i\}$

probability — letter state





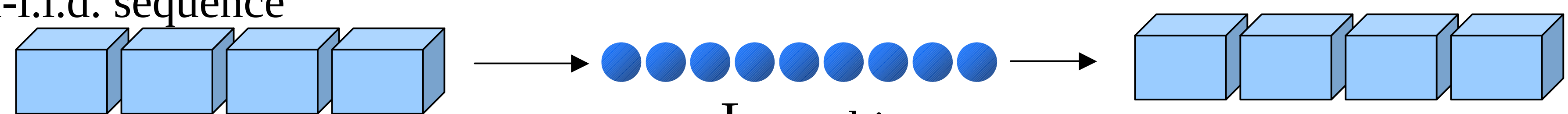
L qubits

(length can be varied)

with success probability (fidelity) 1.

$R^{(1)}_{opt}$: Minimum of expected length $\langle L \rangle$

n-i.i.d. sequence



block coding

L_n qubits

$R^{(n)}_{opt}$: Minimum of expected length per letter, $\langle L_n \rangle / n$

By definition, $R^{(1)}_{opt} \quad R^{(2)}_{opt} \quad \cdots \quad R^{(n)}_{opt} \quad \cdots$

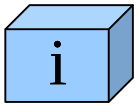
optimal compression rate $R^{(\infty)}_{opt} \equiv \lim_{n \to \infty} R^{(n)}_{opt}$

Fixed-length and asymptotically faithful scenario

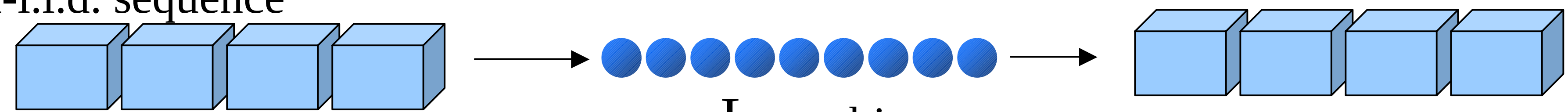
Characterization of a source

$\{ p_i ; \psi_i \}$

probability — letter state



n-i.i.d. sequence



block coding

L_n qubits

(fixed length)

with success probability (fidelity) $1 - \epsilon_n$

Requirement: "Asymptotically faithful" $\lim_{n \to \infty} \epsilon_n = 0$

(vanishing errors in large block length limit)

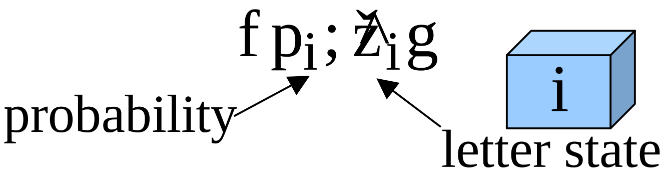
Compression rate $\equiv \lim_{n \to \infty} L_n / n$ (How many qubits are used per letter?)

Minimum of this rate: optimal compression rate R_{opt}^{AF}

Blind and visible scenarios

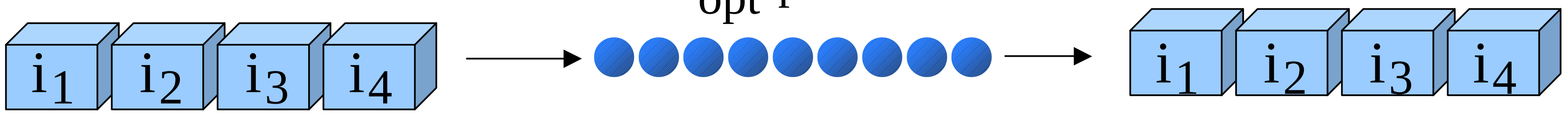
Characterization of a source

$\{p_i; \psi_i\}$



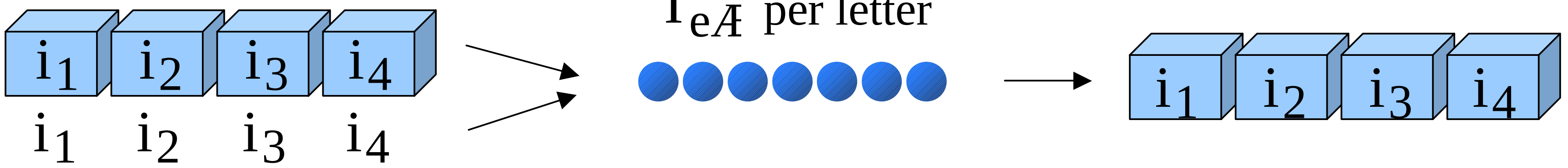
blind scenario

R^{AF}_{opt} per letter

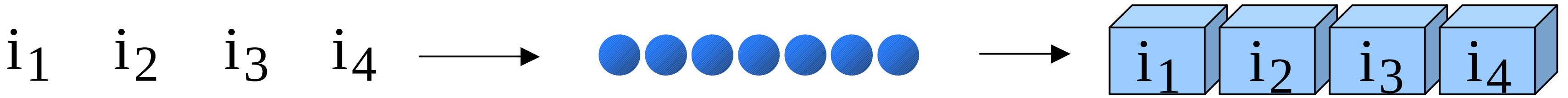


visible scenario

I_{eff} per letter



This is equivalent to directly preparing the state of the qubits.



Different scenarios of compression

Hard

Variable-length
Faithful

Blind

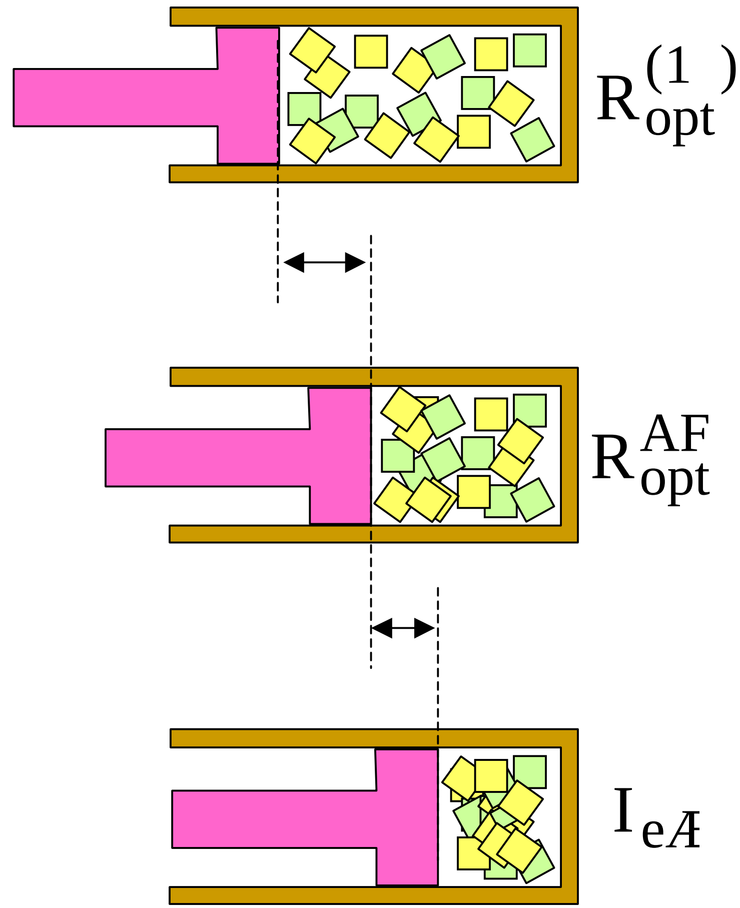
Fixed-length
Asymptotically Faithful

Blind

Fixed-length
Asymptotically Faithful

Visible

Easy



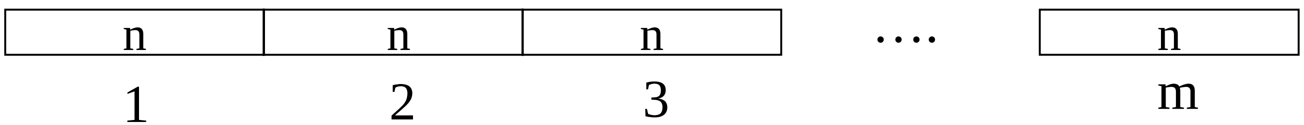
Variable-length, faithful coding $R_{opt}^{(1)}$

more powerful more strict

v.s.

Fixed-length, asymptotically faithful coding R_{opt}^{AF}

Repeating n-block variable-length coding for m times



Expected length per letter is $R_{opt}^{(n)}$

Law of large numbers

When m becomes large, length per letter is almost always close to $R_{opt}^{(n)}$

↓

Fixed-length, asymptotically faithful coding with rate $R_{opt}^{(n)}$

$R_{opt}^{(n)}$ R_{opt}^{AF}

$R_{opt}^{(1)}$ R_{opt}^{AF}

Distinguishable case (Variable-length, faithful coding)

source $\{p_i; i\}$

$i \to C(i)$ length l_i

$$l_i = \left\lceil \log_2 \left(\frac{1}{p_i} \right) \right\rceil$$

Shorter code words for more frequent letters

Kraft inequality

$$\sum_i 2^{-l_i} \le \sum_i 2^{\log p_i} = \sum_i p_i = 1$$

↓

instantaneous code

$$-\log p_i \le l_i < -\log p_i + 1$$

$$R_{opt}^{(1)} < H(\{p_i\}) + 1$$

Applying this to the n-i.i.d. case,

$$L_n < nH(\{p_i\}) + 1$$

$$R_{opt}^{(n)} < H(\{p_i\}) + 1/n$$

$$R_{opt}^{(\infty)} = H(\{p_i\})$$

Shannon entropy

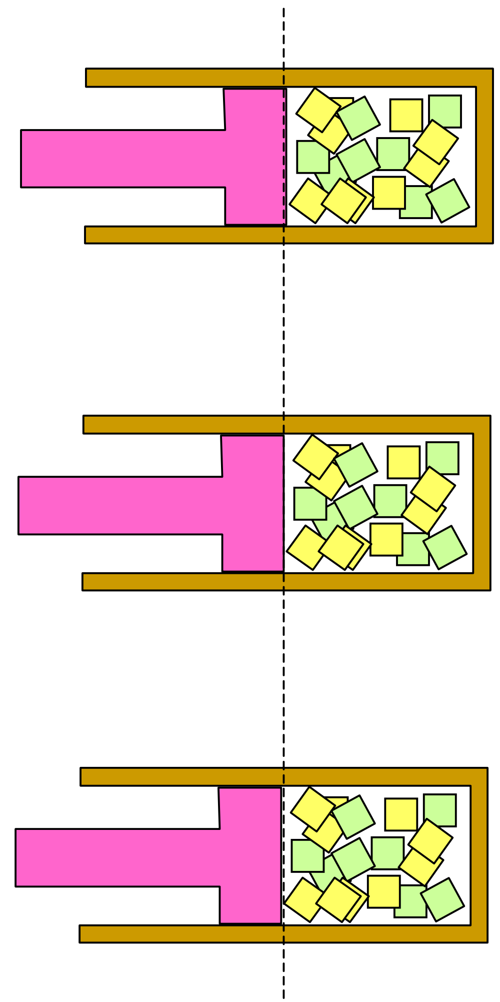
Different scenarios of compression

Distinguishable case f p_i; i g

Hard

Easy

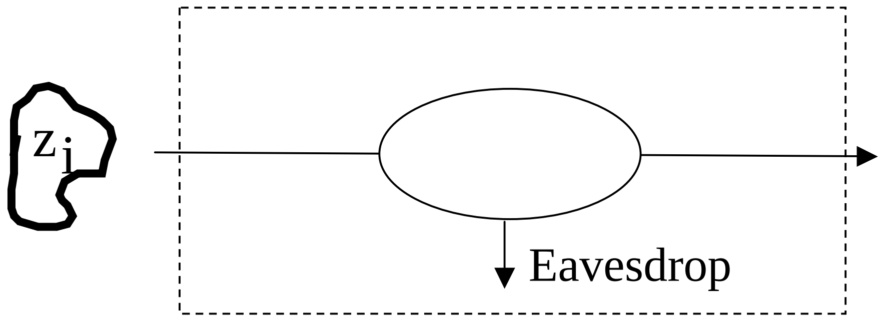
Variable-length Faithful	Blind	$R_{opt}^{(1\ )}$
		=
Fixed-length Asymptotically Faithful	Blind	R_{opt}^{AF}
		=
Fixed-length Asymptotically Faithful	Visible	I_{eA}



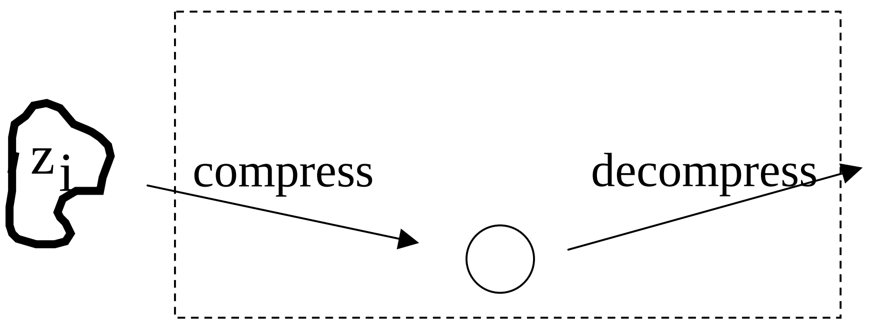
Shannon entropy

The operations that do not disturb f ž$_i$g

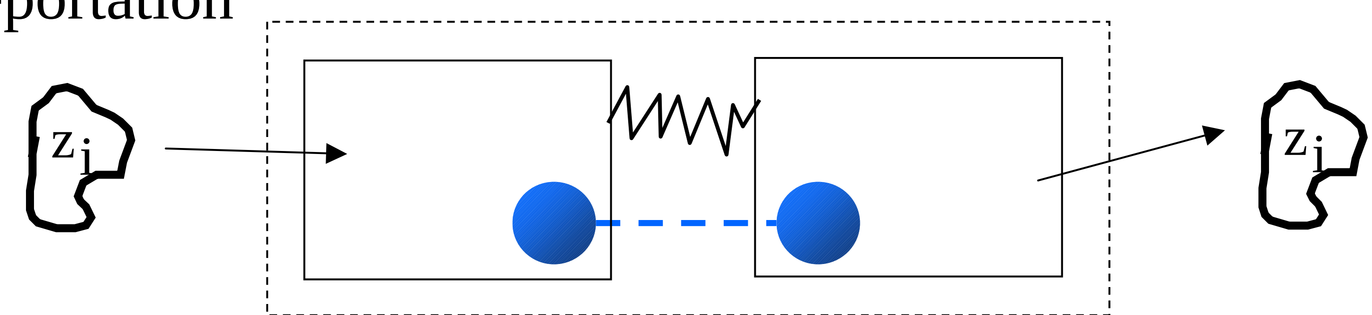
Quantum cryptography



Compression

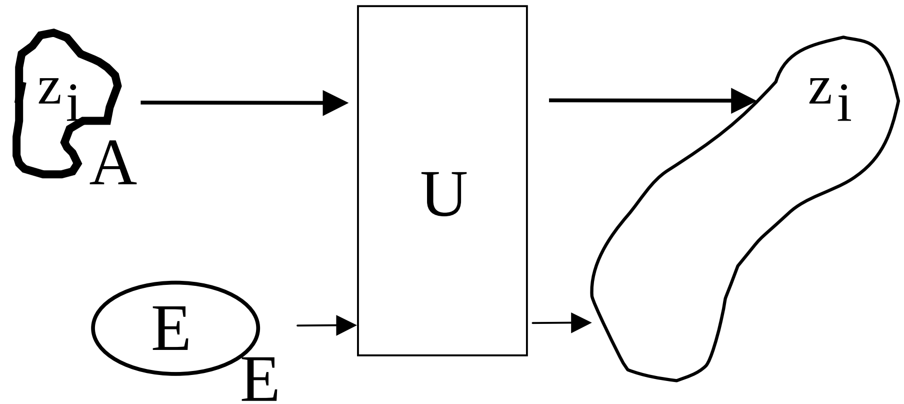


Teleportation



Problem

Given initial states f$ž_i$g , find the condition for the allowed operations.



$$Tr_E[U(ž_i \circ ß) U^y] = ž_i \quad 8i$$

A principle that states what one can do, and what one cannot do, without disturbing the given marginal density operators.

Principle

Koashi & Imoto,
Phys. Rev. Lett. **81**, 4264 (1998)
Phys.Rev. A **66**, 022318 (2002)

$$z_i = \begin{pmatrix} q^{(i;1)}\text{£}\ \check{z}^{(i;1)} & & & & 0 \\ & q^{(i;2)}\text{£}\ \check{z}^{(i;2)} & & & \\ & & q^{(i;3)}\text{£}\ \text{Æ}\check{z}^{(i;3)} & & \\ 0 & & & q^{(i;3)}\text{£}\ \text{Ø}\check{z}^{(i;3)} & \end{pmatrix}$$

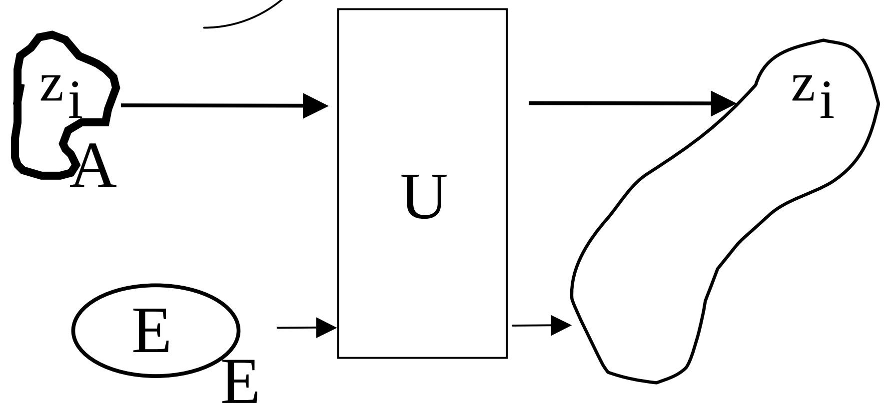
$$\check{z}_J^{(i;3)} = \check{z}^{(i;3)}$$

$$\check{z}_K^{(3)} = \begin{pmatrix} \text{Æ} & 0 \\ 0 & \text{Ø} \end{pmatrix}$$

$$H = \overset{\text{M}}{\underset{l}{}}\ H_J^{(l)} \circ H_K^{(l)}$$

$$\check{z}_i = \overset{\text{M}}{\underset{l}{}}\ q^{(i;l)} \check{z}_J^{(i;l)} \circ \check{z}_K^{(l)}$$

$$\hat{U} = \overset{\text{M}}{\underset{l}{}}\ \hat{1}_J^{(l)} \circ \hat{U}_{KE}^{(l)}$$



Examples

$$H = \bigoplus_l H_J^{(l)} \otimes H_K^{(l)} \qquad \check{z}_i = \bigoplus_l q^{(i;l)} \check{z}_J^{(i;l)} \otimes \check{z}_K^{(l)}$$

$$\check{z}_0 = \frac{1}{12}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix} \qquad \check{z}_0 = 1 \quad \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \otimes \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\check{z}_1 = \frac{1}{15}\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix} \qquad \check{z}_1 = 1 \quad \frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \otimes \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\check{z}_0 = \frac{1}{16}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix} \qquad \check{z}_0 = \frac{1}{4}\; \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \otimes (1) \;\oplus\; \frac{3}{4}\; \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \otimes (1)$$

$$\check{z}_1 = \frac{1}{15}\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix} \qquad \check{z}_1 = \frac{1}{3}\; \frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \otimes (1) \;\oplus\; \frac{2}{3}\; \frac{1}{5}\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \otimes (1)$$

<u>Examples</u>

$$H = \bigoplus_l H_J^{(l)} \otimes H_K^{(l)} \qquad \hat{\rho}_i = \bigoplus_l q^{(i;l)} \hat{\rho}_J^{(i;l)} \otimes \hat{\rho}_K^{(l)}$$

$$\hat{\rho}_0 = \frac{1}{10}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \qquad \hat{\rho}_0 = \frac{3}{10}(1) \otimes \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \oplus \frac{3}{10}(1) \otimes (1) \oplus \frac{4}{10}(1) \otimes (1)$$

$$\hat{\rho}_1 = \frac{1}{10}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \hat{\rho}_1 = \frac{6}{10}(1) \otimes \frac{1}{3}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \oplus \frac{3}{10}(1) \otimes (1) \oplus \frac{1}{10}(1) \otimes (1)$$

$$\hat{\rho}_0 = \frac{1}{10}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \qquad \hat{\rho}_0 = \frac{1}{10}(1) \otimes (1) \oplus \frac{2}{10}(1) \otimes (1) \oplus \frac{3}{10}(1) \otimes (1) \oplus \frac{4}{10}(1) \otimes (1)$$

$$\hat{\rho}_1 = \frac{1}{10}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \hat{\rho}_1 = \frac{4}{10}(1) \otimes (1) \oplus \frac{3}{10}(1) \otimes (1) \oplus \frac{2}{10}(1) \otimes (1) \oplus \frac{1}{10}(1) \otimes (1)$$

$$\hat{z}_i = \sum_{l} q^{(i;l)} \hat{z}_J^{(i;l)} \circ \hat{z}_K^{(l)}$$

Color: l

We know the contents of the box K.

$$\hat{U} = \sum_{l} \hat{1}_J^{(l)} \circ \hat{U}_{KE}^{(l)}$$

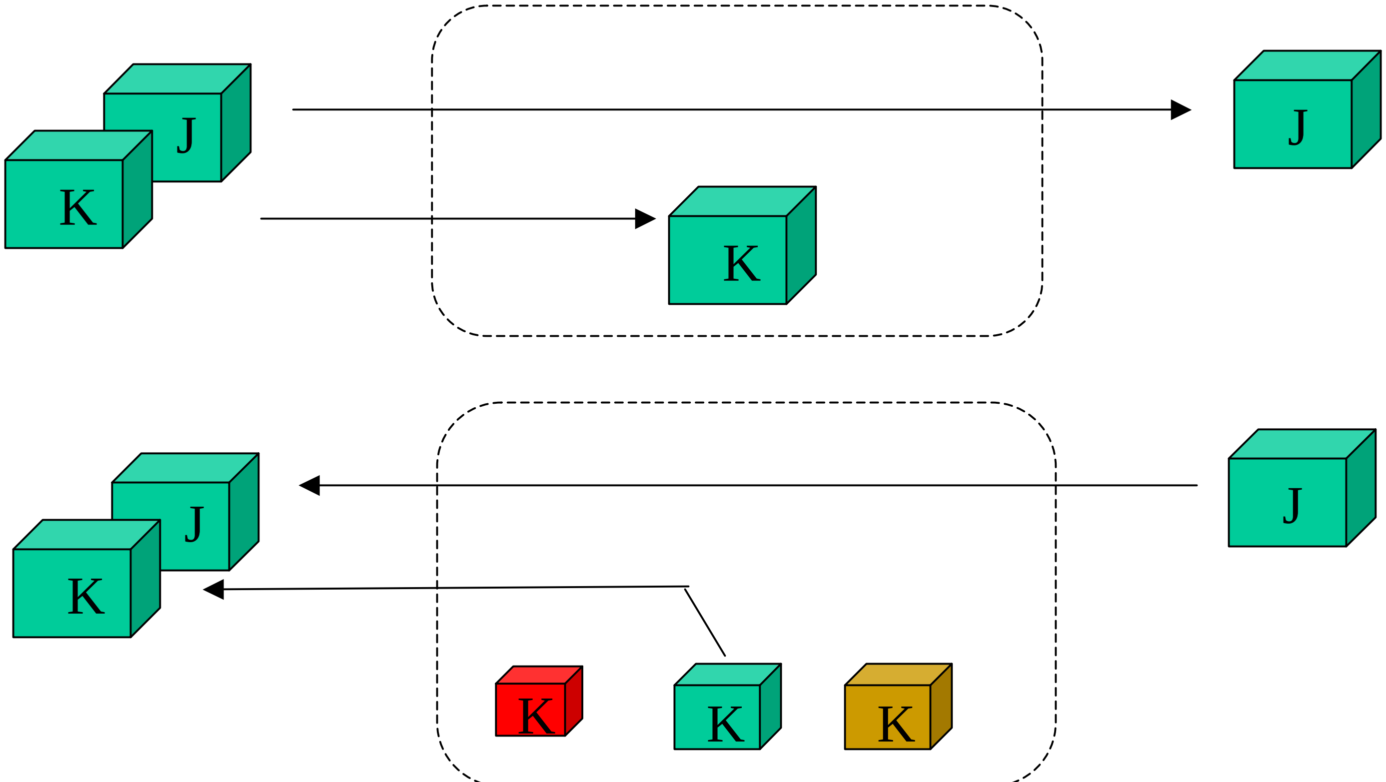
We can see colors, but cannot change colors.

We cannot open the box J.

Reduced ensemble

Original source $\{p_i; \hat{\rho}_i\}$ Reduced source $\{p_i; \hat{\sigma}_i\}$

$$\hat{\rho}_i = \bigoplus_l q^{(i;l)} \hat{\rho}_J^{(i;l)} \otimes \hat{\rho}_K^{(l)} \quad \overset{\text{freely}}{\Longleftrightarrow} \quad \hat{\sigma}_i = \bigoplus_l q^{(i;l)} \hat{\rho}_J^{(i;l)}$$



We know the contents of the box K.

We can see colors.

Original source $\mathrm{f}\ p_i ; \hat{ž}_i\ \mathrm{g}$ Reduced source $\mathrm{f}\ p_i ; \hat{æ}_i\ \mathrm{g}$

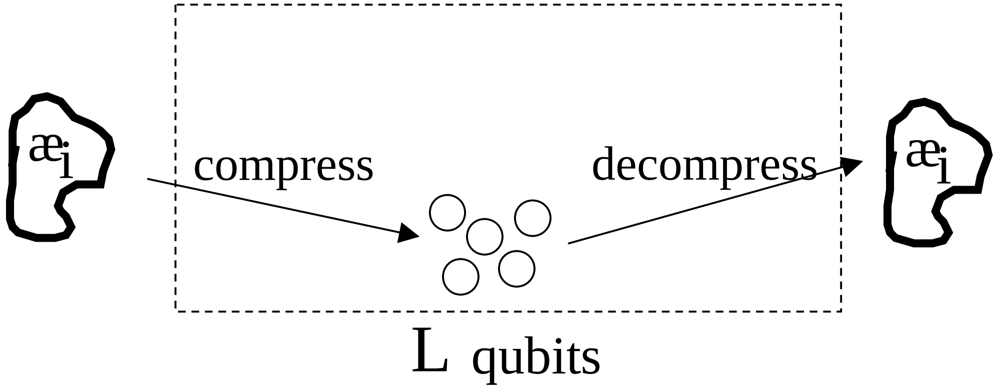
$$\hat{ž}_i = \overset{\mathrm{M}}{\underset{l}{}}\ q^{(i;l)} \hat{ž}_J^{(i;l)} \circ \hat{ž}_K^{(l)} \quad \overset{\text{freely}}{\Longleftrightarrow} \quad \hat{æ}_i = \overset{\mathrm{M}}{\underset{l}{}}\ q^{(i;l)} \hat{ž}_J^{(i;l)}$$

Compressibility of $\mathrm{f}\ p_i ; \hat{ž}_i\ \mathrm{g}$ is equal to that of $\mathrm{f}\ p_i ; \hat{æ}_i\ \mathrm{g}$

Variable-length and faithful scenario

Koashi and Imoto,
Phys. Rev. Lett. **89**, 097904 (2002).

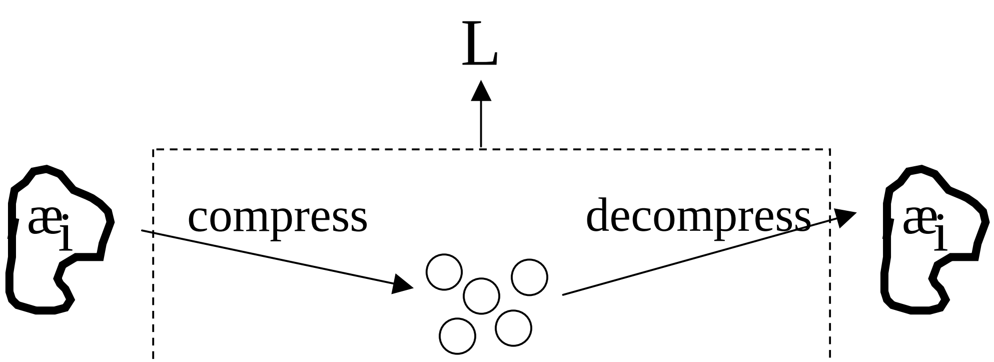
source $E = \mathrm{f}\; p_i ; \hat{ž}_i\, \mathrm{g}$ Reduced source $\mathrm{f}\; p_i ; æ_i\, \mathrm{g}$



What do we mean by saying, “only L qubits are used this time.” ?

“OK, the rest of N-L qubits can be used in any other independent <u>task</u>.”

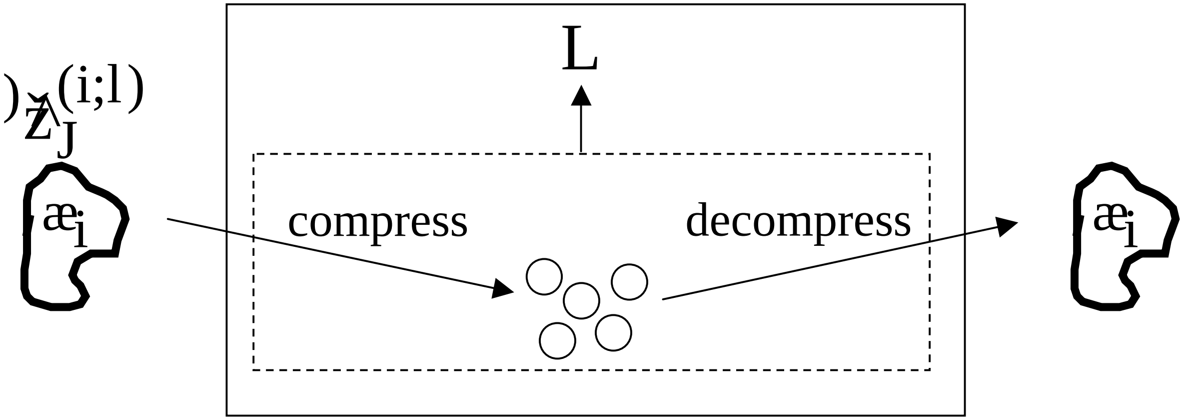
Information of L comes out of the compression machine.



Faithful compression machines preserve $\{\hat{\rho}_i\}$

$\{p_i;\hat{\rho}_i\}$

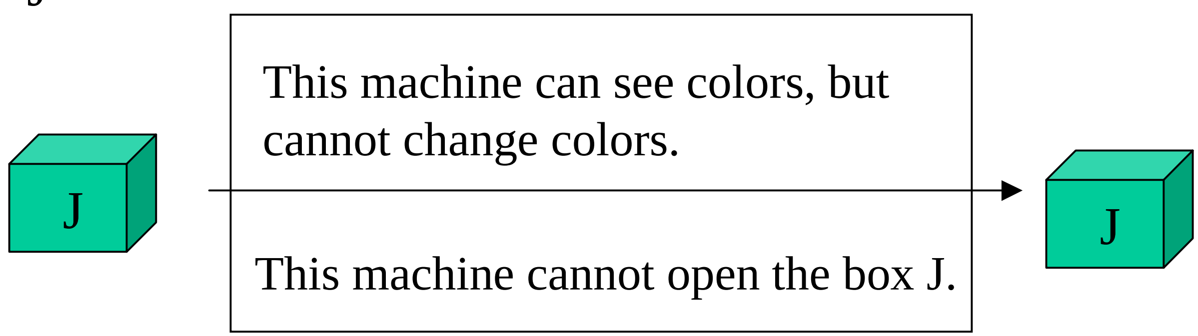
$$\hat{\rho}_i = \bigoplus_l q^{(i;l)} \hat{\check{z}}_J^{(i;l)}$$



$$\hat{U} = \bigoplus_l \hat{1}_J^{(l)} \circ \hat{U}_E^{(l)}$$

Averaged state

$$\hat{\rho} = \bigoplus_l p^{(l)} \hat{\check{z}}_J^{(l)}$$



The expected length $\langle L \rangle$ depends only on the statistics of colors, $p^{(l)}$

A new source with distinguishable letters

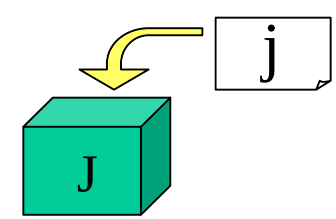
$$\{p(l;j);|l;j\rangle\} \qquad \{|l;j\rangle\}_{j=1,2,\ldots,\dim H_J^{(l)}} \text{ : A basis of } H_J^{(l)}$$

$$p(l;1)=p(l;2)=\cdots=p(l;\dim H_J^{(l)})$$

$$p(l;1)+p(l;2)+\cdots+p(l;\dim H_J^{(l)})=p^{(l)}$$

Pick up the color l with probability $p^{(l)}$

Choose j randomly.



This source should be faithfully compressed with the same rate as the original source $E=\{p_i;\hat{\rho}_i\}$

The rate should never be smaller than $H(\{p(l;j)\})$

$$R^{(1)}_{opt}(E) \geq H(\{p(l;j)\}) = I_C(E) + D_{NC}(E)$$

$$I_C(E) \geq H(\{p^{(l)}\})$$

$$D_{NC}(E) \geq \sum_l p^{(l)} \log_2 \dim H_J^{(l)}$$

A particular compression scheme

$$\rho_i = \sum_\lambda p^{(i;\lambda)} \rho_J^{(i;\lambda)} \otimes \rho_K^{(\lambda)} \qquad \rho = \sum_\lambda p^{(\lambda)} \rho_J^{(\lambda)} \otimes \rho_K^{(\lambda)};$$

Measurement of λ

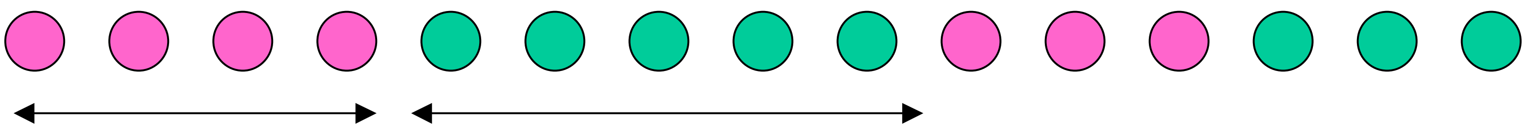
The outcome follows probability $p^{(\lambda)}$

Classical instanteneous code for $\{p^{(\lambda)}; \lambda\}$

Postmeasurement state

$\rho_J^{(i;\lambda)} \otimes \rho_K^{(\lambda)}$

discard



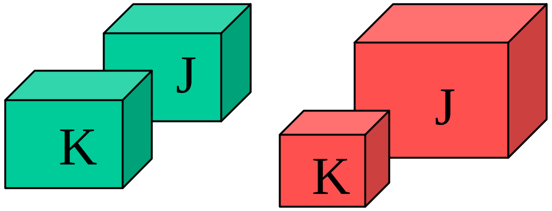
$-\log p^{(\lambda)} + 1$ $\qquad$ $\log \dim H_J^{(\lambda)} + 1$

average $\quad H(\{p^{(\lambda)}\}) + \sum_\lambda p^{(\lambda)} \log_2 \dim H_J^{(\lambda)} + 2$

$= I_C(E) + D_{NC}(E) + 2$

Variable-length and faithful scenario

source $E = \{p_i, \hat{\rho}_i\}$



$$H_A = \bigoplus_l H_J^{(l)} \otimes H_K^{(l)}$$

$$\hat{\rho}_i = \bigoplus_l p^{(i;l)} \hat{\rho}_J^{(i;l)} \otimes \hat{\rho}_K^{(l)}$$

$$\hat{\rho} = \bigoplus_l p^{(l)} \hat{\rho}_J^{(l)} \otimes \hat{\rho}_K^{(l)};$$

$$I_C(E) + D_{NC}(E) \quad R_{opt}^{(1)}(E) \quad I_C(E) + D_{NC}(E) + 2$$

Applying this to the n-i.i.d. case,

$$I_C(E) + D_{NC}(E) \quad R_{opt}^{(n)}(E) \quad I_C(E) + D_{NC}(E) + 2/n$$

$$R_{opt}^{(1\ )}(E) = I_C(E) + D_{NC}(E)$$

$$I_C(E) \approx H(\{p^{(l)}\})$$ Entropy of the color.

$$D_{NC}(E) \approx \sum_l p^{(l)} \log_2 \dim H_J^{(l)}$$ Averaged size of the boxes

Fixed-length and asymptotically faithful scenario

Koashi and Imoto,
Phys. Rev. Lett. **87**, 017902 (2001).

$$\hat{\rho}_i = \bigoplus_l q^{(i;l)} \hat{\rho}_J^{(i;l)} \otimes \hat{\rho}_K^{(l)} \quad \overset{\text{freely}}{\Longleftrightarrow} \quad \hat{\sigma}_i = \bigoplus_l q^{(i;l)} \hat{\rho}_J^{(i;l)}$$

Compressibility of $\{p_i; \hat{\rho}_i\}$ is equal to that of $\{p_i; \hat{\sigma}_i\}$

Averaged state

$$\hat{\sigma} = \bigoplus_l p^{(l)} \hat{\rho}_J^{(l)}$$

By Schumacher compression,
$\{p_i; \hat{\rho}_i\}$ can be compressed into

$$S(\hat{\sigma}) = H(\{p^{(l)}\}) + \sum_l p^{(l)} S(\hat{\rho}_J^{(l)})$$

$$= I_C + I_{NC}$$

(qubits per letter)

$$R_{opt}^{AF}(E) \quad I_C(E) + I_{NC}(E)$$

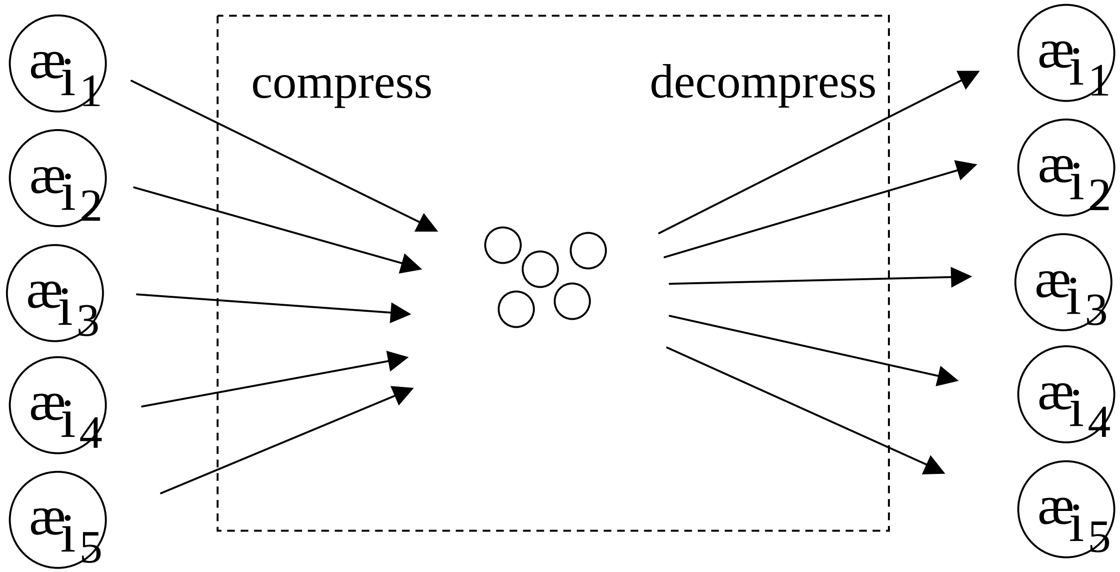
Asymptotically faithful compression machines preserve $f\ \hat{æ}_i\ g$

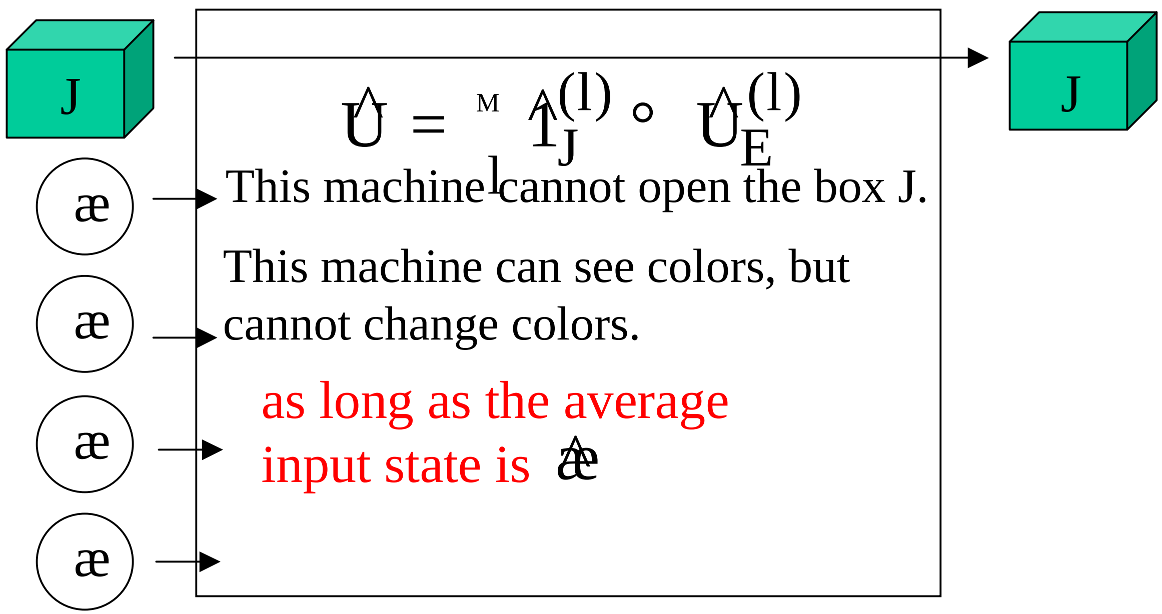
$f\ p_i ; \hat{æ}_i\ g$

$$\hat{æ}_i = \overset{M}{\underset{l}{}} q^{(i;l)} ž \hat{}_J^{(i;l)}$$

Averaged state

$$\hat{æ} = \overset{M}{\underset{l}{}} p^{(l)} ž \hat{}_J^{(l)}$$





$$\hat{U} = \overset{M}{\underset{l}{}} \hat{1}_J^{(l)} \circ \hat{U}_E^{(l)}$$

This machine cannot open the box J.

This machine can see colors, but cannot change colors.

as long as the average input state is $\hat{æ}$

A new source with distinguishable letters

$$\hat{\sigma} = \sum_{l}^{M} p^{(l)} \hat{\rho}_J^{(l)}$$

diagonalize

$$\hat{\rho}_J^{(l)} = \sum_{j} q_j^{(l)} |l;j\rangle\langle l;j|$$

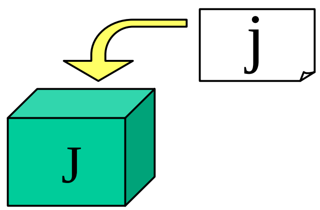
new source

$$\{p^{(l)} q_j^{(l)}; |l;j\rangle\}$$

average state is $\hat{\sigma}$

Pick up the color l with probability $p^{(l)}$

Choose j with probability $q_j^{(l)}$



The states $|l;j\rangle$ should appear at the output asymptotically faithfully.

The machine can transfer $H(\{p^{(l)} q_j^{(l)}\})$ bits per letter of classical information.

The rate should never be smaller than $H(\{p^{(l)} q_j^{(l)}\})$

$$E = \{p_i; \hat{\rho}_i\}$$

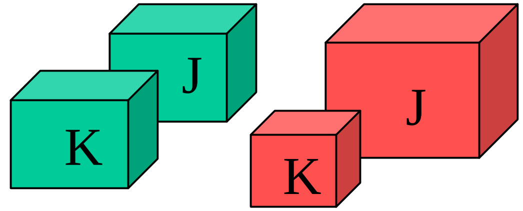
$$R_{opt}^{AF}(E) \qquad H(\{p^{(l)} q_j^{(l)}\}) = S(\hat{\sigma}) = I_C(E) + I_{NC}(E)$$

$$I_C(E) \geq H(\{p^{(l)}\})$$

$$I_{NC}(E) \geq \sum_{l} p^{(l)} S(\hat{\rho}_J^{(l)})$$

Gap between the two scenarios

source $E = \{p_i; \hat{\rho}_i\}$



$$H_A = \bigoplus_l H_J^{(l)} \circ H_K^{(l)}$$

$$\hat{\rho}_i = \bigoplus_l p^{(i;l)} \hat{\rho}_J^{(i;l)} \circ \hat{\rho}_K^{(l)}$$

$$\hat{\rho} = \bigoplus_l p^{(l)} \hat{\rho}_J^{(l)} \circ \hat{\rho}_K^{(l)};$$

$$R_{opt}^{AF}(E) = I_C(E) + I_{NC}(E)$$ Fixed-length, asymptotically faithful

$$R_{opt}^{(1\ )}(E) = I_C(E) + D_{NC}(E)$$ Variable-length, faithful

$$I_C(E) \equiv H(\{p^{(l)}\})$$ — Entropy of the color.

$$I_{NC}(E) \equiv \sum_l p^{(l)} S(\hat{\rho}_J^{(l)})$$ — Averaged entropy of the box contents

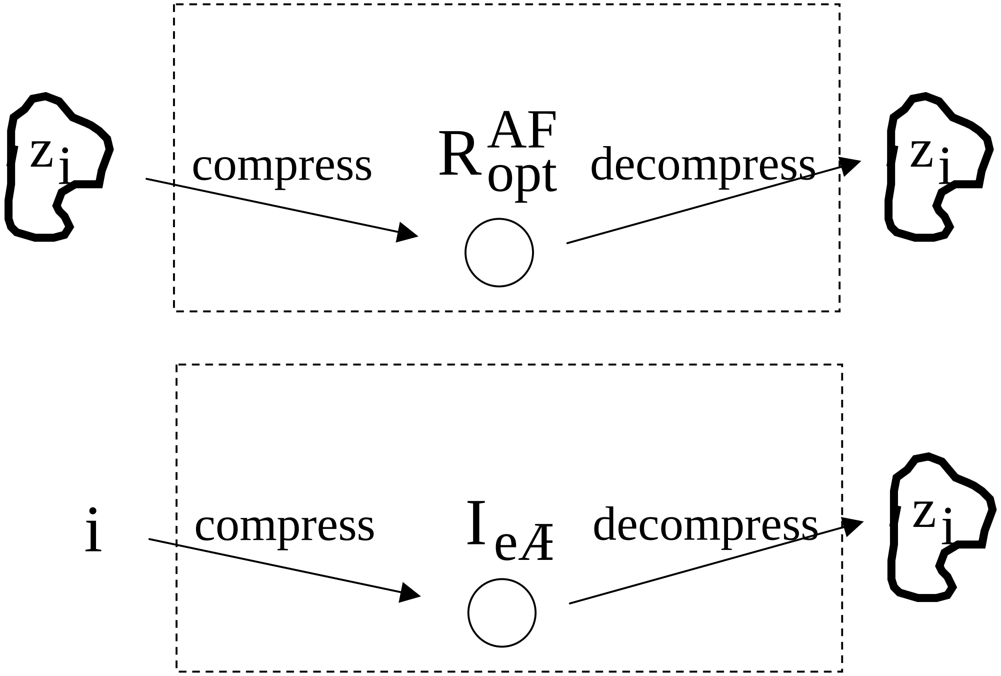
$$D_{NC}(E) \equiv \sum_l p^{(l)} \log_2 \dim H_J^{(l)}$$ — Averaged size of the boxes

$$R_{opt}^{(1\ )} \circ R_{opt}^{AF} = \sum_l p^{(l)} [\log_2 \dim H_J^{(l)} \circ S(\hat{\rho}_J^{(l)})]$$

$R_{opt}^{(1\ )} = R_{opt}^{AF}$ if all $\hat{\rho}_i$ commute.

This gap has the genuinely quantum origin.

Information defect



blind scenario

visible scenario

$$I_{e\text{Æ}} \quad I_{LH} \text{ ¥ } S(\hat{ž}) \circ \sum_i p_i S(\hat{ž}_i)$$

$$¢_{b\circ v} \text{ ¥ } R^{AF}_{opt} \circ I_{e\text{Æ}} \quad I_C + I_{NC} \circ I_{LH}$$

$$= \sum_i p_i S(\hat{ž}_i) \circ \sum_l p^{(l)} S(\hat{ž}^{(l)}_K) = \sum_i p_i S(\hat{æ}_i)$$

Averaged entropy of reduced letters.

$¢_{b\circ v} = 0$ if $f \hat{æ}_i g$ are all pure.

$¢_{b\circ v}$ can be nonzero even when $f \hat{æ}_i g$ commute.
(classical data)

Nonzero information defect: Example

$$\hat{ž}_0 = \frac{1}{10}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

$$\hat{ž}_0 = \frac{1}{10}(1)°(1) \; © \; \frac{2}{10}(1)°(1) \; © \; \frac{3}{10}(1)°(1) \; © \; \frac{4}{10}(1)°(1)$$

$$\hat{ž}_1 = \frac{1}{10}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\hat{ž}_1 = \frac{4}{10}(1)°(1) \; © \; \frac{3}{10}(1)°(1) \; © \; \frac{2}{10}(1)°(1) \; © \; \frac{1}{10}(1)°(1)$$

$$p_0 = p_1 = \frac{1}{2}$$

$$\hat{ž} = \hat{æ} = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

blind scenario $\quad R^{AF}_{opt} = S(\hat{æ}) = 2$

visible scenario $\quad I_{eÆ} \quad H(f\, p_i\, g) = 1$

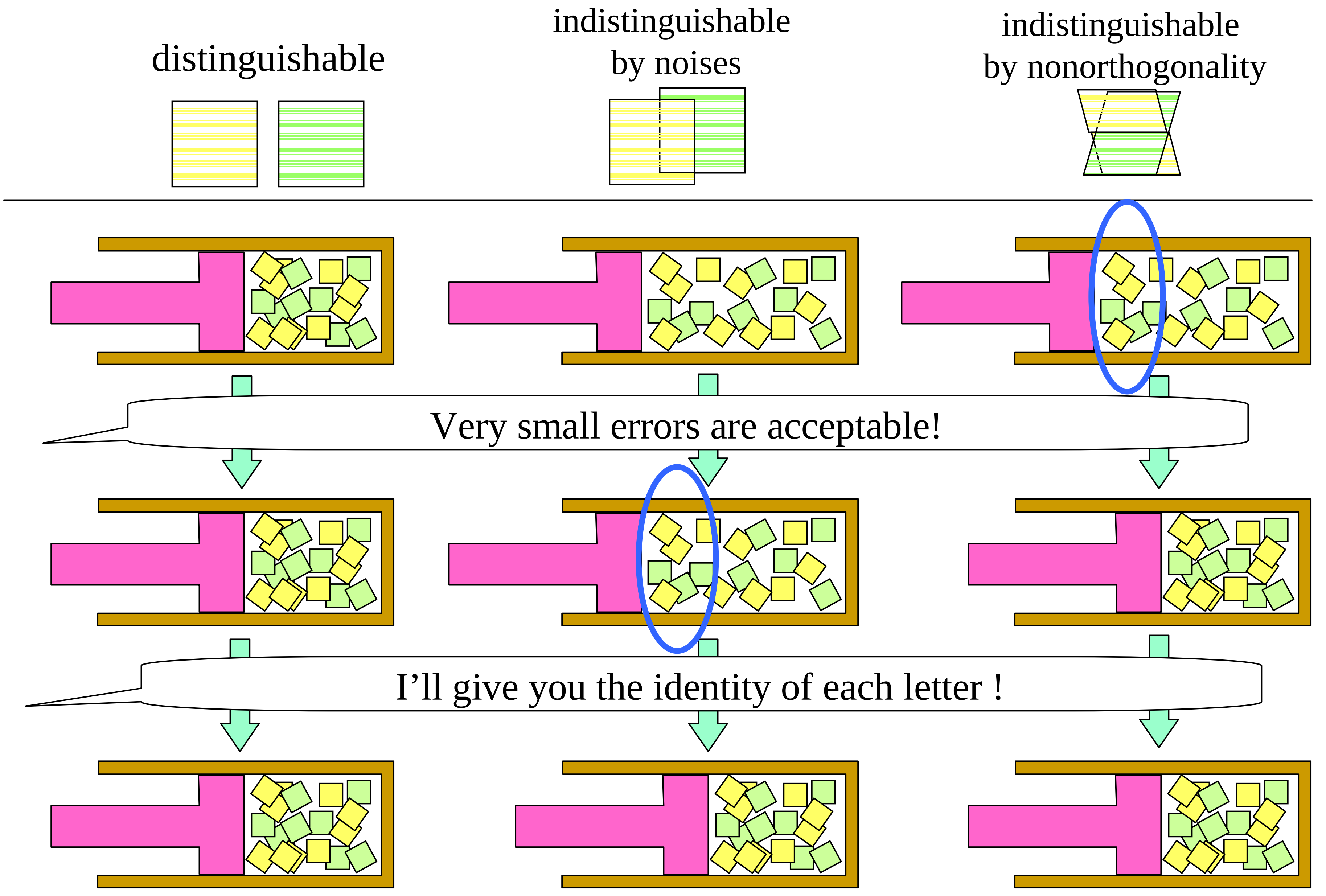
<u>Nature of information and compressibility</u>

Information	classical $([\hat{ž}_i ; \hat{ž}_j] = 0)$	quantum
pure $(S(\hat{æ}_i) = 0)$	$R^{(1)}_{opt} = R^{AF}_{opt} = I_{eÆ}$	$R^{(1)}_{opt} \quad R^{AF}_{opt} = I_{eÆ}$
mixed	$R^{(1)}_{opt} = R^{AF}_{opt} \quad I_{eÆ}$	$R^{(1)}_{opt} \quad R^{AF}_{opt} \quad I_{eÆ}$

$R^{(1)}_{opt}$	R^{AF}_{opt}	$I_{eÆ}$
variable-length faithful	fixed length asymptotically faithful	
blind		visible

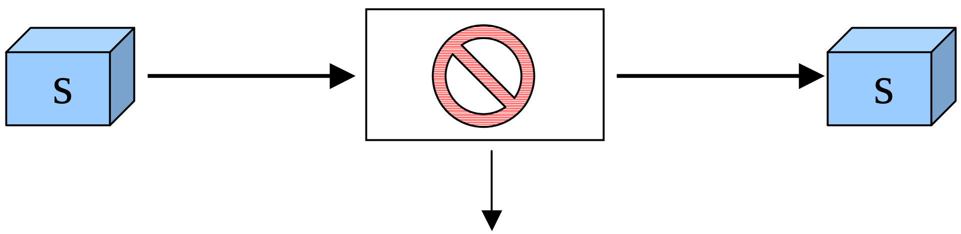
Koashi and Imoto, Phys. Rev. Lett. **89**, 097904 (2002).

Distinguishablity and compression rates



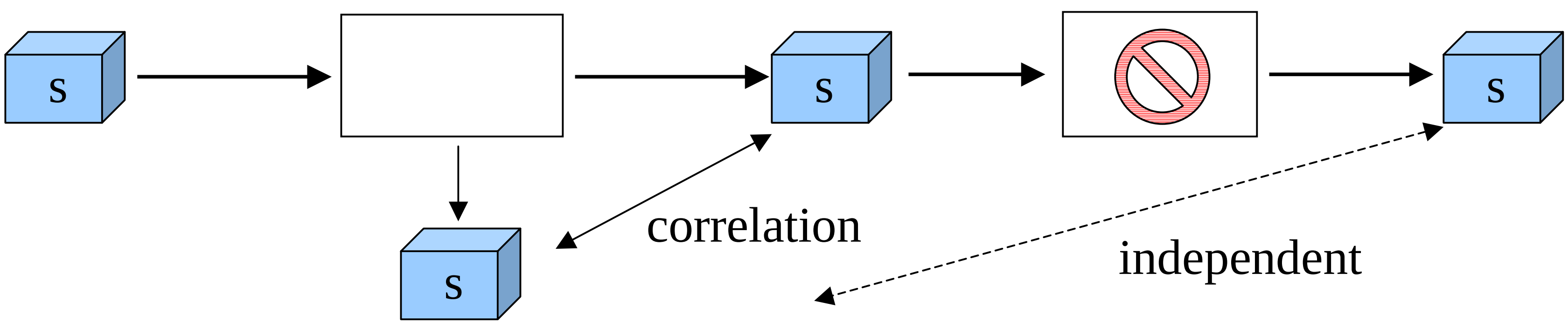
Restriction arising from indistinguishability

Pure & indistinguishable



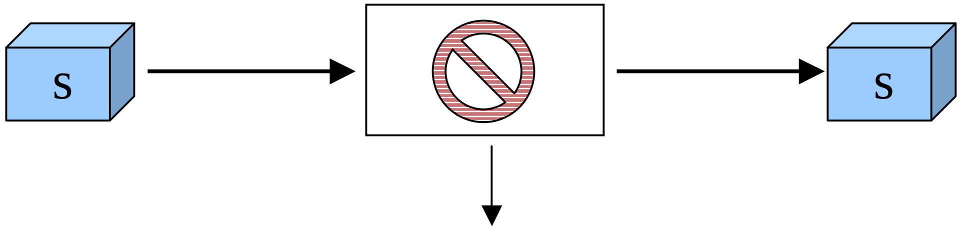
010100...
The outcome depends on s.

Classical & indistinguishable

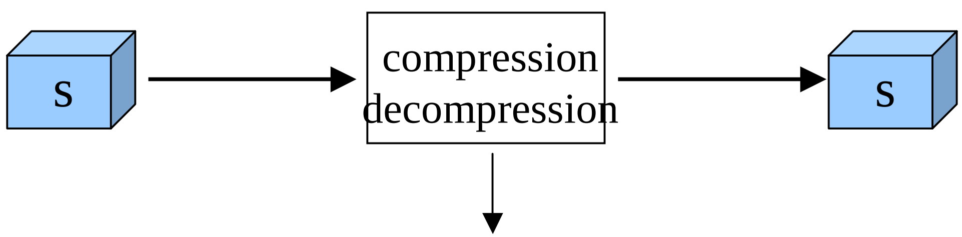


Restriction arising from indistinguishability

Pure & indistinguishable



010100...
The outcome depends on s.



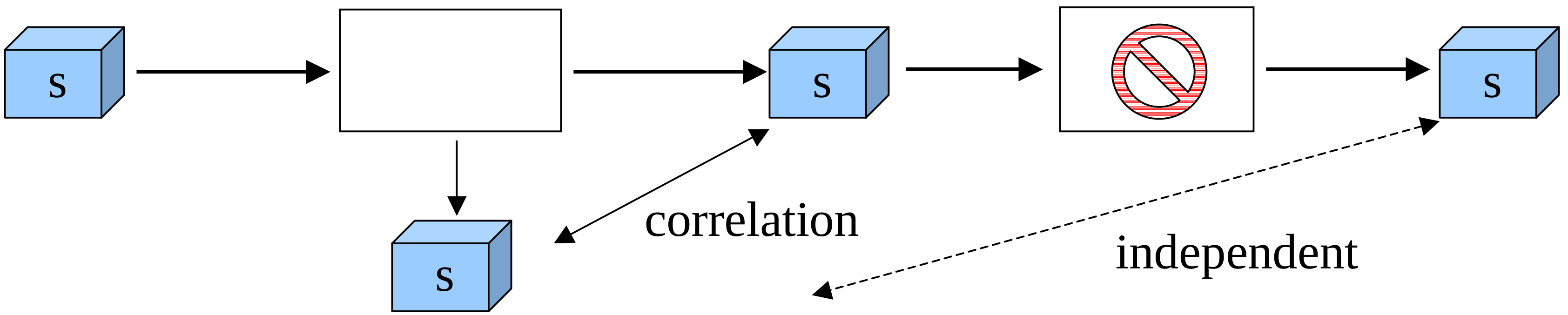
L
The outcome independent of s.

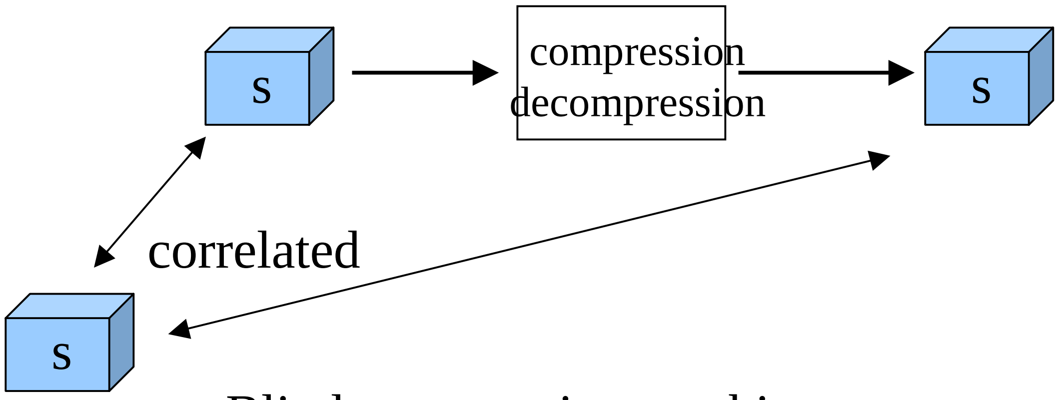
Faithful compression machines cannot vary the length according to the input.

Variable length coding doesn’t work.

Restriction arising from indistinguishability

Classical & indistinguishable





Blind compression machines preserve not only the original signals but also the possible correlation to a broadcast system.

Conclusion

The compressibility of the source with indistinguishable letters depends on scenarios of compression.

There are at least two different kinds of indistinguishability, one with quantum origin (nonorthogonality) and another with classical origin (mixing). This difference shows up in the compressibility.