Quantum Authentication

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joint work with

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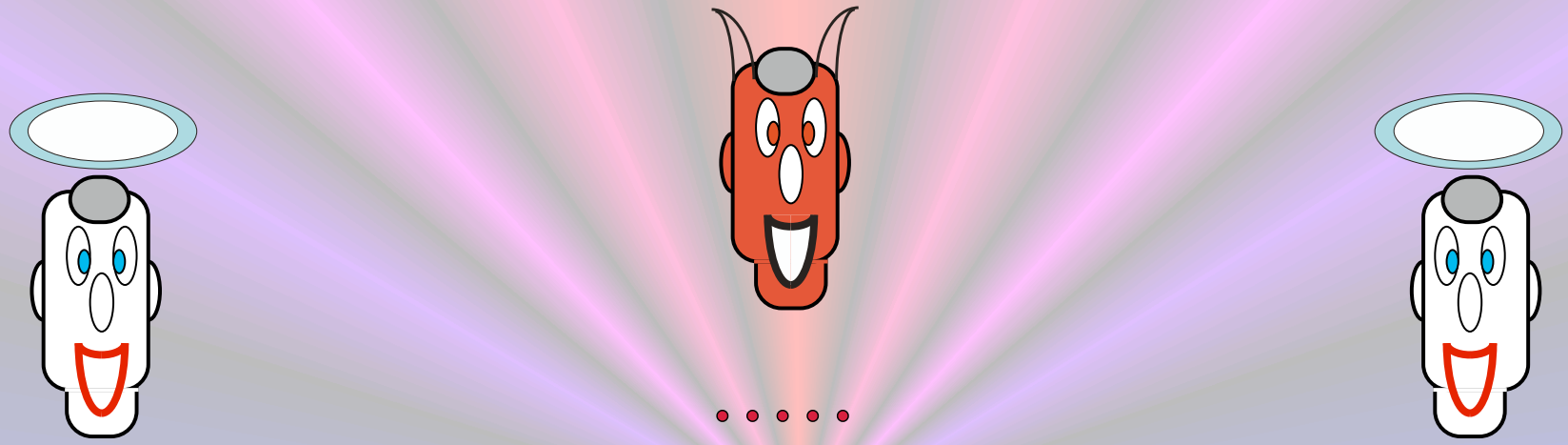
(1)
Classical Cryptography

(1.1) Information Theoretical Cryptography

(1.1) Information Theoretical Cryptography

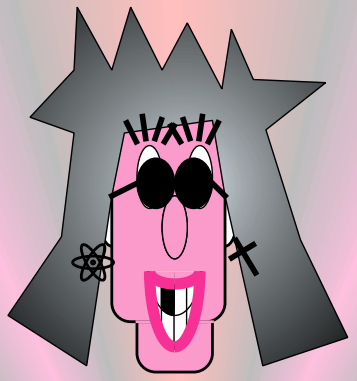


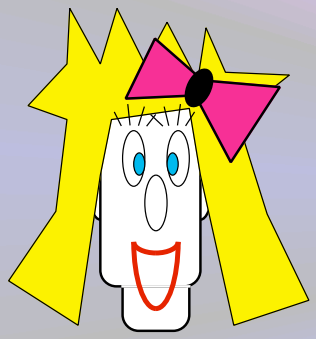
(1.1.1) key distribution

(1.1.2) Encryption

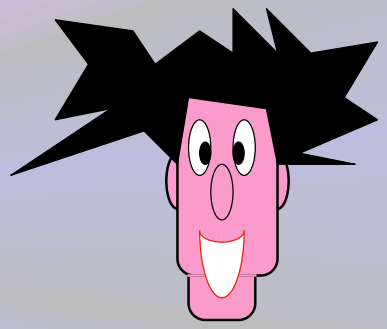
(1.1.3) Authentication

. . . . .





Will you marry me ?

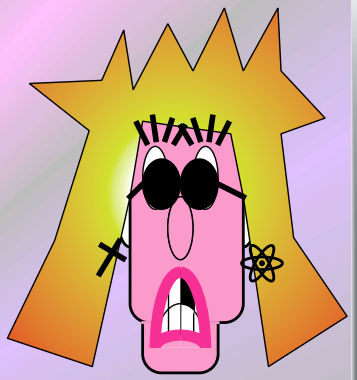


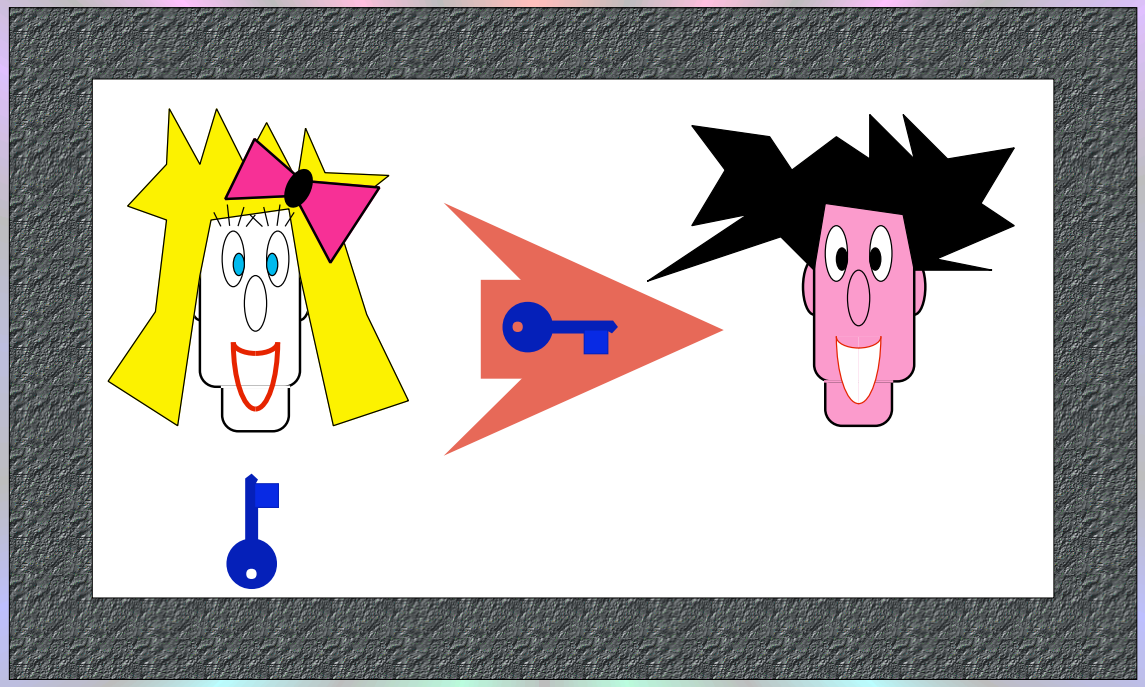
Divorce your wife first !

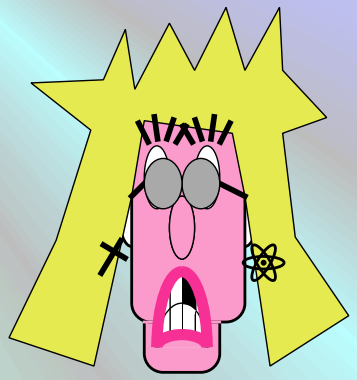
The papers are in the mail...

OK, I will !

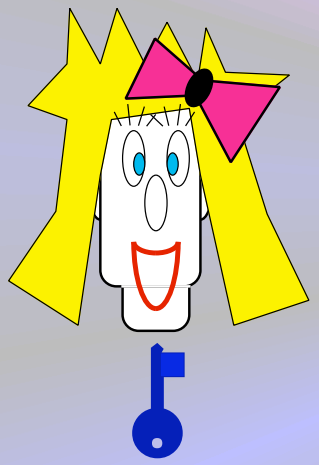
(1.1.1) key distribution



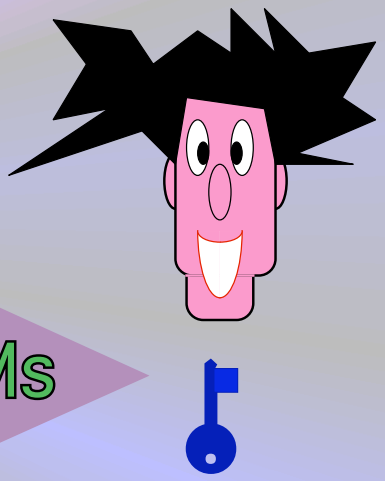




(1.1.2) Encryption



8RdewtU5qkLa$es!T9@

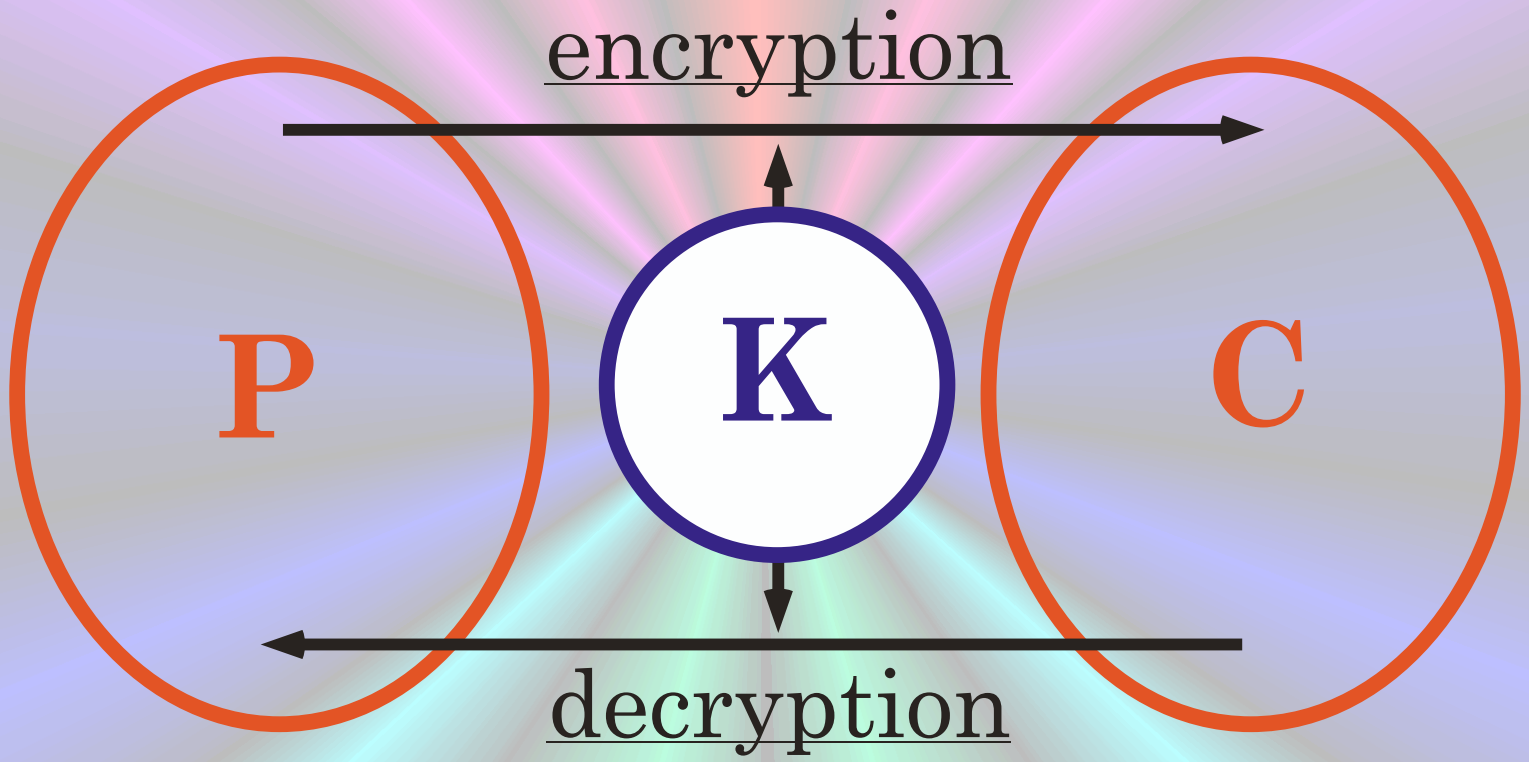


I(D%eXhDqIiykl#2cV7dEwnMs

H&fs@tyHvFGhaOKpTrGbl.Z/rUih*

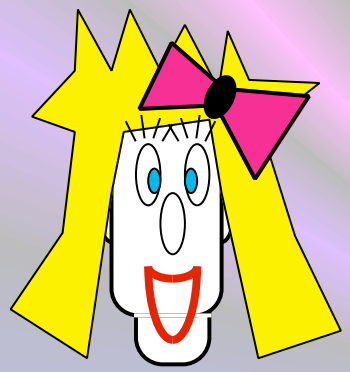
B7B3tdsjUila

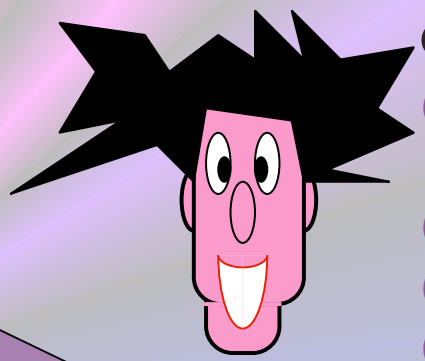
symmetric encryption

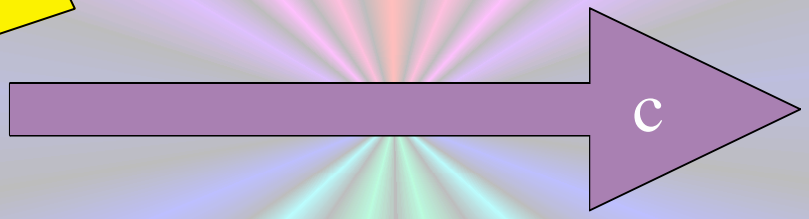


Information Theoretical Security

Vernam's One-Time-Pad







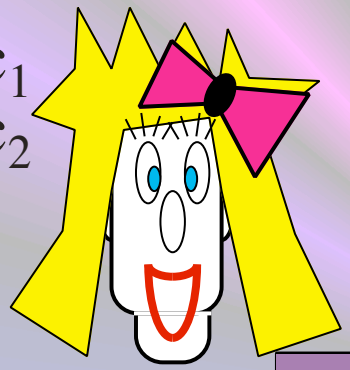
m		k	=	c
1		1		0
0		1		1
1		1		0
0		0		0
0		0		0
1		1		0
0		1		1
0		0	=	0
1		1	=	0
1		1		0
1		0		1
1		1		0
1		0		1
0		1		1
0		1		1
1		1		0

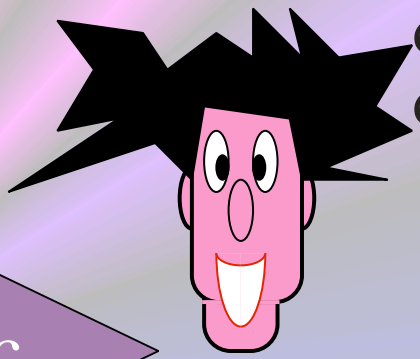
c		k	=	m
0		1		1
1		1		0
0		1		1
0		0		0
0		0		0
0		1		1
1		1		0
0		0	=	0
0		1	=	1
0		1		1
1		0		1
0		1		1
1		0		1
1		1		0
1		1		0
0		1		1

Information Theoretical Security

Vernam's One-Time-Pad

$m_1 \oplus k = c_1$
$m_2 \oplus k = c_2$





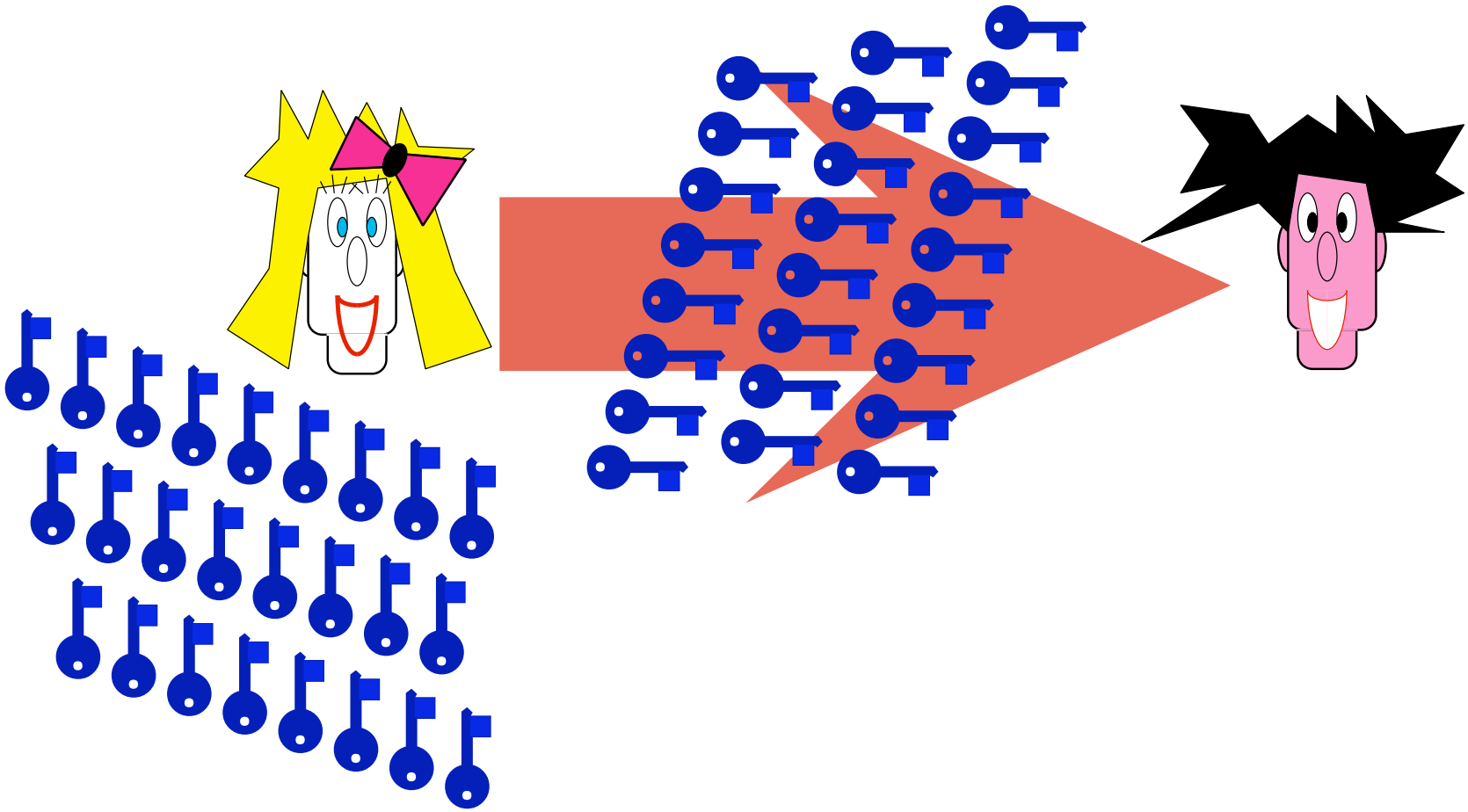
$c_1 \oplus k = m_1$
$c_2 \oplus k = m_2$

c_1

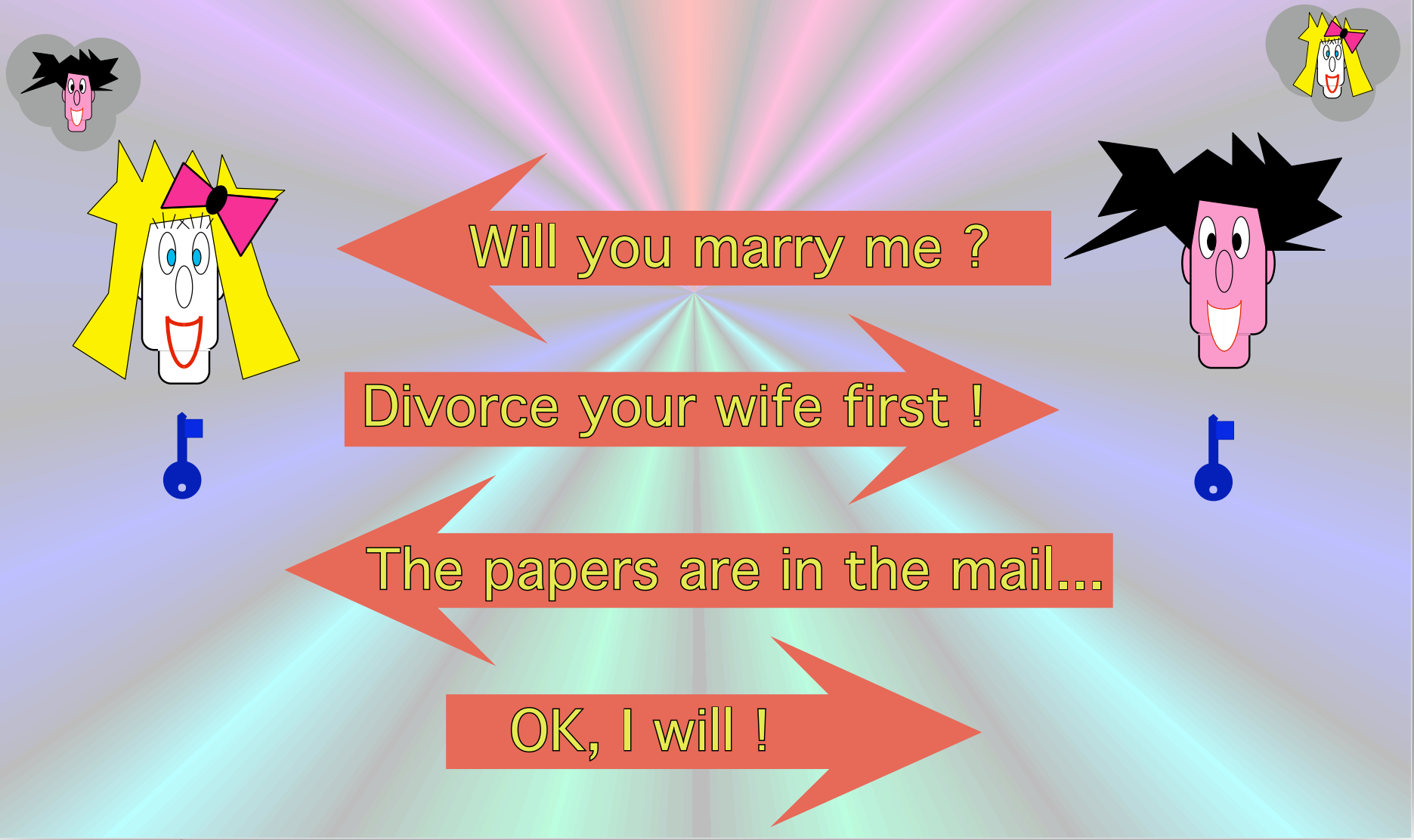
c_2

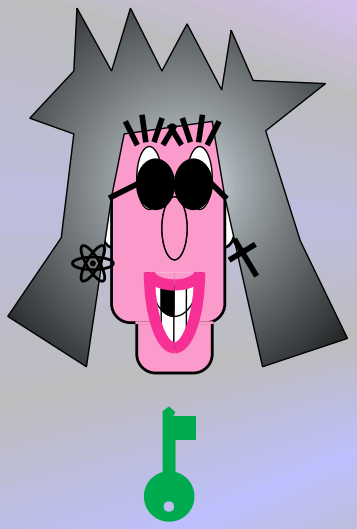
$c_1 \oplus c_2 = m_1 \oplus m_2$

(1.1.1) key distribution PROBLEM



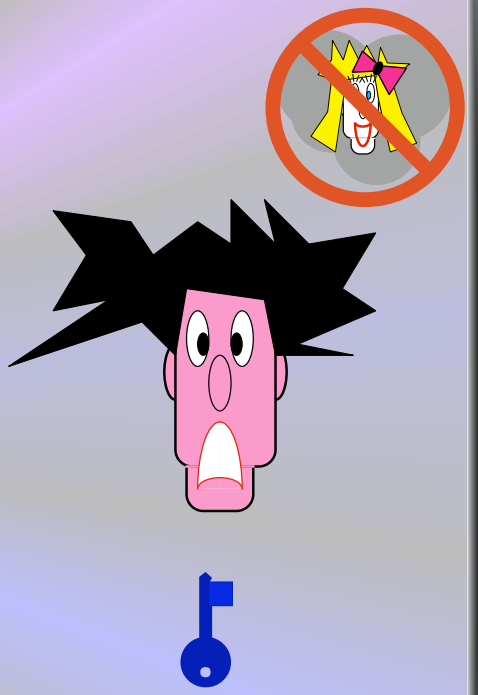
(1.1.3) Authentication



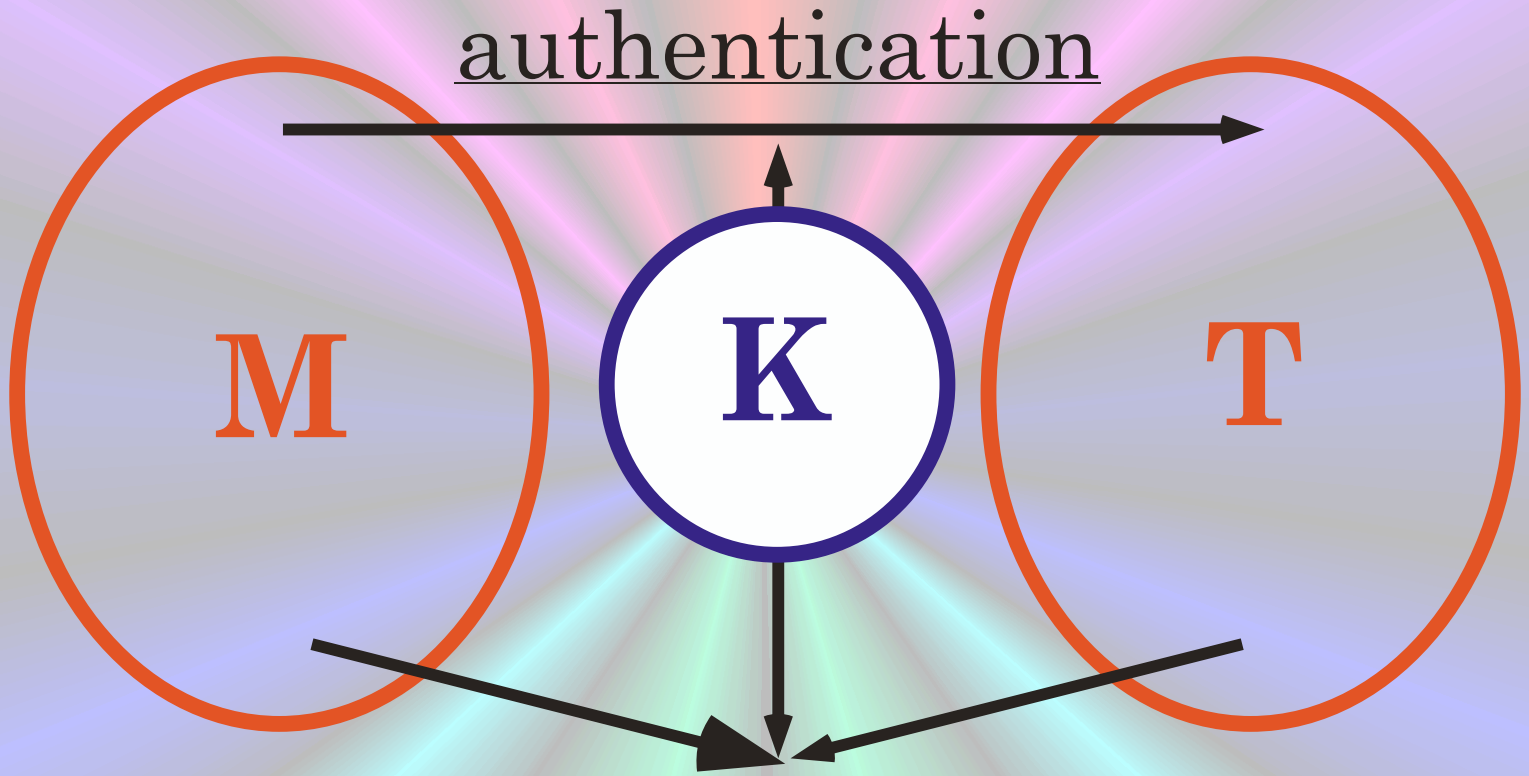


Will you marry me ?

No, I never will !

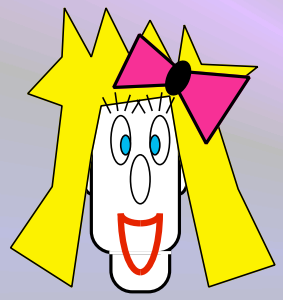


symmetric authentication

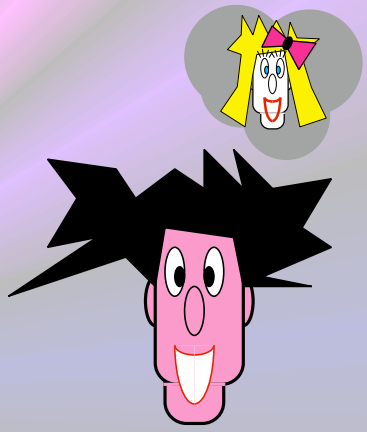


Information Theoretical Security

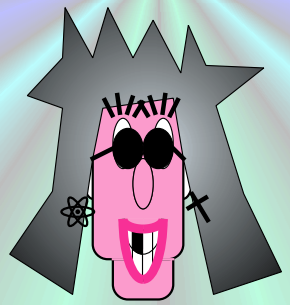
Authentication



(m,t)



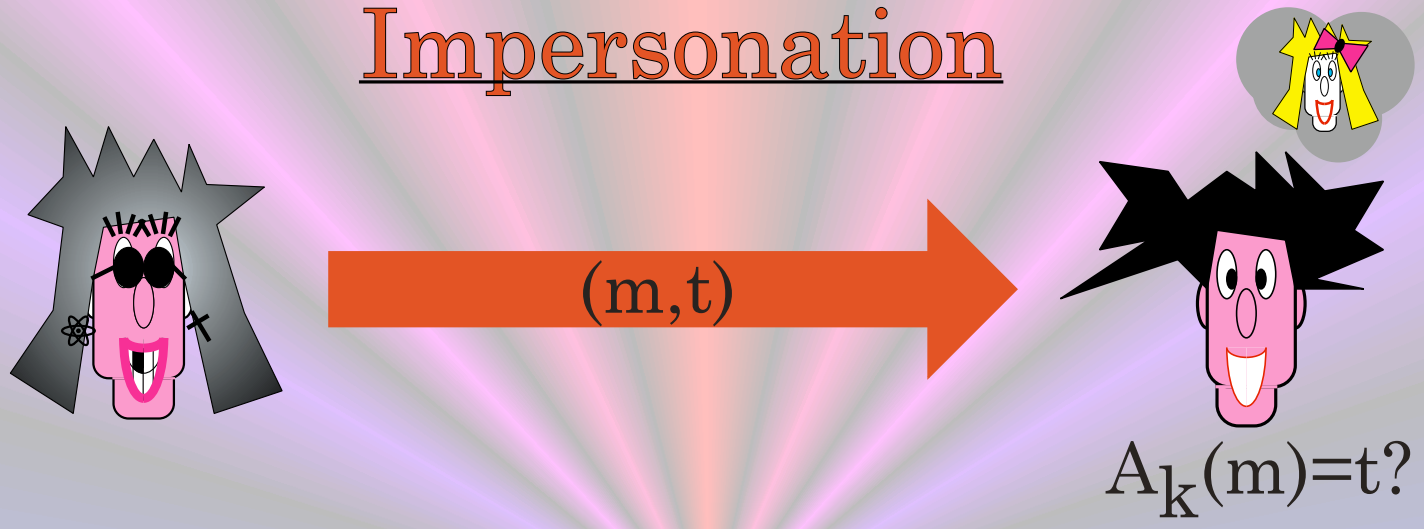
$t=A_k(m)$



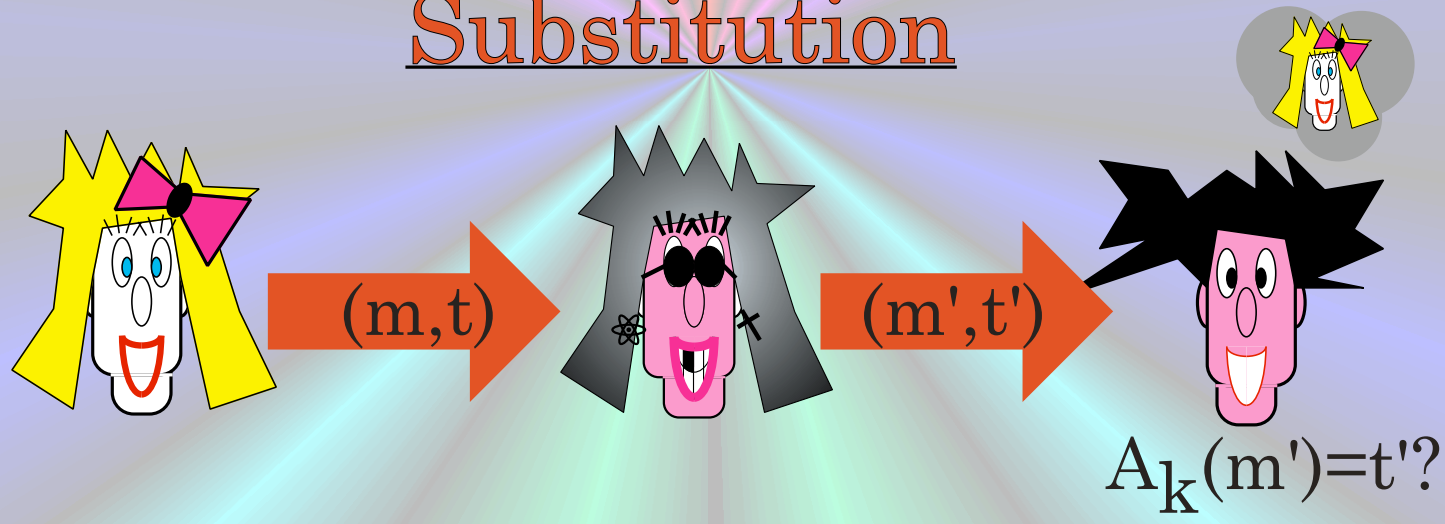
$A_k(m)=t?$

Information Theoretical Security

Impersonation



Substitution



Information Theoretical Security

WC One-Time-Authentication

$$A_{\mathbf{M},b}(x) = \mathbf{M}x \oplus b$$

$|x| = n,\ |\mathbf{M}| = n \cdot s,\ |t| = |b| = s$

$\forall m \in M, \forall t \in T$

$\Pr(A_{\mathbf{M},b}(m)=t) = 1/|T| = 1/2^s$

$\forall m \neq m' \in M, \forall t,t' \in T$

$\Pr(A_{\mathbf{M},b}(m')=t' \mid A_{\mathbf{M},b}(m)=t) = 1/|T| = 1/2^s$

WC One-Time-Authentication and (linear) error correction

$$A_{\mathbf{M},b}(x) = \mathbf{M}x \oplus b$$

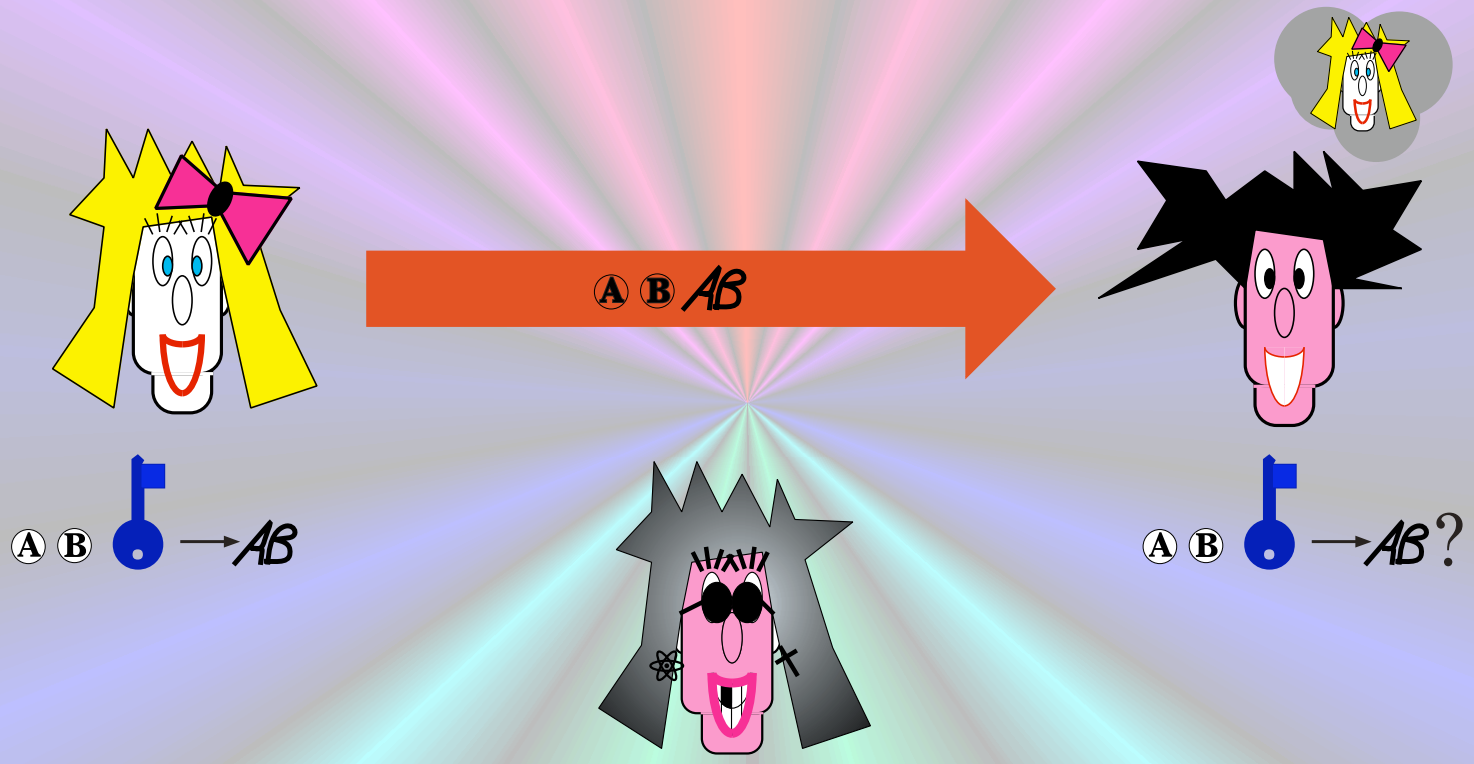
[**I**:**M**]m⊕[0:b]=[m:t]

G=[**I**:**M**] (systematic) generating matrix of error correcting code

[0:b] error syndrome = one-time pad encryption of tag

[m:t] systematic form of (message,tag)

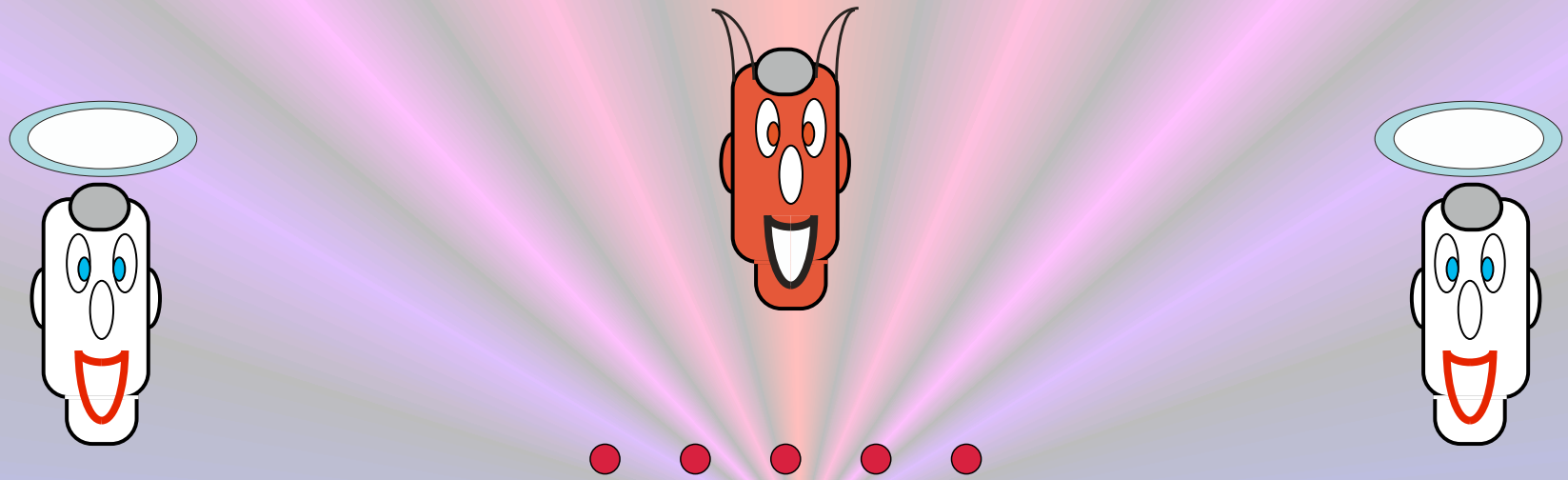
Authentication



Information Theoretical Security

(1.2) Complexity Theoretical Cryptography

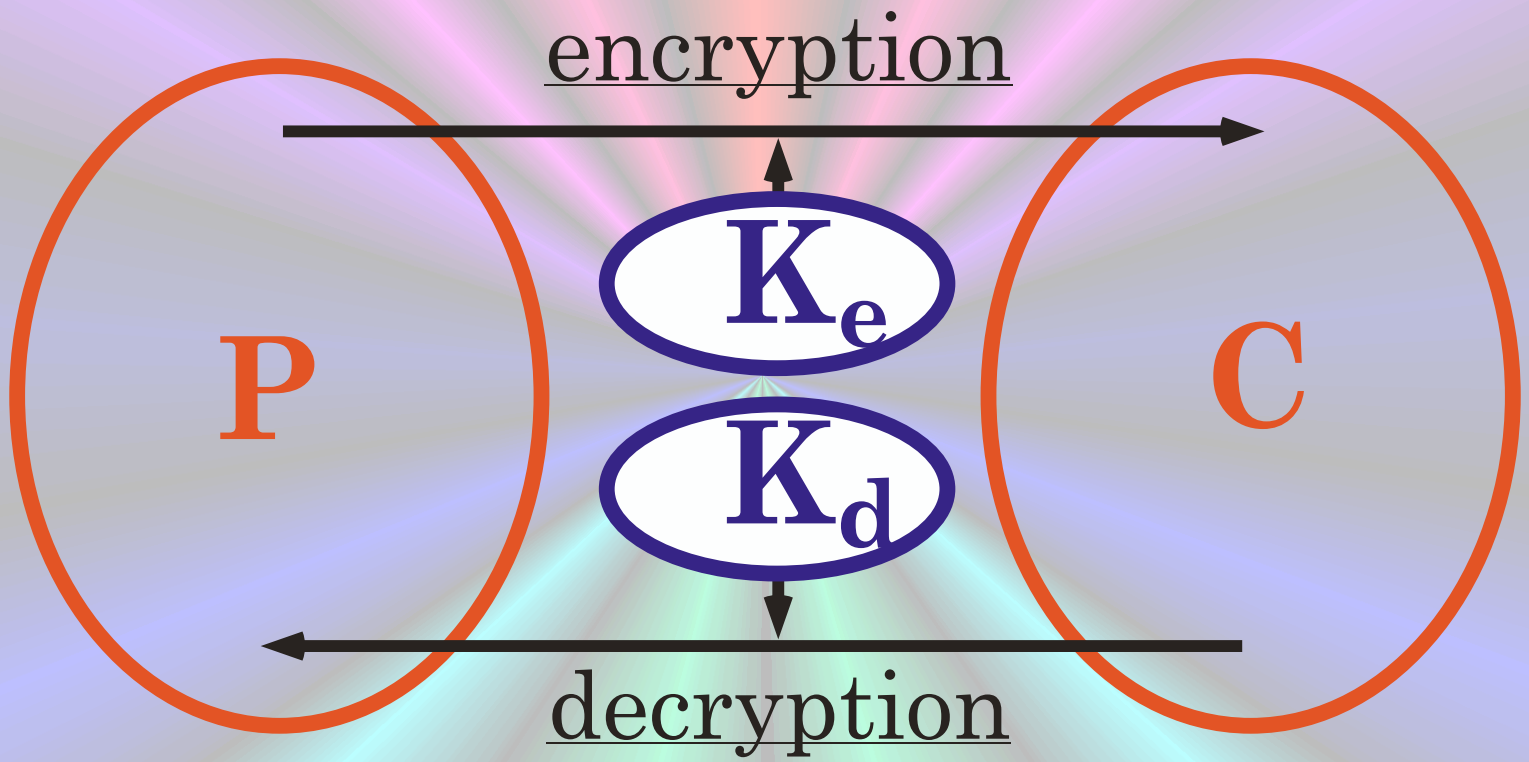
(1.2) Complexity Theoretical Cryptography



(1.2.1) Public key cryptosystem

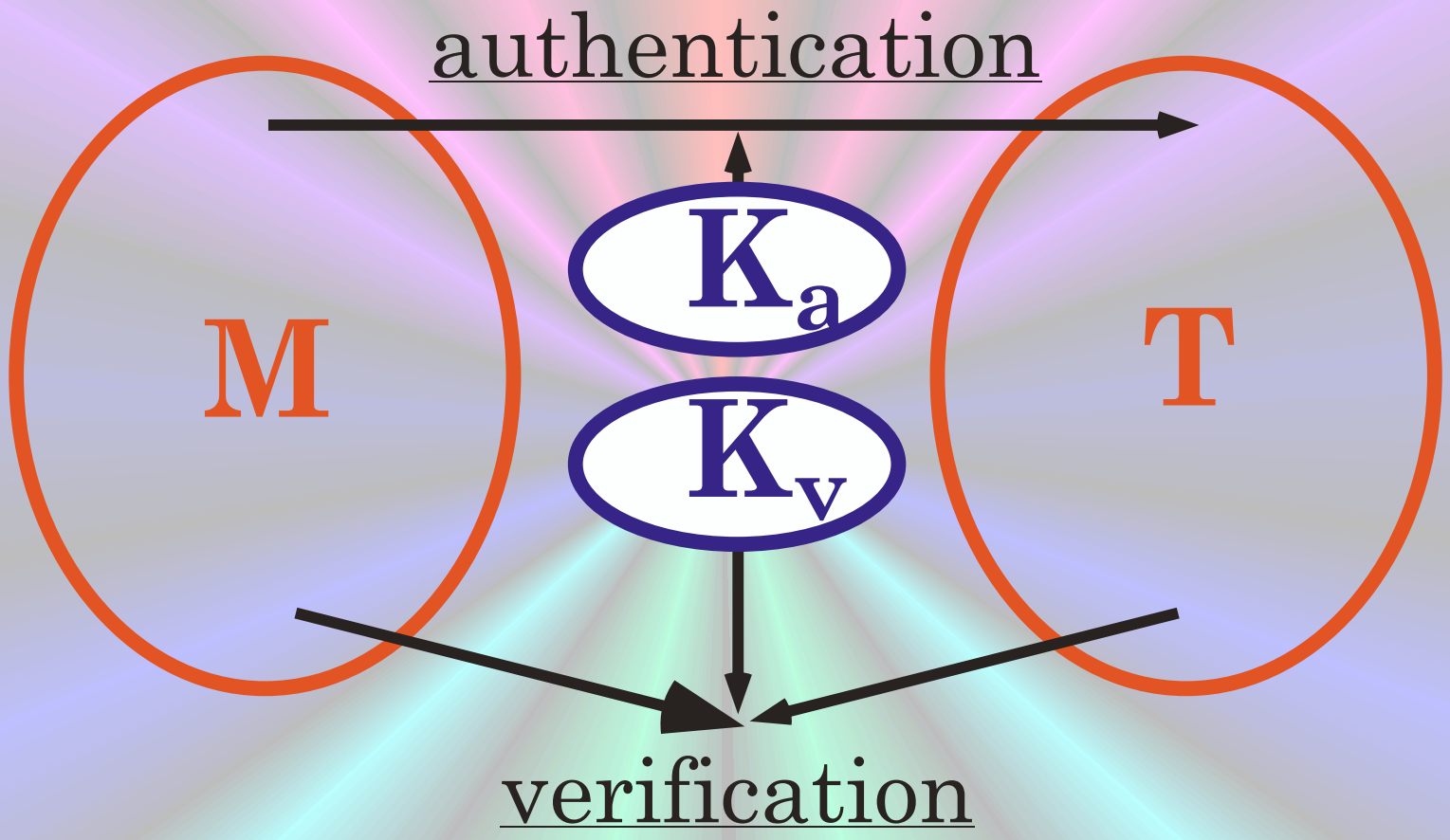
(1.2.2) Digital signature scheme

asymmetric encryption
(public-key cryptography)



Complexity Theoretical Security

asymmetric authentication
(digital signature schemes)



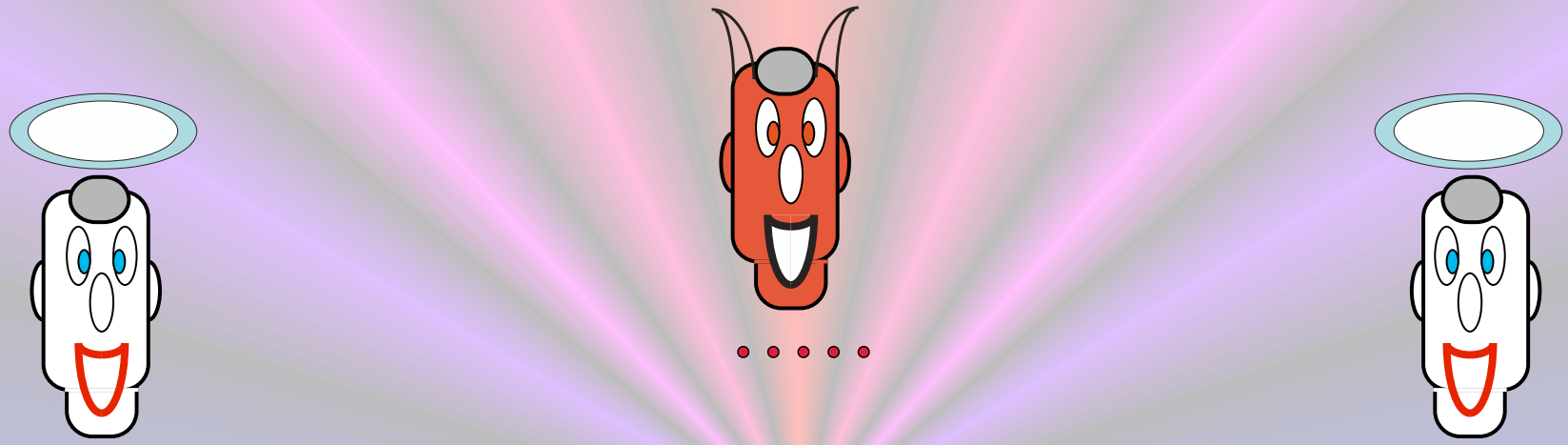
Complexity Theoretical Security

(2)
Quantum Information & Computations

(3)
Quantum Cryptography

(3.1)
Information Theoretical Quantum Cryptography

(3.1) Information Theoretical Cryptography



(3.1.1) Key distribution : Q-key distribution +
Q-distillation (formerly purification)

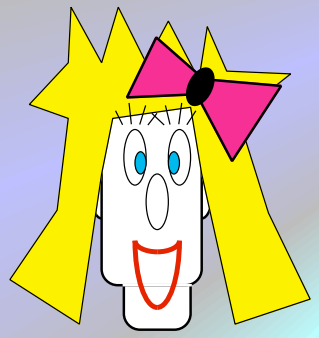
(3.1.2) One-time pad : one-time Q-pad (Q-teleportation)
Vernam Q-cipher

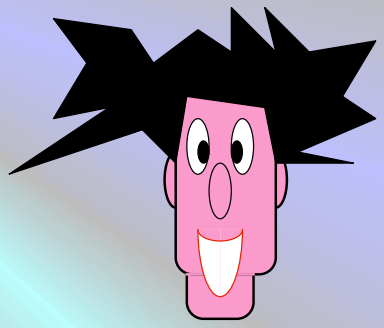
(3.1.3) one-time authentication : 1x authenticated Q-pad +
1x Q-authentication

(3.1.1) Key distribution

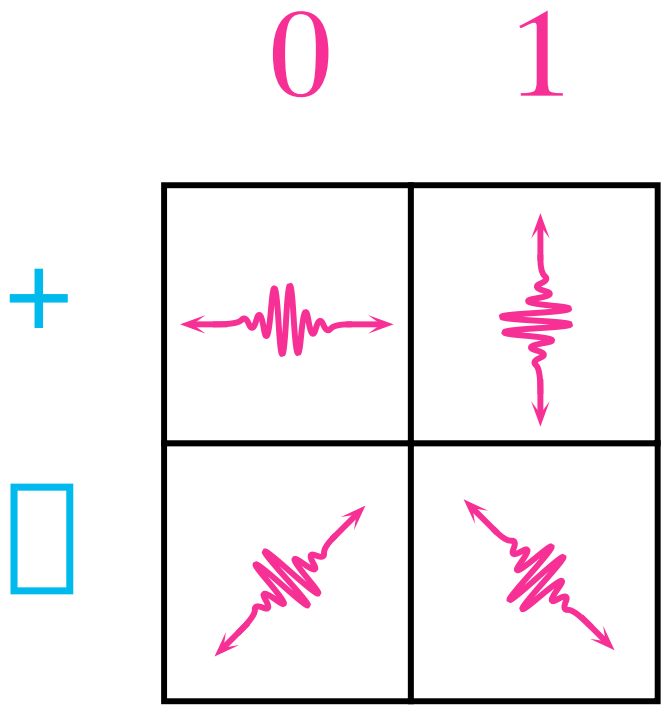
Classical key: Q-distribution of keys(BB84)
+ error-correction
+ privacy amplification

Quantum key : Q-key distribution(Ekert/Lo-Chau)
+ Q-error-correction or
+ Q-Distillation (Purification)

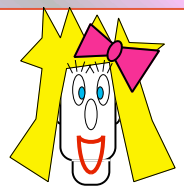


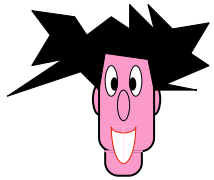


Ambiguous Coding Scheme



(3.1.1C) Quantum distribution of Keys



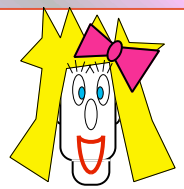


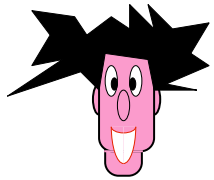
:	0	1	1	0	0	1	0	0	1	1	0	1	0	0	0	1	1	1	0	1	1	0	0	0
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
:			+	+		+	+	+		+	+				+				+	+		+		+
	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	1	0	0	0	1	1	0	0	0
:	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
:	0	←	←	0	←	1	←	←	1	←	0	←	←	←	←	1	0	←	←	1	←	0	0	0
:	0			0		1			1		0					1	0			1		0	0	0
:	0			0		1			1		0					1	1			1		0	0	0
:	0					1					0						1					0		
:	=			=					=													=		
:				0					1							1				1		0	0	
:				0					1							1				1		0	0	

20%

Bennett- Brassard

(3.1.1C) Quantum distribution of Keys





:	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?
	□	+	□	+	+	+	□	□	□	□	+	+	+	+	□	□	□	+	□	+	+	+	□	+
:	¿	¿	¿	¿	¿	¿	?	¿	¿	¿	¿	¿	¿	¿	¿	¿	?	¿	¿	¿	¿	¿	¿	¿
	□	□	+	+	□	+	+	+	□	+	+	□	□	□	+	□	□	□	+	+	□	+	□	+
	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	1	1	0	0	0	1	1	0	0
:	□	+	□	+	+	+	□	□	□	□	+	+	+	+	□	□	□	+	□	+	+	+	□	+
:	0	↓	↓	0	↓	1	↓	↓	1	↓	0	↓	↓	↓	↓	1	0	↓	↓	1	↓	0	0	0

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:	1					0					1						0							1
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20%

Ekert

(3.1.1C) Quantum distribution of Keys

• • • • •

- Produces raw classical key

- Observed error rate indicates amount of eavesdropper information

- Error-correction is used to fix errors

- Random hash function is used to distill a smaller secret classical key

• • • • •

(3.1.1) Key distribution

Classical key : Q-distribution of keys(BB84)

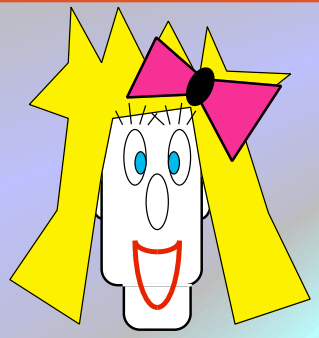
+ error-correction

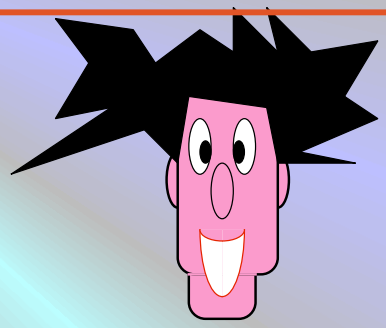
+ privacy amplification

Quantum key : Q-key distribution(Ekert/Lo-Chau)

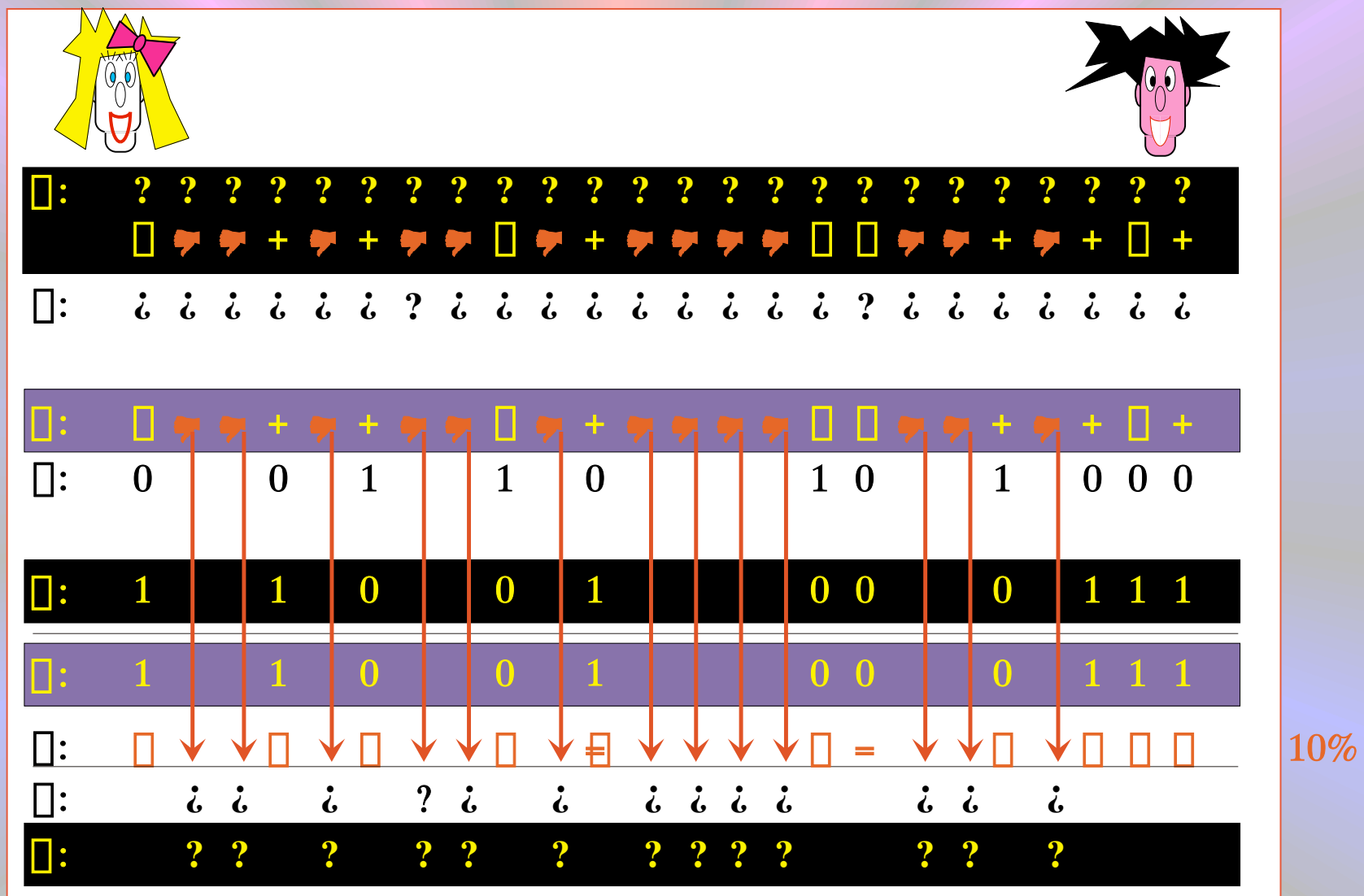
+ Q-error-correction (CSS) or

Q-Distillation (Purification)



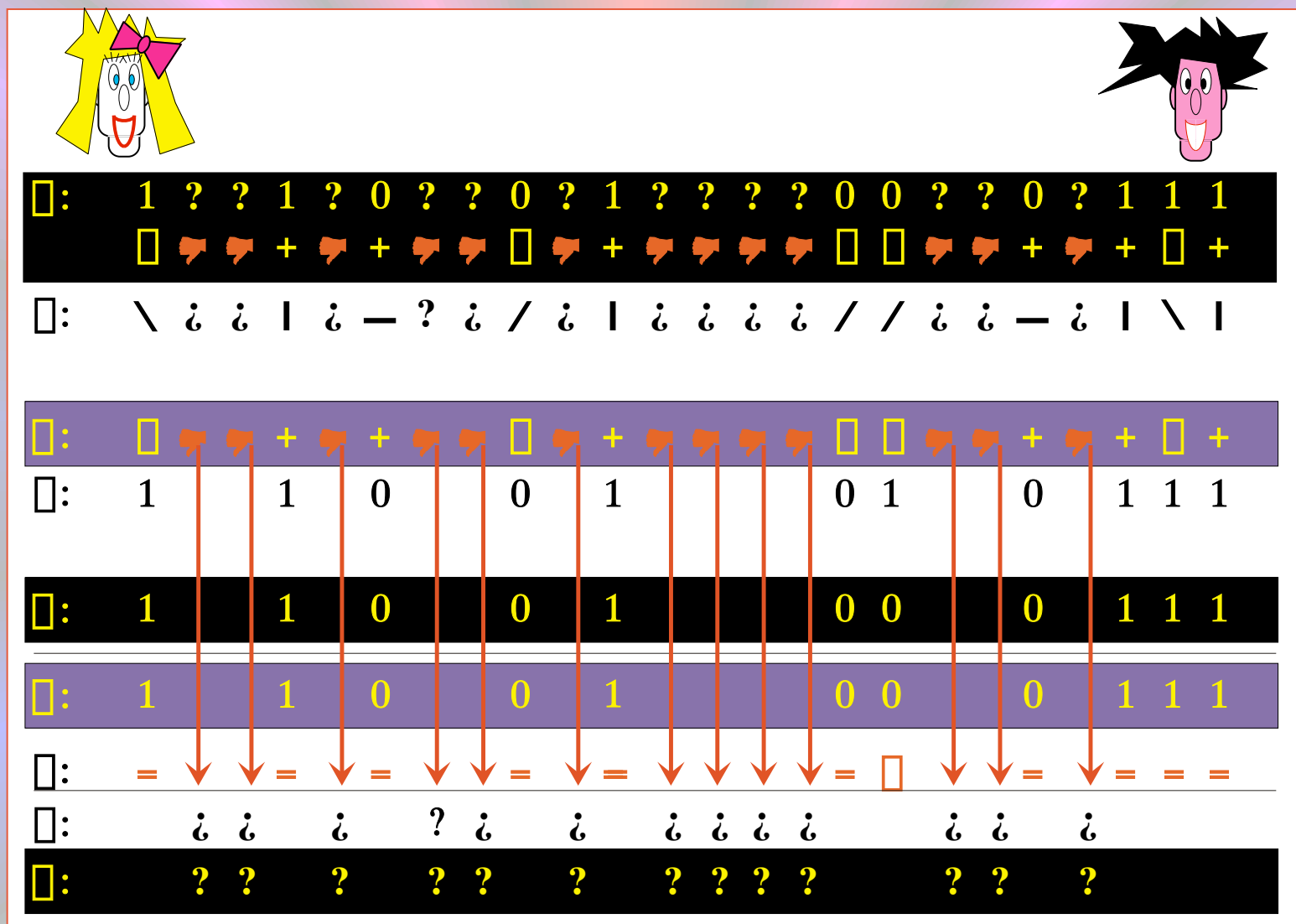


(3.1.1Q) Quantum-Key distribution



Ekert + Lo-Chau

(3.1.1Q) Quantum-Key distribution



Shor-Preskill

(3.1.1Q) Quantum-Key distribution

• • • • •

- Produces raw quantum key (EPR states)

- Observed error rate indicates amount of impurity of EPR states

- Quantum error-correction is used to purify raw EPR states into a smaller pure set

• • • • •

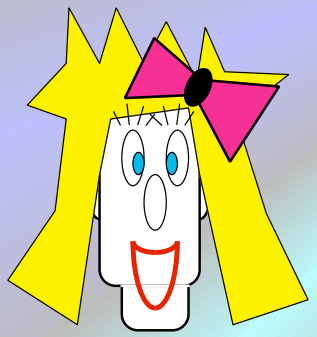
(3.1.2) One-time pad

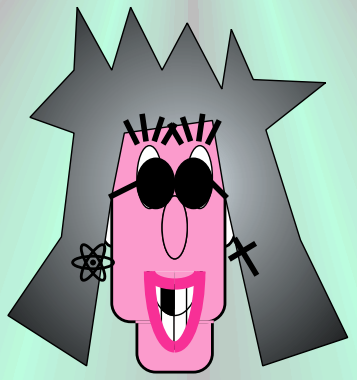
Classical key : Vernam Q-cipher (various sources)

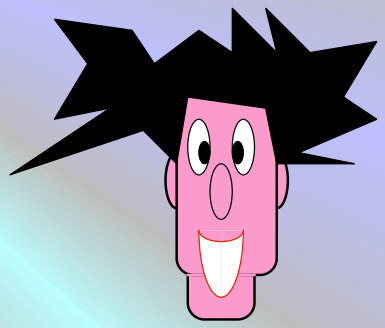
Quantum Ciphertext

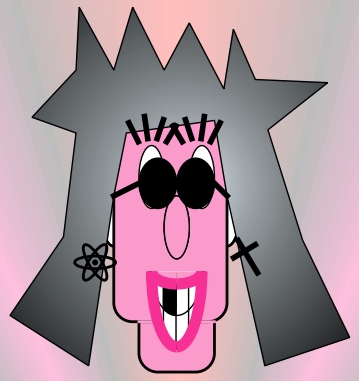
Quantum key : one-time Q-pad (Q-teleportation)

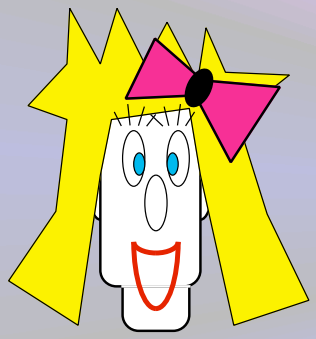
Classical Ciphertext (BBCJPW)



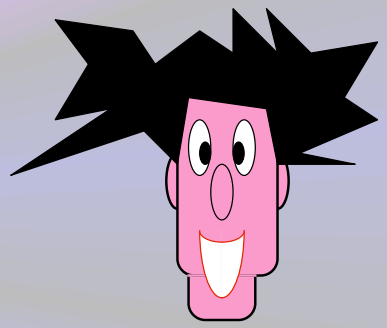








|Ωιλλ ψου μαρρψ με ?⟩

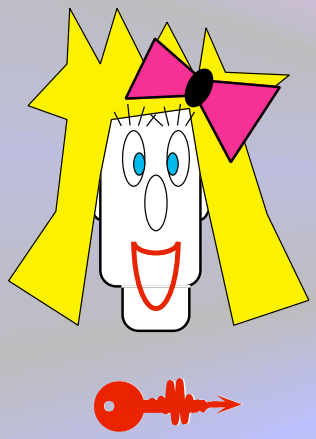


|Διωορχε ψουρ ωιφε φιρστ !⟩

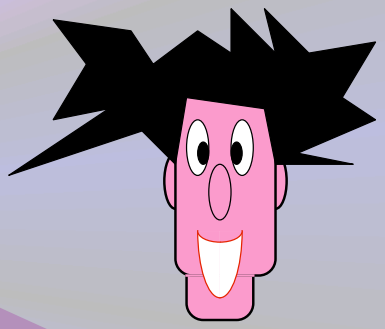
|Τηε παπερσ αρε ιν τηε μαιλ...⟩

|OK, I ωιλλ !⟩

(3.1.2Q) One-time Q-pad



8RdewtU5qkLa$es!T9@

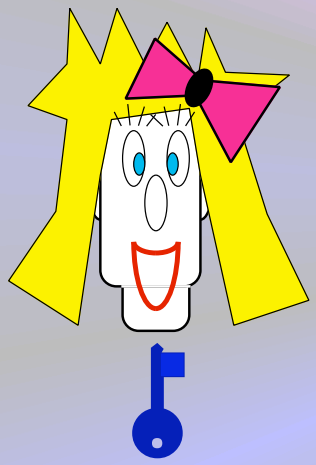


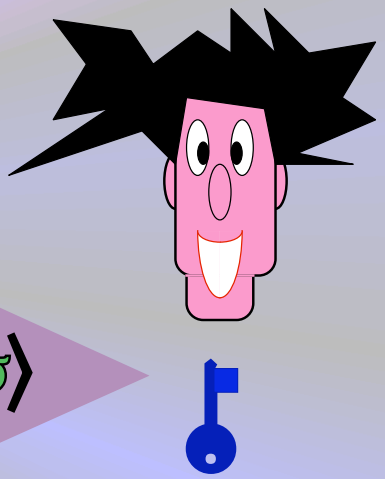
I(D%eXhDqIiykl#2cV7dEwnMs

H&fs@tyHvFGhaOKpTrGbl.Z/rUih*

B7B3tdsjUila

(3.1.2C) Vernam Q-cipher





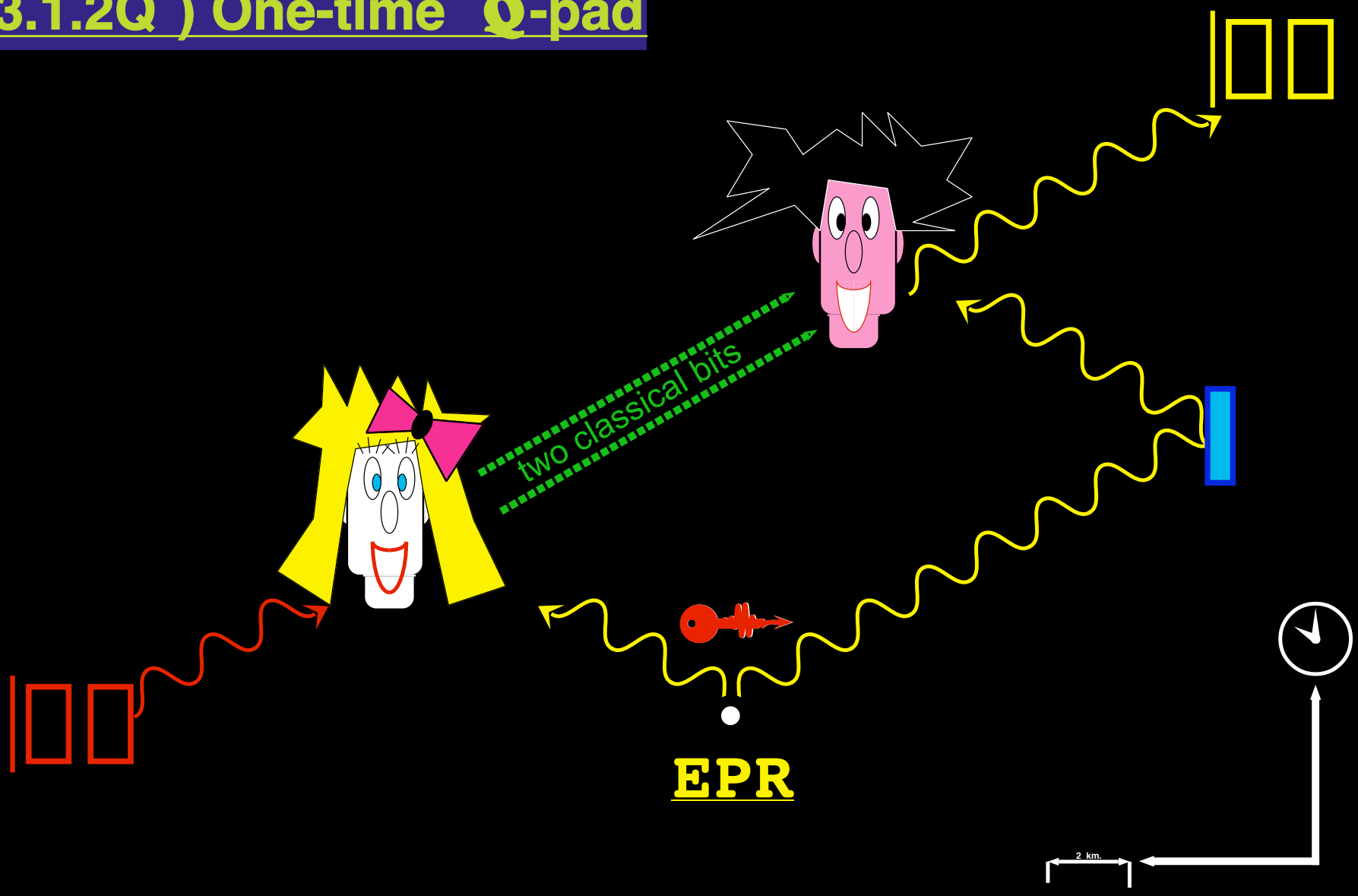
|8PδεωτY5θκΛαΞεσ!T9≅⟩

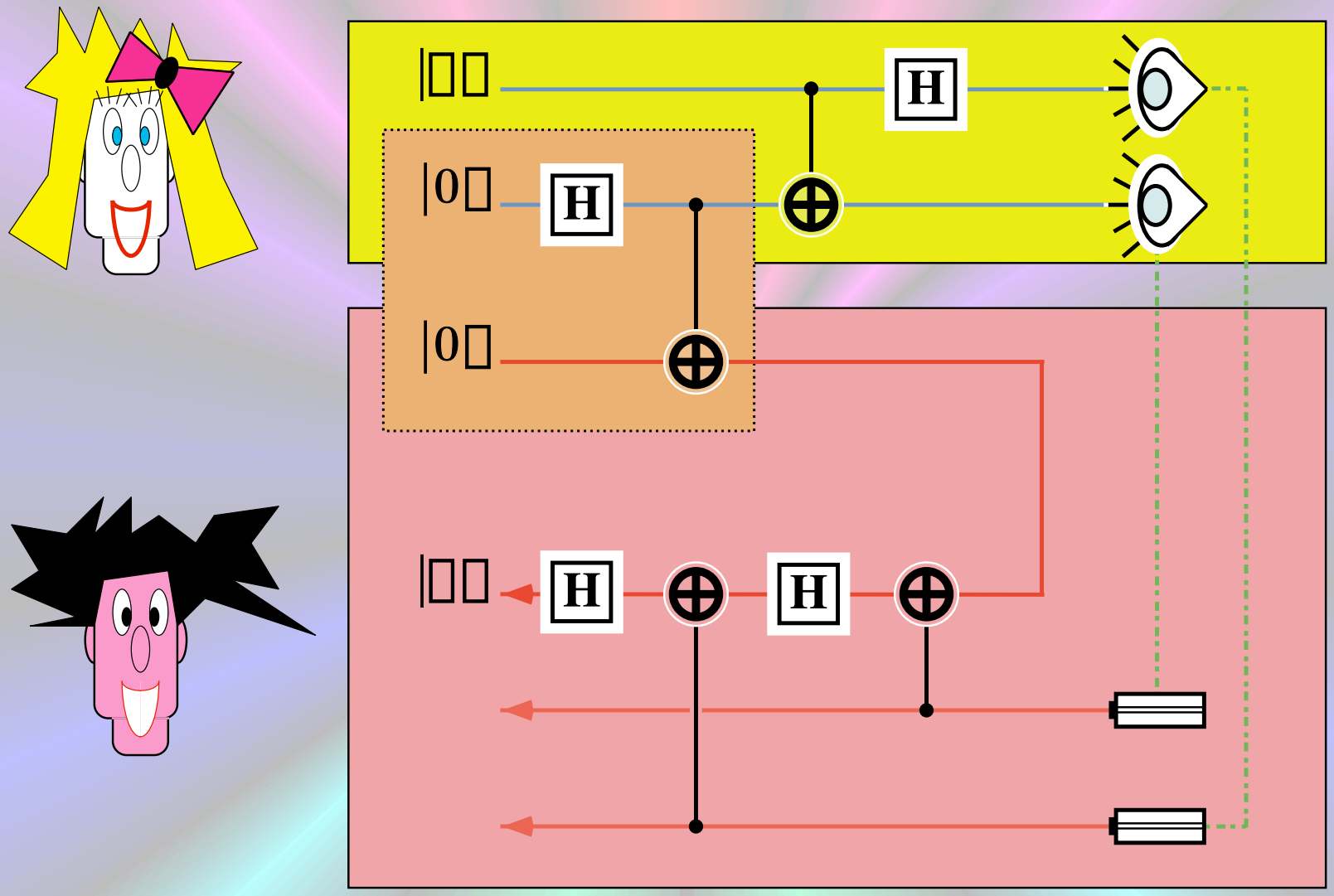
|I(Δ%εΞηΔθIιψκλ#2χς7δEωνMσ⟩

|H&φσ≅τψϖΦηαOKπTρΓβλ.Z/ρYιη*⟩

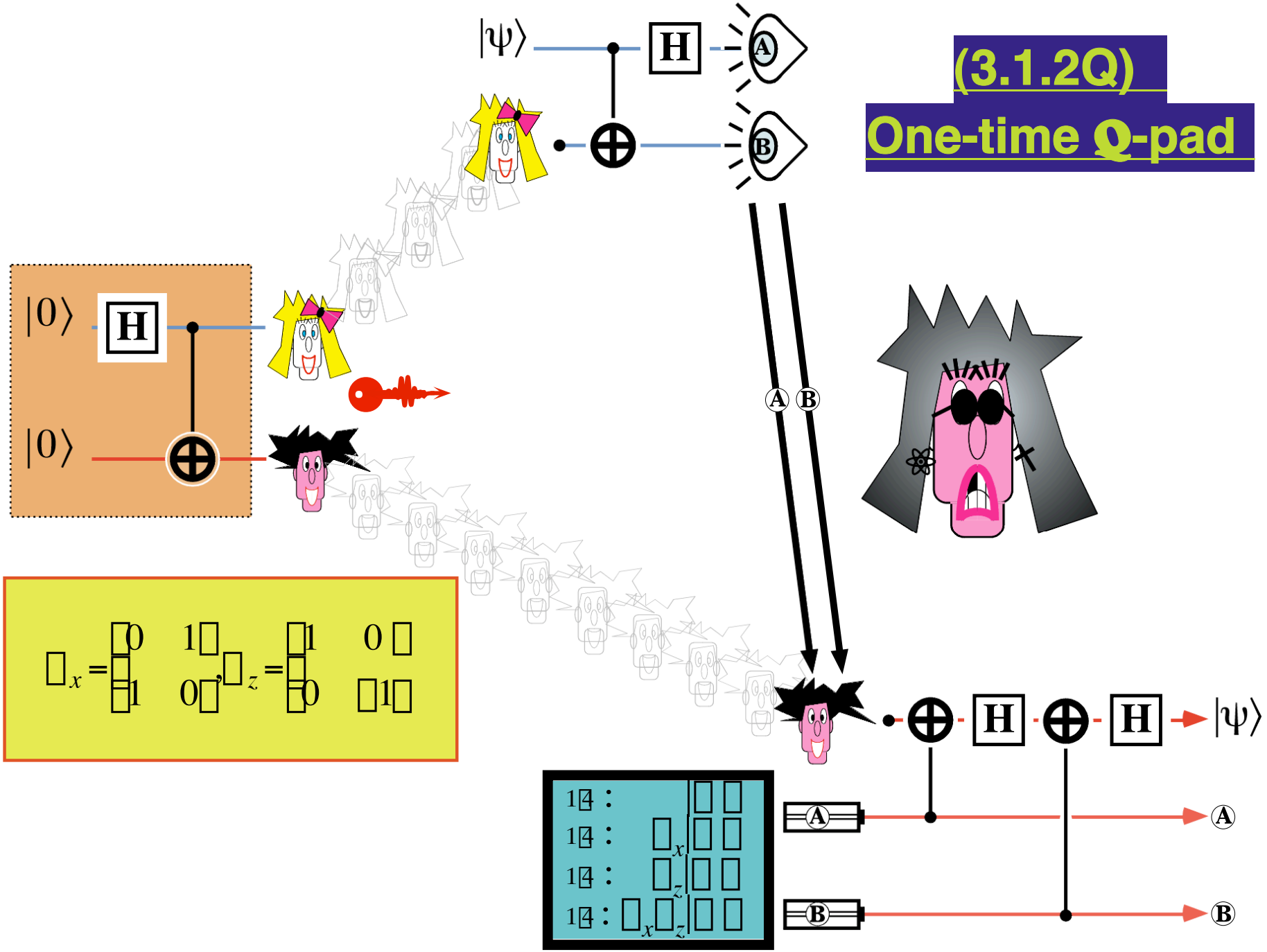
|B7B3τδσφYιλα⟩

(3.1.2Q) One-time Q-pad





(3.1.2Q) One-time Q-pad

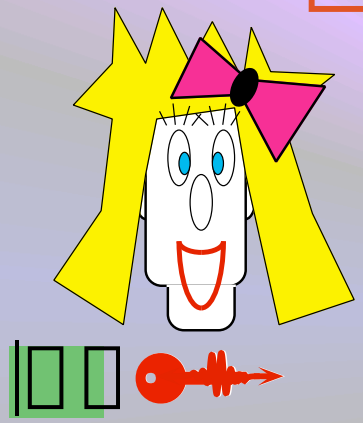


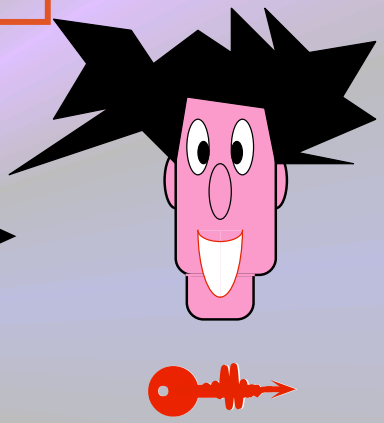
$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(3.1.2Q) One-time Q-pad

Quantum key : one-time Q-pad

Classical Ciphertext





Ⓐ Ⓑ

two random bits

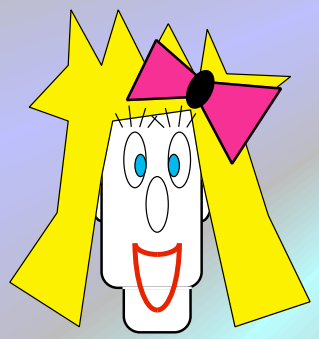
(3.1.2) One-time pad

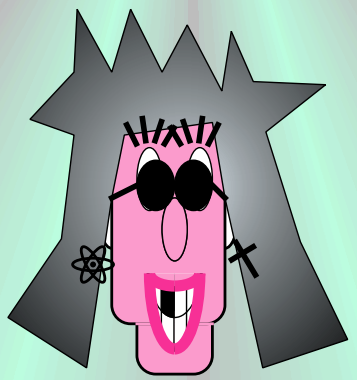
Classical key : Vernam Q-cipher (various sources)

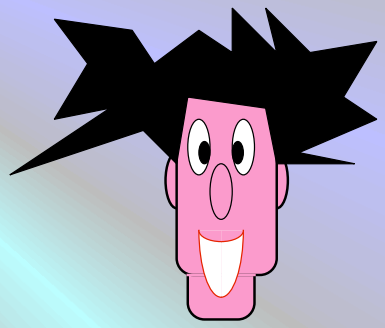
Quantum Ciphertext

Quantum key : one-time Q-pad (BBCJPW)

Classical Ciphertext







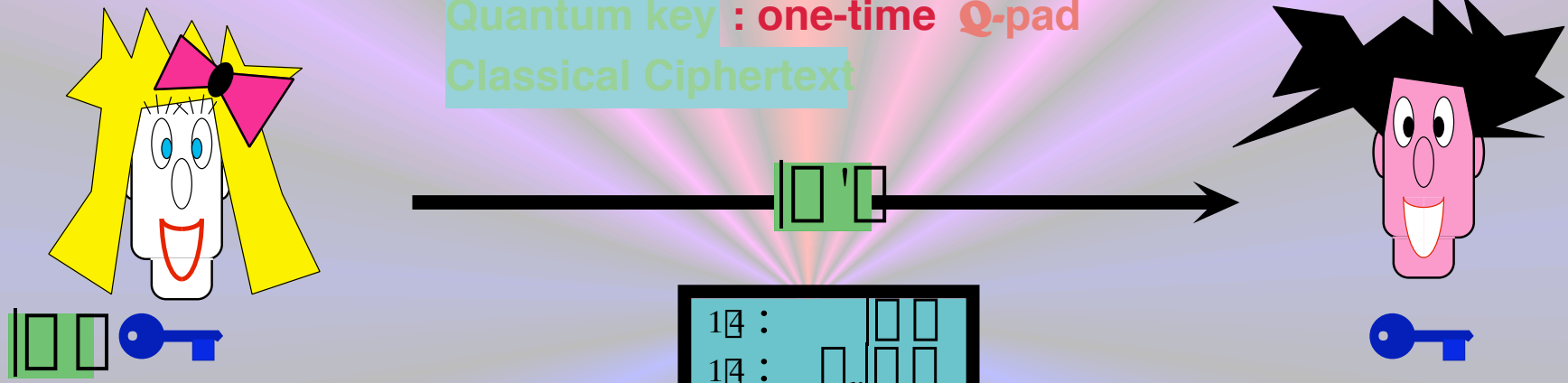
(3.1.2C) Vernam Q-cipher

Classical key: Vernam Q-cipher

Quantum Ciphertext

Quantum key : one-time Q-pad

Classical Ciphertext



a,b random bit key

$|\psi'\rangle = (\sigma_x)^a (\sigma_z)^b |\psi\rangle$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

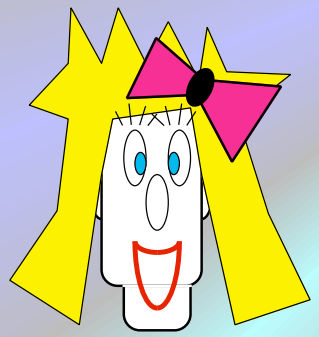
a,b random bit key

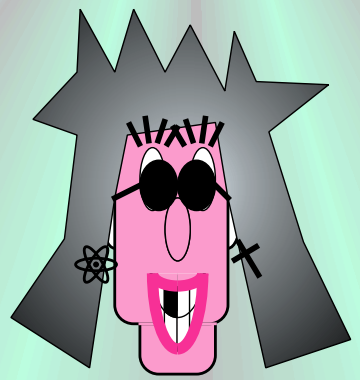
$|\psi\rangle = (\sigma_z)^b (\sigma_x)^a |\psi'\rangle$

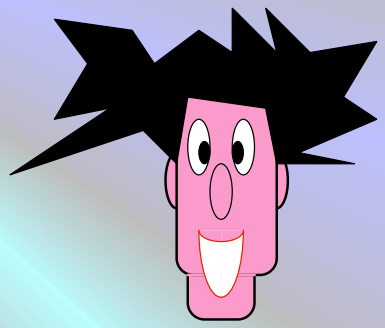
(3.1.3) One-time Authentication

Classical key : 1x Q-Authentication (BCGST)
Quantum message+tag

Quantum key : 1x Authenticated Q-pad
Classical message+tag (BBCJPW)

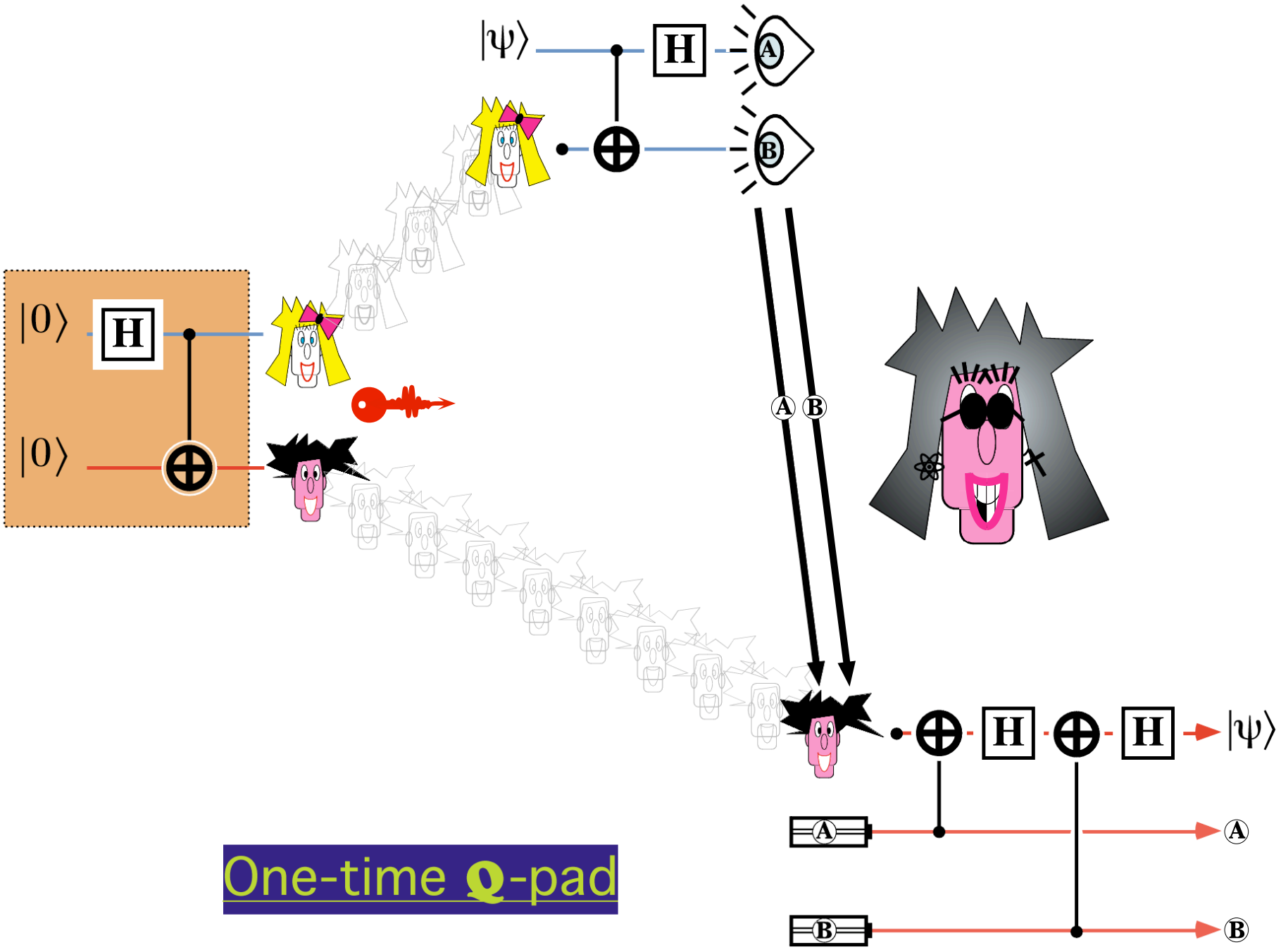




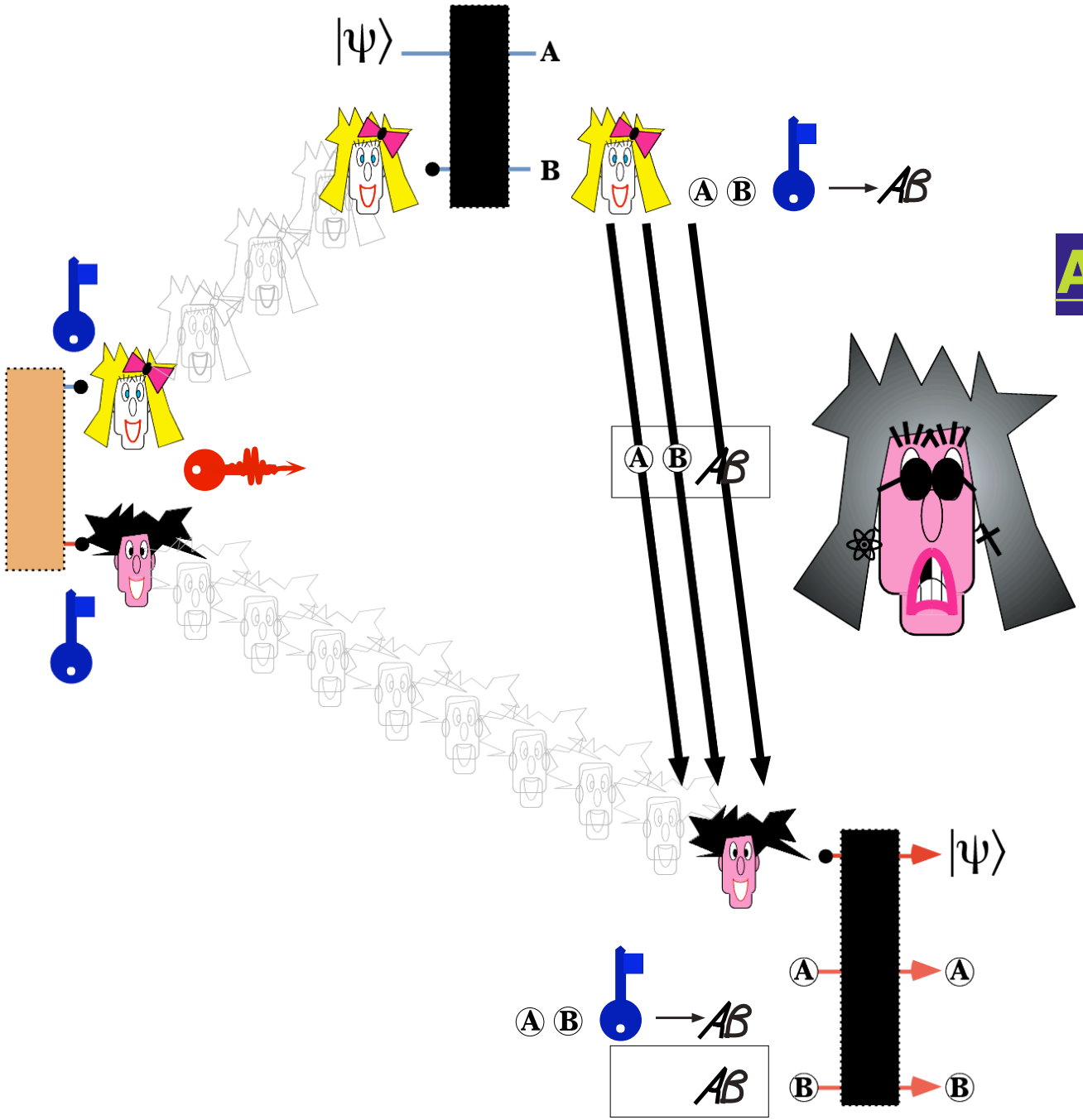


(3.1.3Q) One-time Authenticated Q-pad

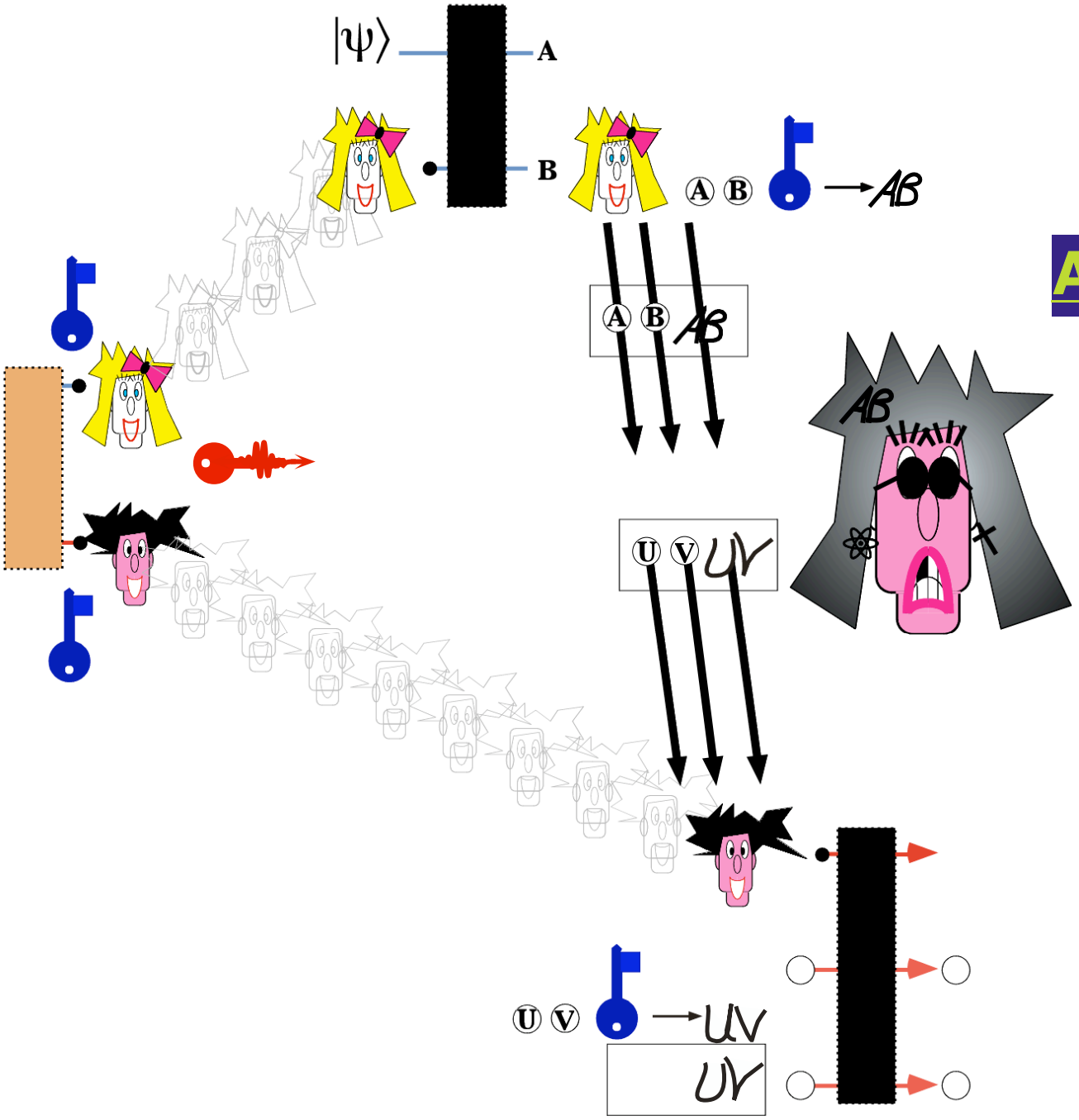
(3.1.3C) One-time Q-Authentication



One-time Q-pad



(3.1.3Q) One-time Authenticated Q-pad



(3.1.3Q) One-time Authenticated Q-pad

(3.1.3Q) One-time Authenticated Q-pad

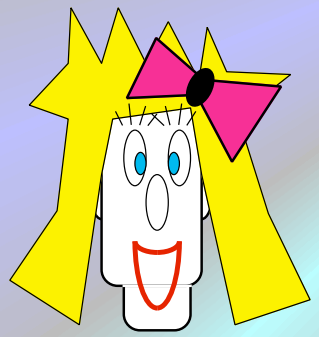
Quantum key : 1x Authenticated Q-pad

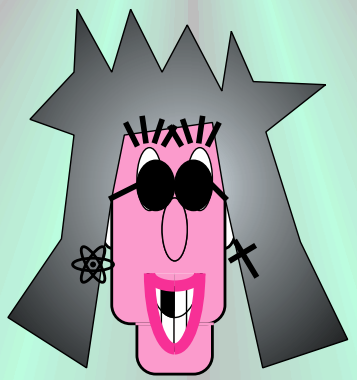
Classical message+tag

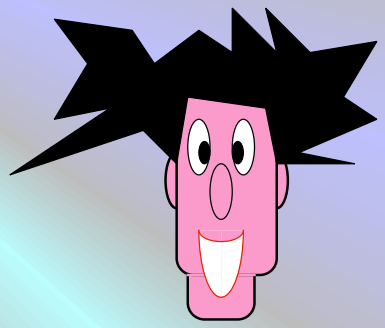
(3.1.3) One-time Authentication

Classical key: 1x Q-Authentication (BCGST)
Quantum message+tag

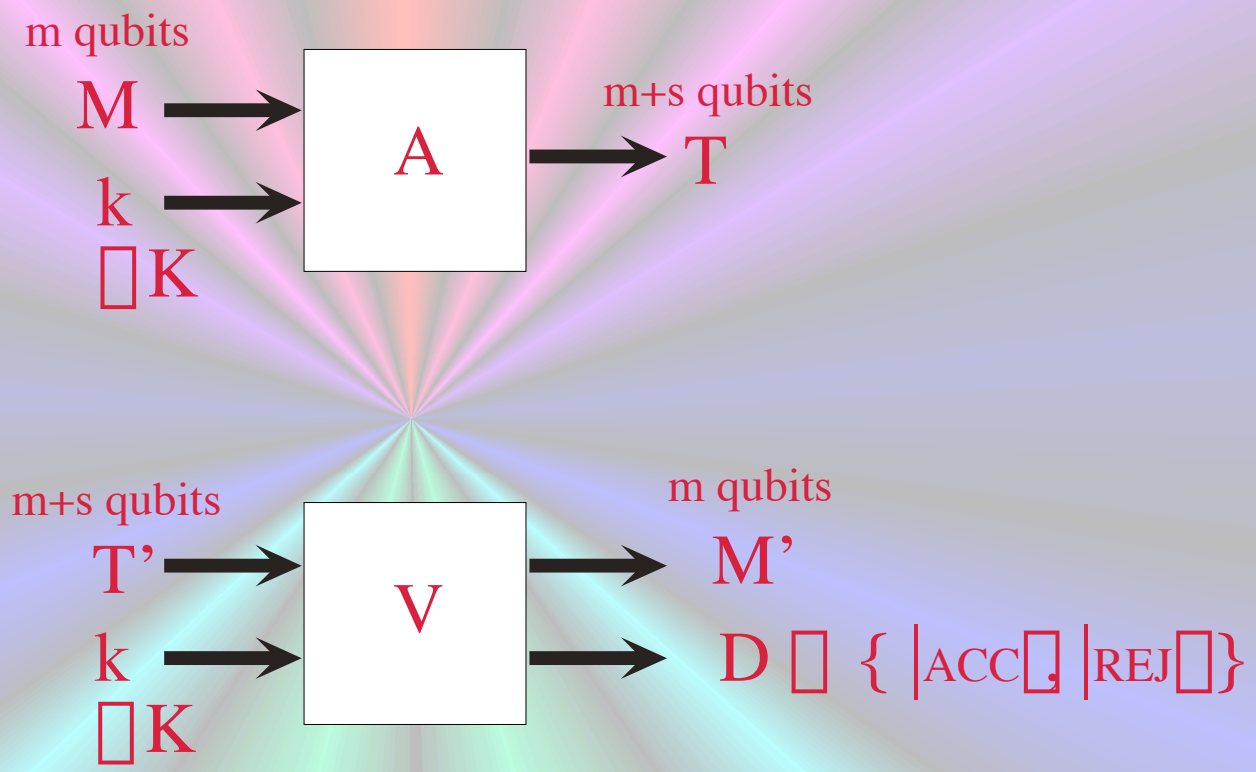
Quantum key : 1x Authenticated Q-pad
Classical message+tag (BBCJPW)







(3.1.3C) One-time Q-Authentication



(3.1.3C) One-time Q-Authentication

For any pure state $|\psi\rangle$ consider the measurement on (M',D) such that

- output Right if M'=$|\psi\rangle$ or if D=$|\mathrm{REJ}\rangle$
- output Wrong otherwise

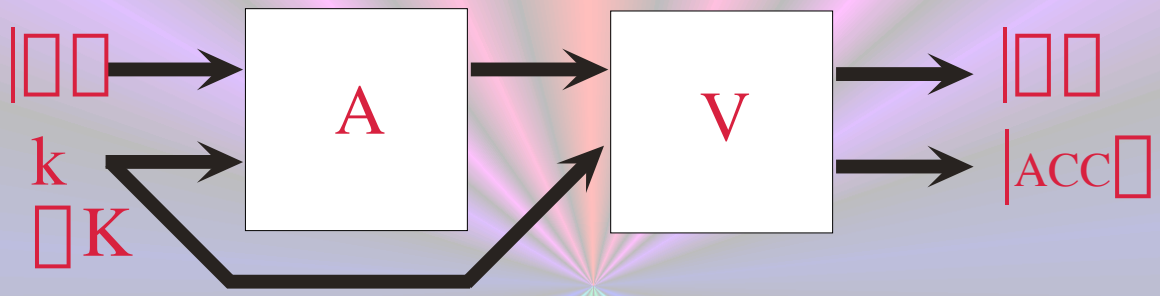
The corresponding projectors are

$$R_{|\psi\rangle} = |\psi\rangle\langle\psi| \otimes I_D + I_{M'} \otimes |\mathrm{REJ}\rangle\langle\mathrm{REJ}| - |\psi\rangle\langle\psi| \otimes |\mathrm{REJ}\rangle\langle\mathrm{REJ}|$$

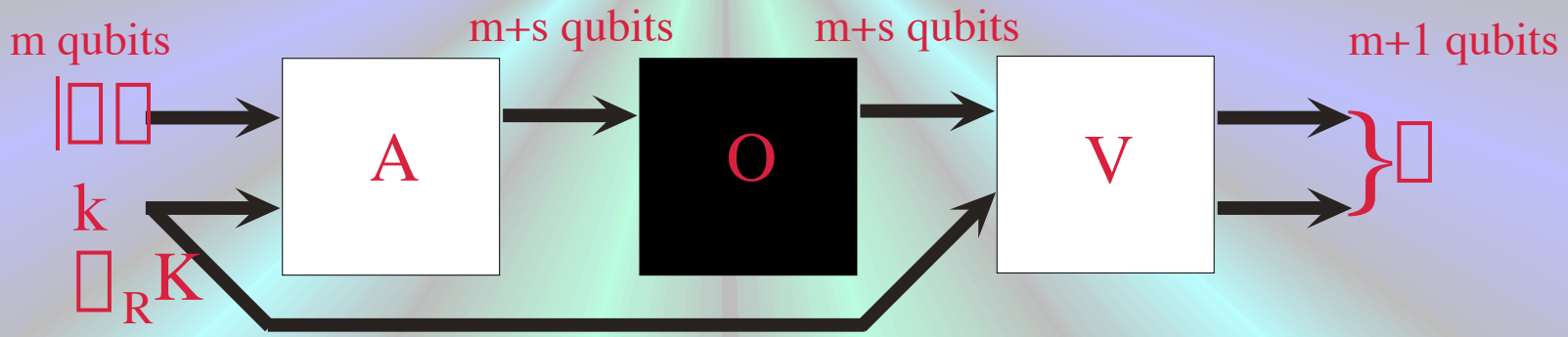
$$W_{|\psi\rangle} = (I_{M'} - |\psi\rangle\langle\psi|) \otimes |\mathrm{ACC}\rangle\langle\mathrm{ACC}|$$

(3.1.3C) One-time Q-Authentication

Completeness:

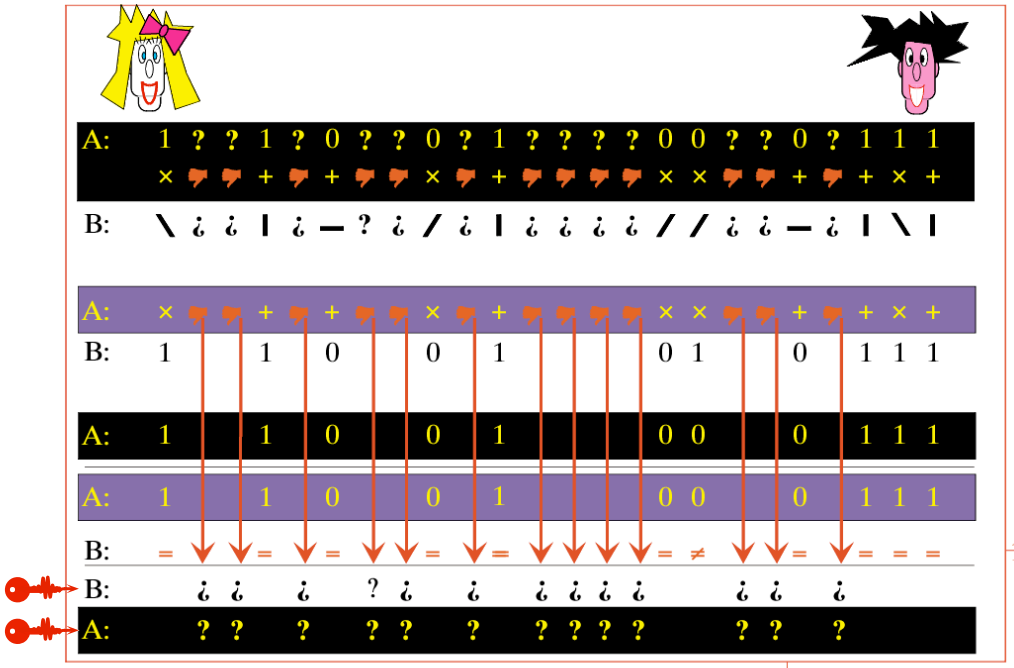


Soundness:



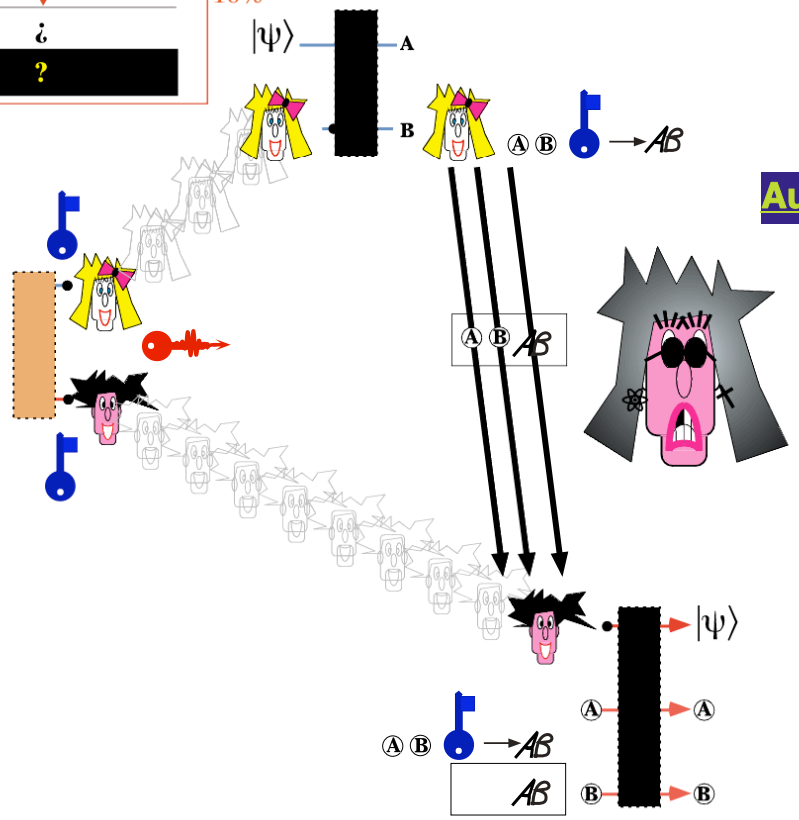
$$Tr(R_{|\Psi\rangle}) \geq 1-2^{-s}$$

(3.1.1Q) Quantum-Key distribution



Shor-Preskill

10%



(3.1.3Q) One-time Authenticated Q-pad

(3.1.3C) One-time *interactive* Q-Authentication

. . . . .

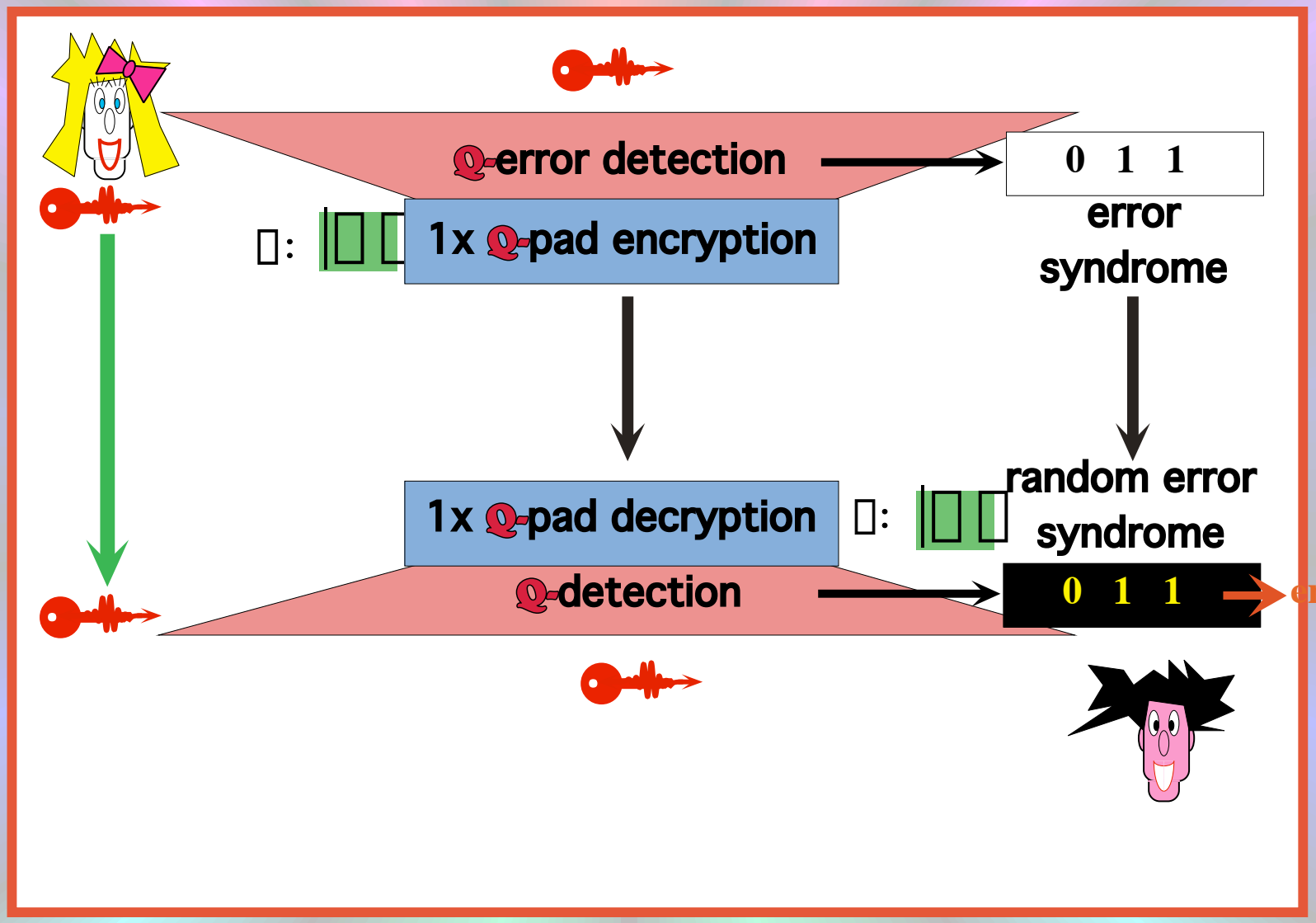
• Transmit quantum key (EPR states)

• Quantum error-correction is used to purify (or test purity of) EPR states to form a smaller pure set

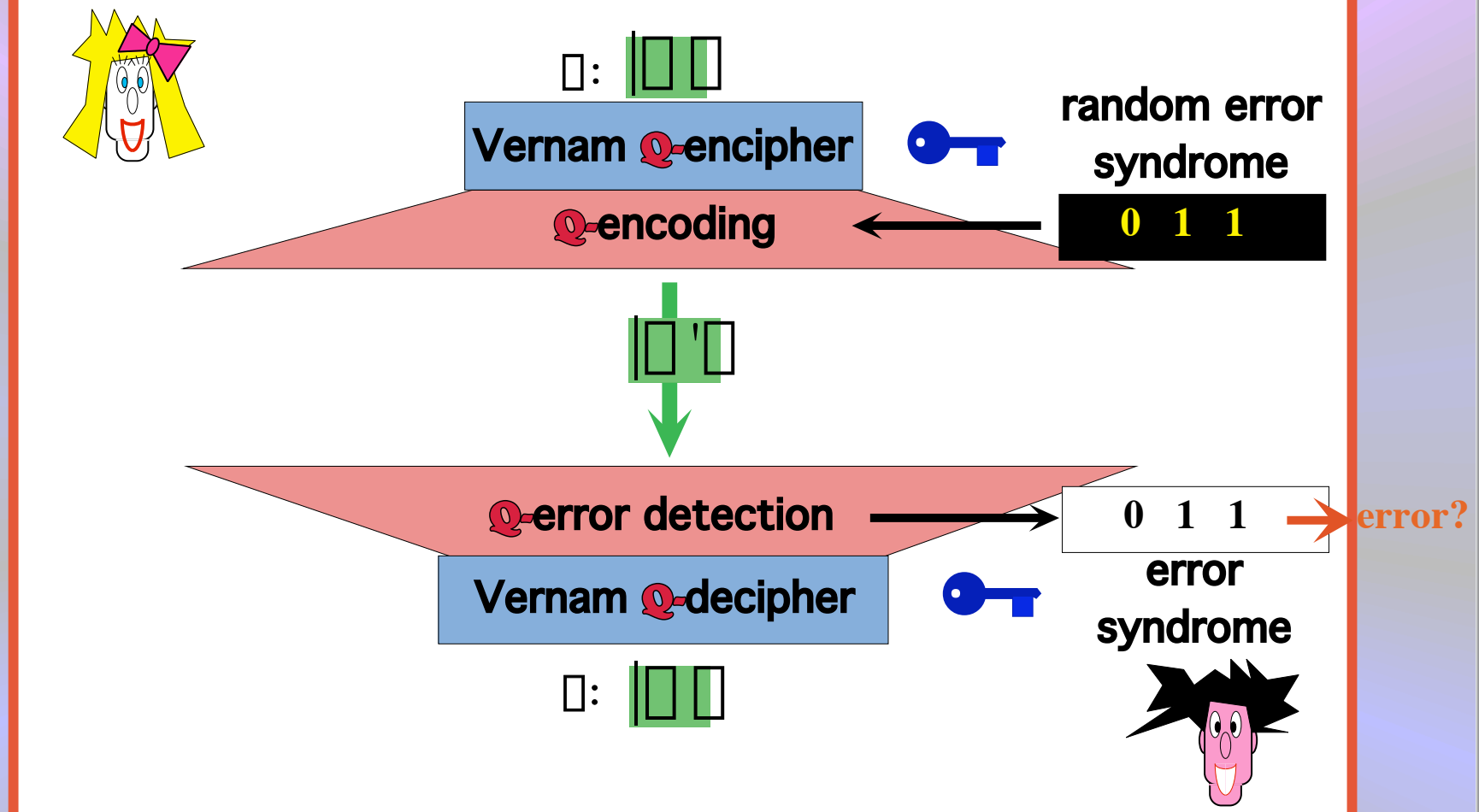
• one-time Authenticated Quantum pad is used to send message

. . . . .

(3.1.3C) One-time *interactive* Q-Authentication



(3.1.3C) One-time Q-Authentication



Barnum-Crépeau-Gottesman-Smith-Tapp

(3.1.3C) One-time Q-Authentication

. . . . .

• Quantum Vernam cipher message

• encode using Quantum error-correction with random syndrome

• Transmit result

. . . . .

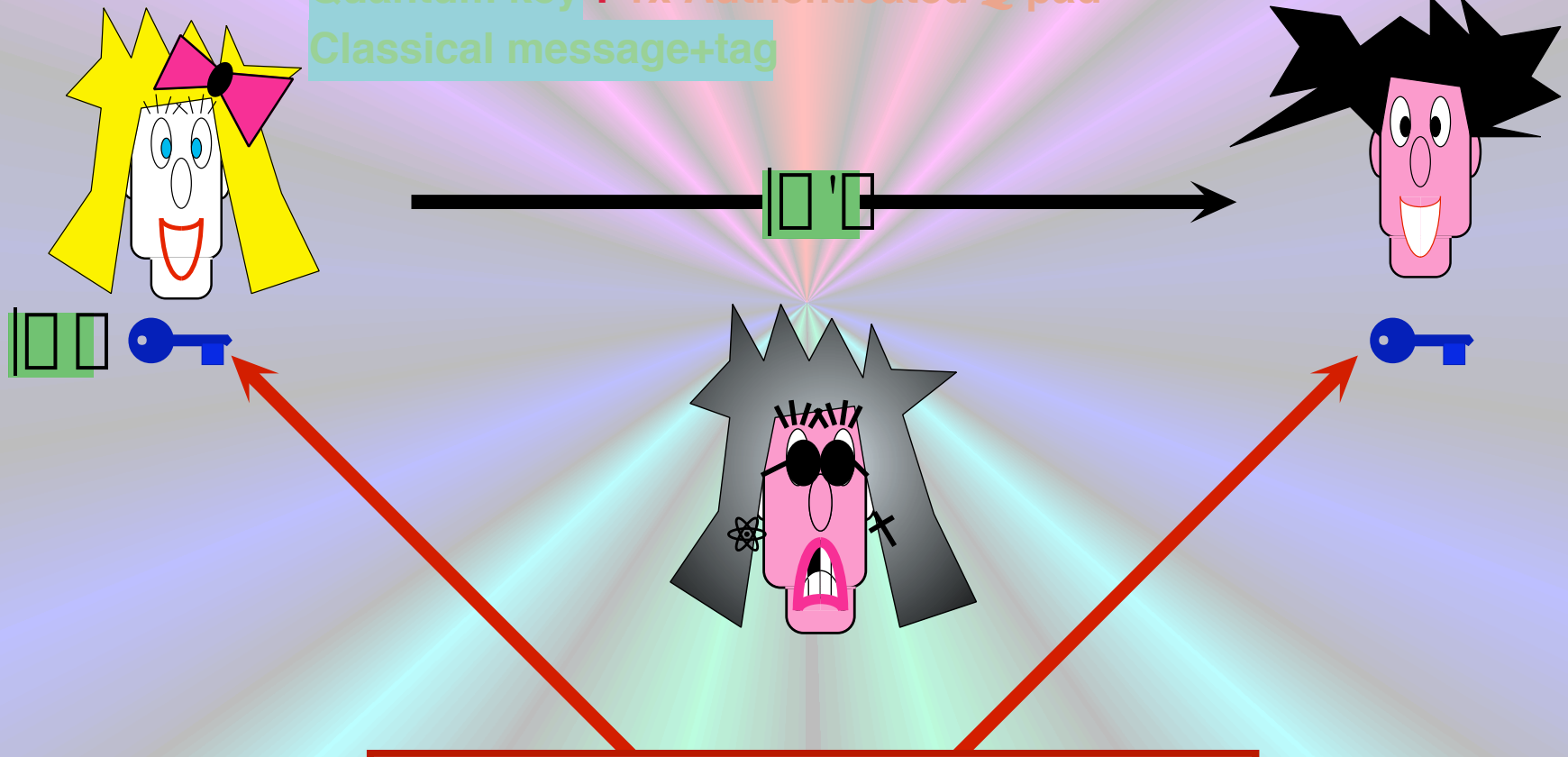
(3.1.3C) One-time Q-Authentication

Classical key : 1x Q-authentication

Quantum message+tag

Quantum key : 1x Authenticated Q-pad

Classical message+tag



- Q-error-correcting code
- secret key for encryption & syndrome

Barnum-Crépeau-Gottesman-Smith-Tapp

one-time Q-authentication

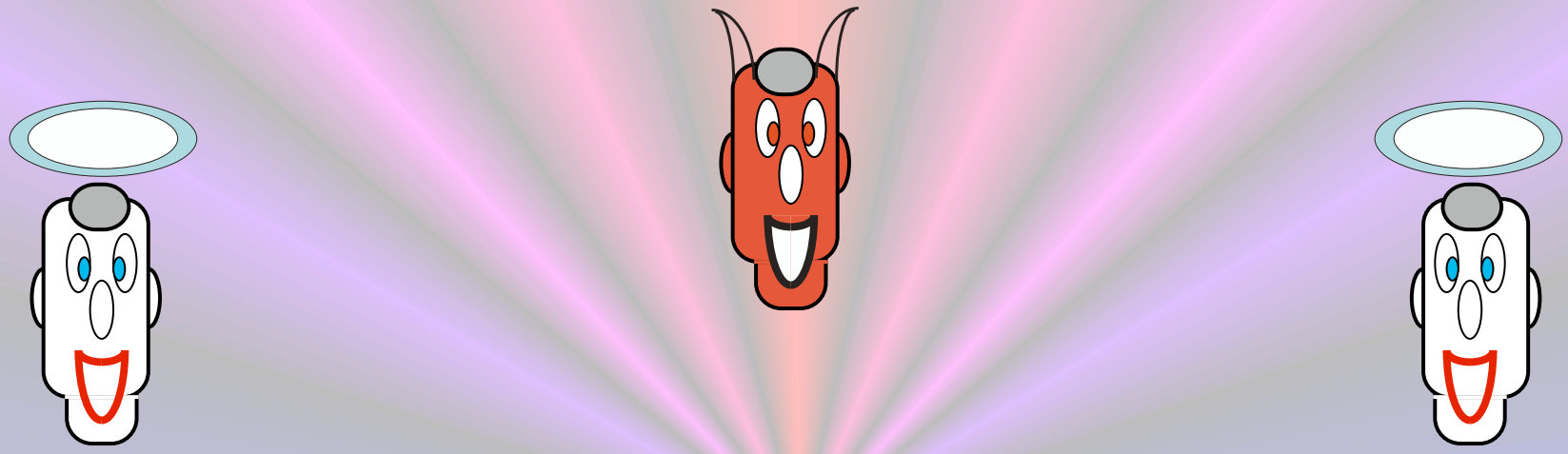
⟹

Vernam Q-cipher

(authenticated messages must be encrypted ;
this is false with classical messages!)

(3.2)
Complexity Theoretical Quantum Cryptography

(3.2) Complexity Theoretical Cryptography



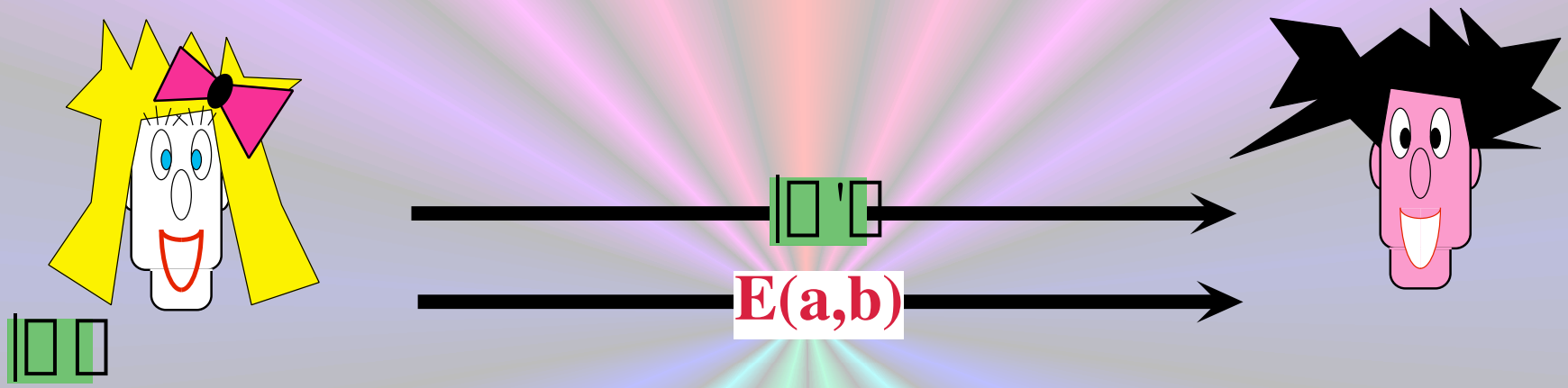
(3.2.1) Public key cryptosystem : public-key $\mathbb{Q}$-cryptosystem

(3.2.2) Digital signature scheme : public-key $\mathbb{Q}$-Authentication

$\mathbb{Q}$-digital signature scheme

(3.2.1) Public-Key Q-Cryptosystem

Assuming Classical Public Key Cryptography

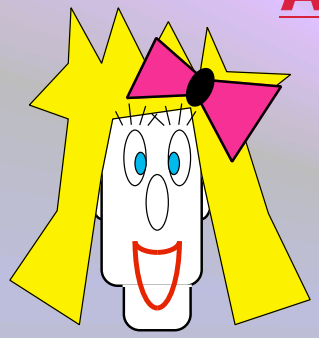


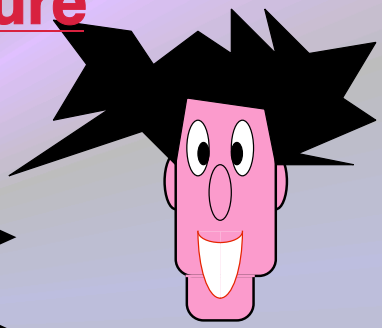
a,b random bits

(a,b) := D(E(a,b))

(3.2.2A) Public-Key Q-Authentication

Assuming Classical Public Key Cryptography
Assuming Classical Digital Signature





$|\Box'\Box$

Sign(E(k))

$|\Box\Box$

verify sign(E(k))
k := D(E(k))

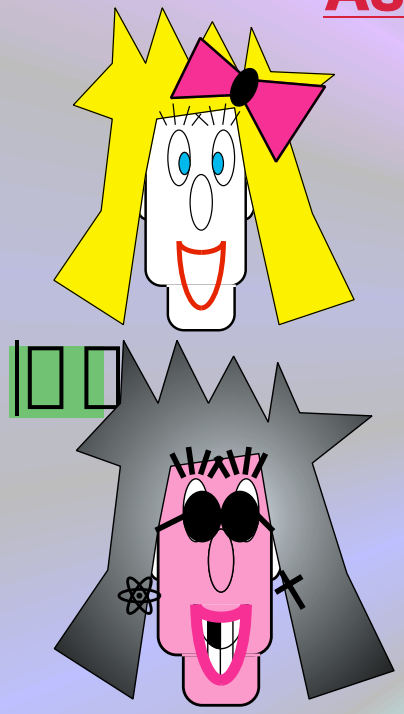
$k \Box_R K$

$|\Box'\Box := A_k(|\Box\Box)$

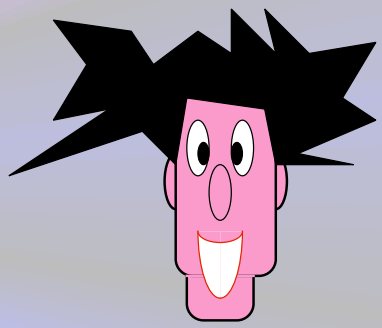
$|\Box\Box := A_k^{-1}(|\Box'\Box)$

(3.2.2S) Q-Digital Signature Scheme

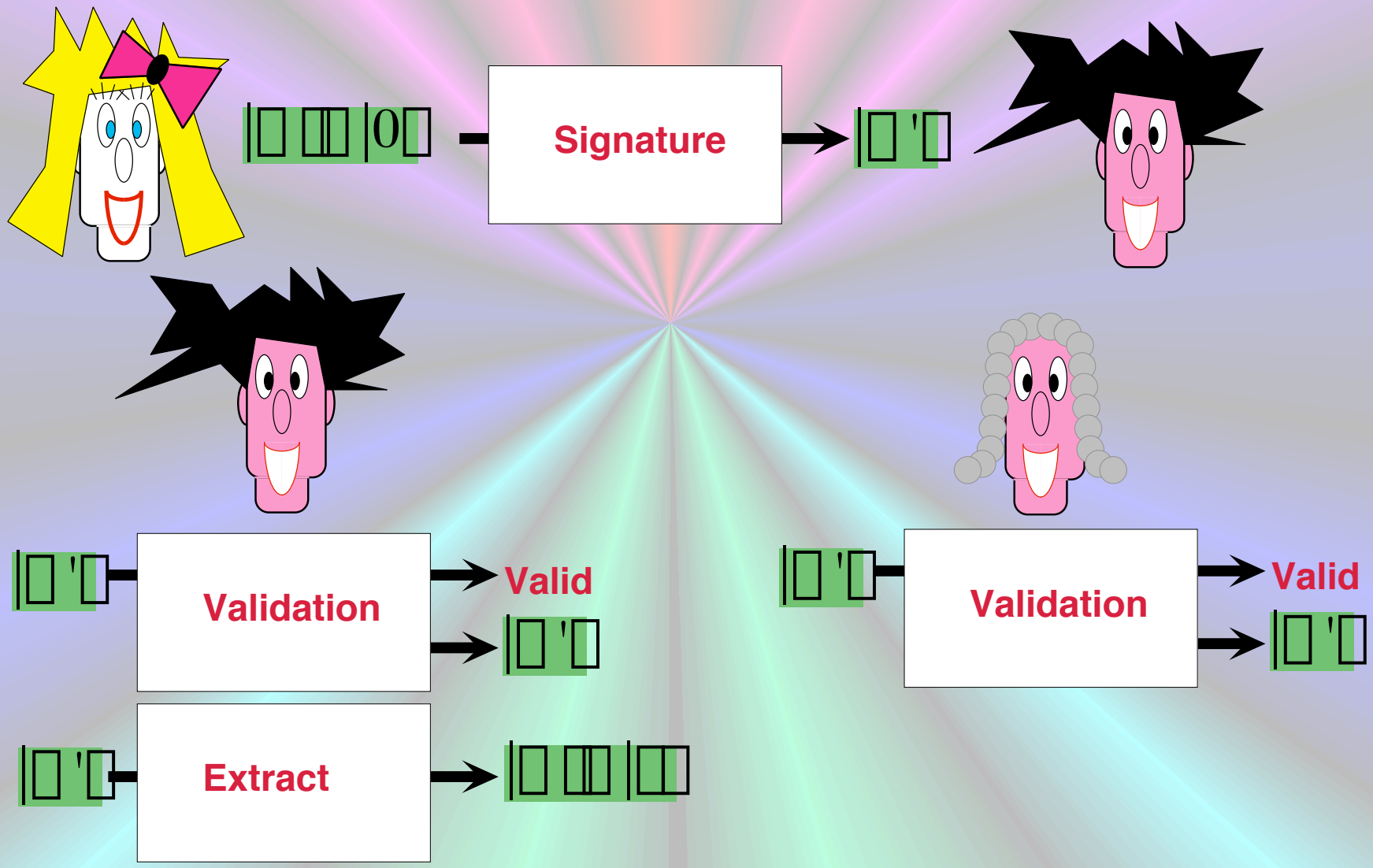
Assuming Classical Public Key Cryptography
Assuming Classical Digital Signature



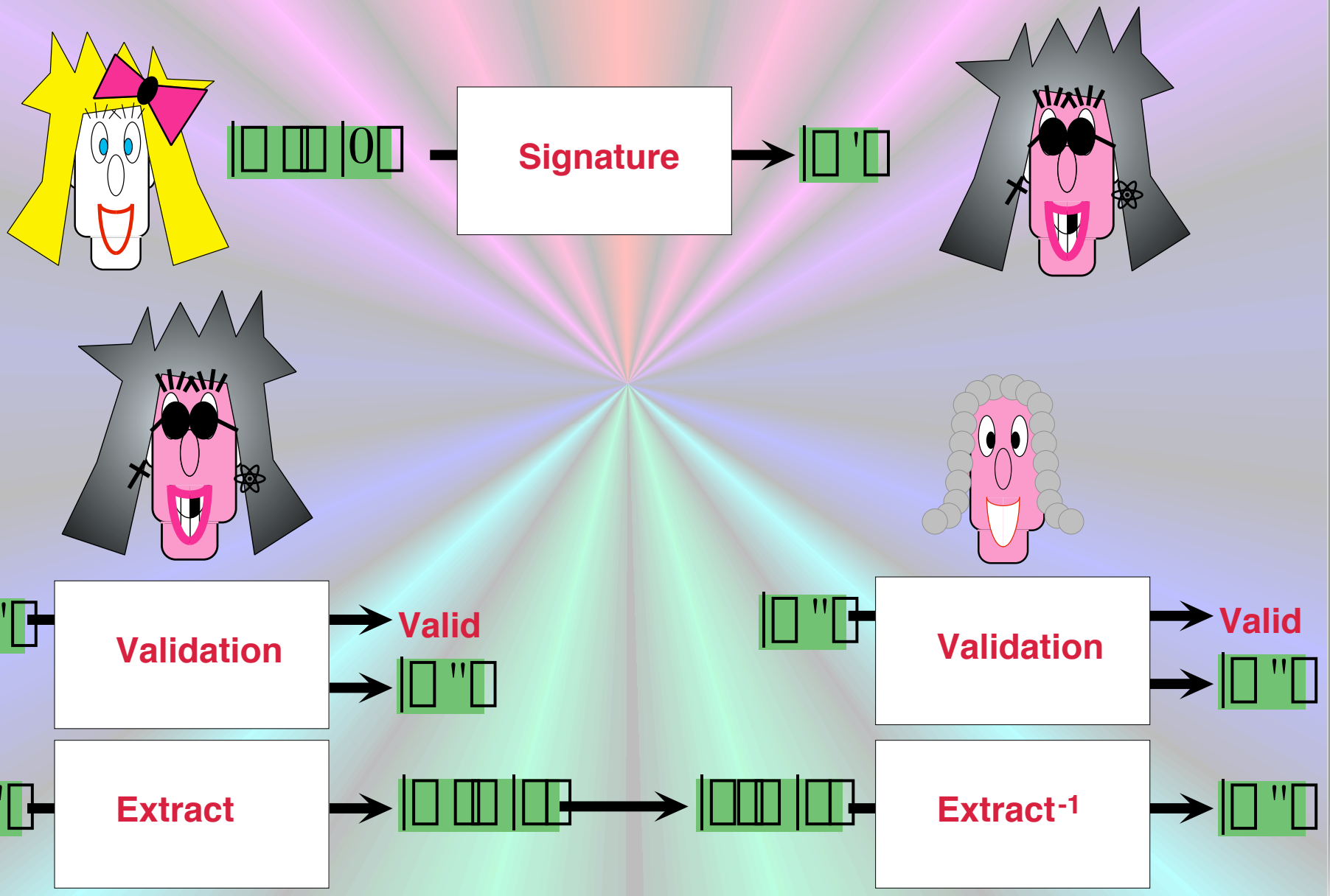
IMPOSSIBLE



(3.2.2S) Q-Digital Signature Scheme



(3.2.2S) Q-Digital Signature Scheme



Further Applications of one-time Q-Authentication

- Uncloneable Encryption
(Gottesman)

- Length n QECC correcting (n-1)/2 arbitrary errors
(with exponentially small probability)
(Crépeau,Gottesman,Smith)

- Achieving classical bounds for VQSS and MPQC
(Crépeau,Gottesman,Smith)

Quantum Authentication

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McGill University