ADIABATIC COMPUTATION: UNIVERSALITY & TOOLS

DORIT AHARONOV (HEBREW U)

JOINT WORK WITH

AMNON TA-SHMA (TEL-AVIV U)

PAPER ON QUANT-PH

QUANTUM ALGORITHMS

ADIABATIC COMPUTATION [FARHI et.al 200

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

e.s: $|\psi(t)\rangle = e^{-iEt}|\psi(0)\rangle$

(UNITARY)

ADIABATIC THEOREM (KATO 51)

$|\psi_0\rangle = |GS(0)\rangle \longrightarrow |\psi_T\rangle = |GS(T)\rangle$

DELAY SCHEDULE.

$$i\hbar \frac{d}{ds}|\psi(s)\rangle = \tau(s)\, H(s)\, |\psi(s)\rangle$$

$s \in [0,1]$

ADIABATIC CONDITION:

$$\tau(s) \gg \left|\frac{d}{ds} H(s)\right| / \Delta^2(H(s))$$

MOST PAPERS: STRAIGHT LINE

$H(s) = (1-s)H_0 + sH_1$

BACKGROUND - ADIABATIC

- NUMERICAL RESULTS FOR NPC PROBLEM

  [FARHI, GUTMANN, GOLDSTONE, SIPSER
  LAPAN, LANDGREN, PREDA,.. 2000-200

- LOWER BOUNDS

  [VAN DAM, MOSCA, VAZIRANI 2001]

- GROVER'S SPEED UP ADIABATICALLY

  [V. DAM, MOSCA, VAZIRANI 2001
  ROLAND, CERF 2001]

- ADIABATIC VS. SIMULATED ANNEALING

  [FARHI, GOLDSTONE, SIPSER 2002]

HOW POWERFUL IS ADIABATIC COMPUTATION? WHAT CAN BE DONE?

OUR ADIABATIC RESULTS

1) **ADIABATIC ≡ CIRCUIT MODEL**

WHAT PROBLEMS? STATE GENERATION

2) **SZK ⟶ QSAMPLING** $\sum_i \sqrt{p_i}|i\rangle$

WHAT STATES CAN WE GENERATE?

MARKOV CHAINS & APPROX COUNTING

3) **APPROX COUNTING ⇒ QSAMPLING**

- PERFECT MATCHING
- CONVEX BODY

TOOLS:

WHAT HAMILTONIANS?

THE "SPARSE HAMILTONIAN" LEMMA:

H SPARSE, ROW-COMPUTABLE ⇒ SIMULATABLE

HOW TO GUARANTEE LARGE Δ?

THE "JAGGED ADIABATIC PATH" LEMMA:

ADIABAT ON  $\langle\alpha_i|\alpha_{i+1}\rangle > \frac{1}{\text{poly}(n)}$

ADIABATIC ≡ CIRCUIT MODEL [A. 02]

GIVEN: $U_1, \cdots U_T$ (GATES)

$x_1 \cdots x_n$ (INPUT STRING)

$$|x, 0\rangle \xrightarrow[\text{ADIABAT}]{} U_T \cdots U_1 |x, 0\rangle$$

$$H_0 = \Pi_{|x,0\rangle^\perp}$$

$$H_1 = \Pi_{U_T \cdots U_1 |x,0\rangle^\perp} = U_T \cdots U_1 H_0 U_1^+ \cdots U_T^+$$

LITTLE LEMMA?:

$$\Pi_{|\alpha\rangle^\perp} \longrightarrow \Pi_{|\beta\rangle^\perp}$$

$$\Delta_{min} = |\langle\alpha|\beta\rangle|$$

BUT THIS MIGHT BE 0!

ADIABATIC ≡ CIRCUIT MODEL (CONTD)

GIVEN: $U_1 \cdots U_T$ (GATES)

$X_1 \cdots X_n$ (INPUT STRING)

INSTEAD:

$$|x,0\rangle \longrightarrow |h_x\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} U_t \cdots U_1 |x\rangle \otimes |t\rangle$$

$$H_0 = (I - |x\rangle\langle x|) \otimes I + I \otimes (I - |0\rangle\langle 0|)$$

$$H_1 = (I - |x\rangle\langle x|) \otimes |0\rangle\langle 0| + H_{prop}$$

$$H_{prop} = \frac{1}{2} \sum_t \left[I \otimes |t\rangle\langle t| + I \otimes |t-1\rangle\langle t-1| - U_t \otimes |t\rangle\langle t-1| - U_t^\dagger \otimes |t-1\rangle\langle t| \right]$$

LITTLE LEMMA 2:
NON NEGLIGIBLE Δ $\Rightarrow$ PROJECTION

BUT WHY IS $\Delta(H_{prop})$ NON NEGLIGIBLE?

ADIABATIC ≡ CIRCUIT MODEL (conti

$$H_{prop} = \frac{1}{2} \sum_t \Big[I \otimes |t\rangle\langle t| + I \otimes |t-1\rangle\langle t-1| - U_t \otimes |t\rangle\langle t-1| - U_t^\dagger \otimes |t-1\rangle\langle t| \Big]$$

$$H'_{prop} = I \otimes \begin{pmatrix} 1 & -\frac{1}{2} & & & 0 \\ -\frac{1}{2} & 1 & \ddots & & \\ & \ddots & \ddots & -\frac{1}{2} & \\ & & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & & & -\frac{1}{2} & 1 \end{pmatrix} = I \otimes (I - M)$$

M IS MARKOV CHAIN MATRIX

FOR RANDOM WALK ON $[0, \ldots, T]$

RANDOM WALKS MIX TO LIMITING DIST

THM: [ALON] MIXING TIME $\sim \frac{1}{1-\lambda_2}$

MIX IN TIME $O(T^2) \longrightarrow \Delta > \frac{1}{T^2}$

ADIABATIC ≡ CIRCUIT MODEL (cont'd)

LITTLE LEMMA 1: (two PROJECTIONS)

$$H(s) = (1-s)(I - |\alpha\rangle\langle\alpha|) + s(I - |\beta\rangle\langle\beta|)$$

$$\rightarrow \Delta(H(s)) \geq |\langle\alpha|\beta\rangle|$$

LITTLE LEMMA 2: (HAMILTONIAN → PROJECTION)

IF H CAN BE SIMULATED,

AND $\Delta(H)$ NON NEGLIGIBLE

$$\Downarrow$$

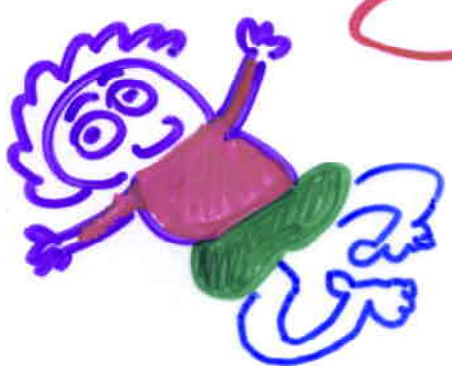
$I - |gs\rangle\langle gs| = \Pi_{\overline{gs}}$ CAN BE SIMULATED

TO FINISH PROOF:

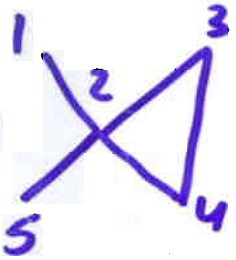
$$H_{prop} \xrightarrow{2} \Pi_{prop} \xrightarrow{1} \Delta(H_1) \text{ IS N.N}$$

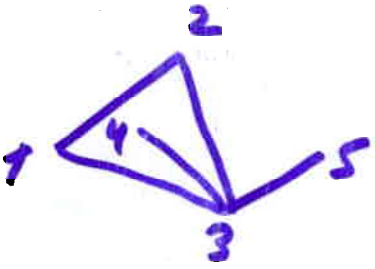
$$\Delta(H_s) \text{ N.N} \xleftarrow{2} \Pi_H$$

WHAT PROBLEMS TO ATTACK ADIABATICALLY ?



GRAPH ISOMORPHISM

$G_0 =$ 

$G_1 =$ 

$$A_{G_0} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad A_{G_1} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

INPUT: G_0, G_1 on n nodes

OUTPUT: IS THERE $\sigma \in S_n$ s.t $\sigma(G_0) = G_1$?

REDUCIBLE TO:

INPUT: $|G\rangle$

OUTPUT $|G\rangle|\alpha_G\rangle$

$$|\alpha_G\rangle = \frac{1}{\sqrt{n!}} \sum_{\sigma \in S_n} |\sigma(G)\rangle$$

SIMPLE ALGORITHM:

$A_0 \longrightarrow |\alpha_0\rangle$

$A_1 \longrightarrow |\alpha_1\rangle$

$\langle\alpha_0|\alpha_1\rangle = ?$

$$\frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \xrightarrow{A} \frac{|0\rangle|\alpha_0\rangle + |1\rangle|\alpha_1\rangle}{\sqrt{2}}$$

$$H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad |0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\longrightarrow \frac{(|0\rangle+|1\rangle)|\alpha_0\rangle + (|0\rangle-|1\rangle)|\alpha_1\rangle}{2}$$

$$= |0\rangle\left(\frac{|\alpha_0\rangle+|\alpha_1\rangle}{2}\right) + |1\rangle\left(\frac{|\alpha_0\rangle-|\alpha_1\rangle}{2}\right)$$

$$Pr(0) = \left|\frac{|\alpha_0\rangle + |\alpha_1\rangle}{2}\right|^2 = \frac{1 + \langle\alpha_0|\alpha_1\rangle}{2}$$

SIMPLE ALGORITHM ?

HOW TO GENERATE

$$|\alpha_G\rangle = \sum_{G \in S_n} |G(G)\rangle \quad ?$$

CAN SAMPLE A RANDOM $G(G)$!

CAN GENERATE

$$\sum_{G \in S_n} |G\rangle \otimes |G(G)\rangle$$

HOW TO FORGET ?

QUANTUM SAMPLING

INPUT : C (A CIRCUIT)

ON UNIFORMLY DISTRIBUTED

x'

$C(x') = x \sim \pi(x)$.

WANTED : $|C\rangle = \sum \sqrt{\pi(x)}\, |x\rangle$

↑

THE QUANTUM SAMPLING OF THE DISTRIBUTION π.

QUANTUM SAMPLING & SZK

SZK $\Longrightarrow$ QSAMPLING

WHAT KIND OF STATES CAN WE GENERATE ?

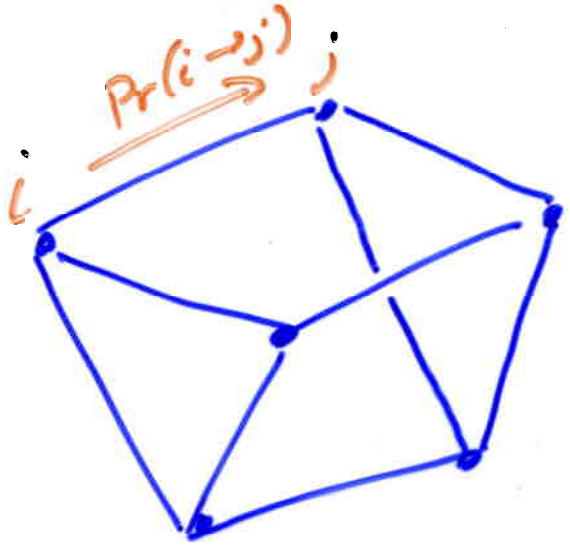
WHAT DISTRIBUTIONS ?

APPROXIMATE COUNTING => QSAMPLING

CAN EFFICIENTLY APPROXIMATE

- ALL PERFECT MATCHINGS IN A BIPARTITE GRAPH G
- $\sum \sqrt{\pi(a)}\,|a\rangle$ IF $\pi(a)$ IS LOG CONCAVE AND EASY TO COMPUTE
- ALL GRID POINTS INSIDE A CONVEX BODY IN HIGH DIM
- ALL EXTENTIONS OF A GIVEN PARTIAL ORDER
- SUPER POSITION OVER THE GIBBS DISTRIBUTION FOR VARIOUS STAT. MECH MODELS (POTTS ISING, ETC ..)

MARKOV CHAINS



STATE SPACE = Ω

$|\Omega| = EXP(n)$

M_{ij} = TRANSITION MATRIX

$$P_t = M P_{t-1}$$

π = LIMITING DISTRIBUTION

$$\pi = \lim_{t \to \infty} M^t P_0 \qquad \forall P_0$$

RAPID MIXING IF $\lambda_2 < 1 - \frac{1}{n^c}$

(LARGE SPETRAL GAP)

APPROXIMATE COUNTING

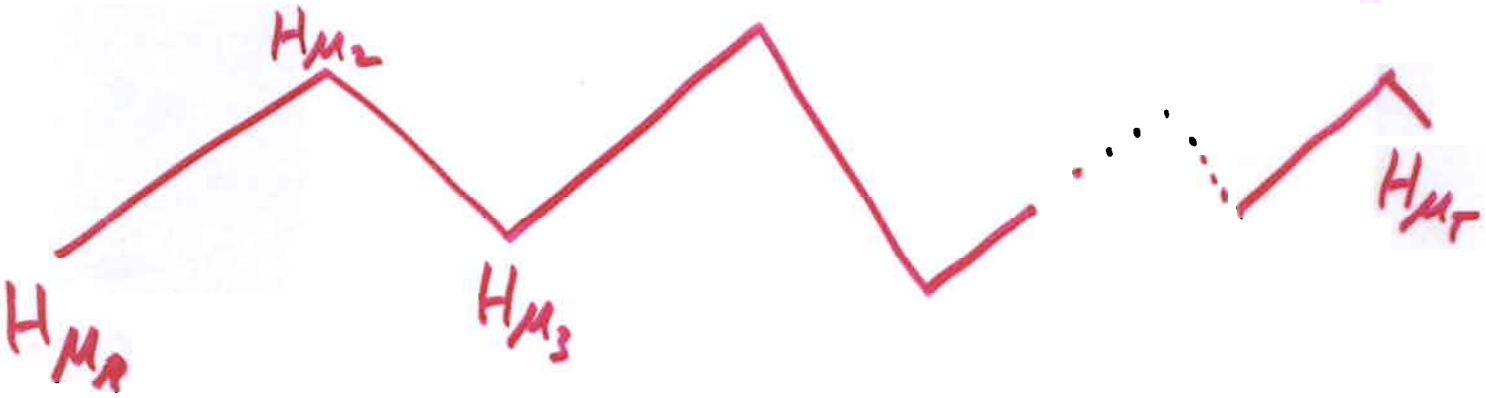
SEQUENCES OF RAPIDLY MIXING MARKOV CHAINS

THM:

APPROX COUNTING → QSAMPLING

MARKOV CHAINS ARE "STRONGLY SAMPLABLE"
(SPARSE, ROW COMPUTIBLE, REVERSIBLE,
& EASY TO CALCULATE $\pi(i)/\pi(j)$)

PROOF OF THEOREM:



WALK ADIABATICALLY ON THE BROKEN LINE CONNECTING THE H_{M_t} IN HAMILTONIAN SPACE

IF 1) Δ_{min} poly big

2) $\frac{dH}{ds}$ poly big

3) CAN APPLY $e^{iH(s)\Delta t}$ BY A QC

HAMILTONIANS FROM MC's

HAMILTONIAN : HERMITIAN MATRIX

$$H \longleftrightarrow M$$

$$\pi(i) M_{ij} = \pi(j) M_{ji} \qquad \text{REVERSIBLE}$$

$$\begin{pmatrix} \frac{1}{\sqrt{\pi_1}} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\sqrt{\pi_N}} \end{pmatrix} M \begin{pmatrix} \sqrt{\pi_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\pi_N} \end{pmatrix}$$

$$H = I - \frac{1}{\sqrt{\pi}} M \sqrt{\pi}$$

$$\lambda \longleftrightarrow 1-\lambda$$

$$\sum \sqrt{\pi(x)}\,|x\rangle \longleftrightarrow \pi \qquad \text{LIMITING DIST}$$

GROUND STATE

THE SPARSE HAMILTONIAN LEMMA

H IS SPARSE, $\forall i$ $H_{ij} \neq 0$ CAN BE COMPUTED EFFICIENTLY

e^{iHt} CAN BE APPROXIMATED TO WITHIN POLY ACCURACY EFFICIENTLY

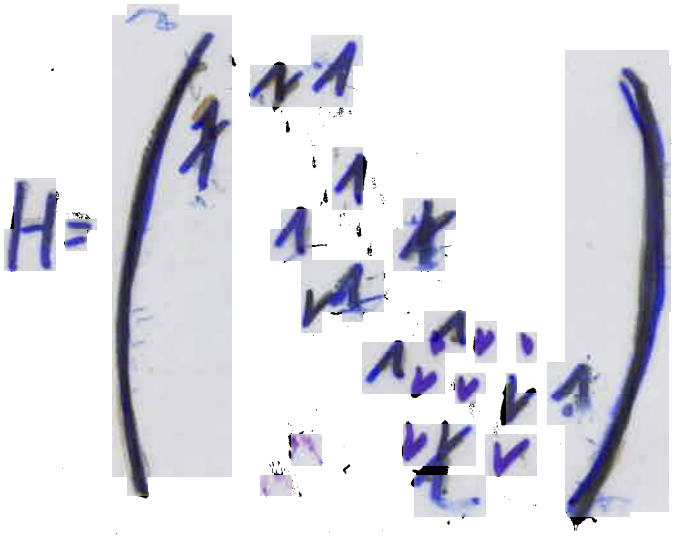
(IMPLICATIONS TO QWALKS ETC.)

PROOF (OF APPLYING A SPARSE H)

1. WRITE $H = \sum_{b=1}^{poly} H_b$.

TROTTER: $e^{i(A+B)} \approx \left(e^{i\frac{A}{r}} e^{i\frac{B}{r}}\right)^r$

$$e^{iH\Delta t} = e^{i\sum_{b=1}^{poly} H_b \Delta t} \approx \left[\prod_{b=1}^{poly} e^{i\frac{H_b \Delta t}{r}}\right]^r$$

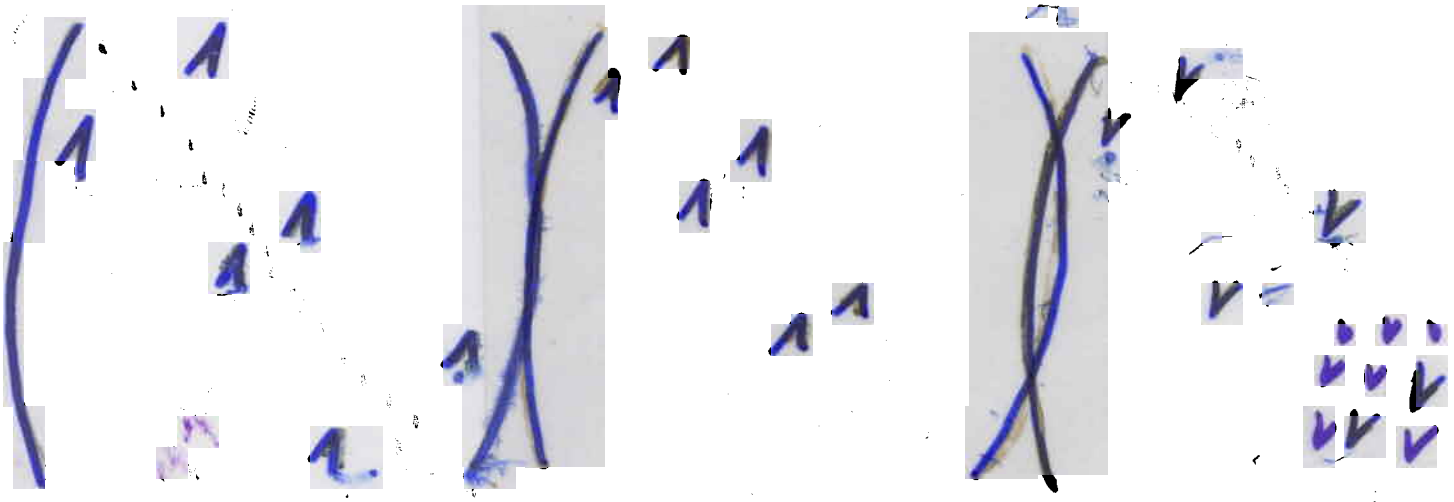
$H =$ 

PROOF (OF APPLYING A SPARSE H)

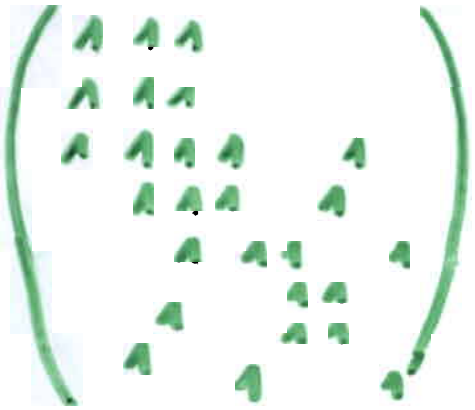
1. WRITE $H = \sum_{b=1}^{poly} H_b$.

TROTTER: $e^{i(A+B)} \approx \left(e^{i\frac{A}{r}} e^{i\frac{B}{r}}\right)^r$

$$e^{iH\Delta t} = e^{i\sum_{b=1}^{poly} H_b \Delta t} \approx \left[\prod_{b=1}^{poly} e^{iH_b \frac{\Delta t}{r}}\right]^r$$

$H =$ 

DECOMPOSING H

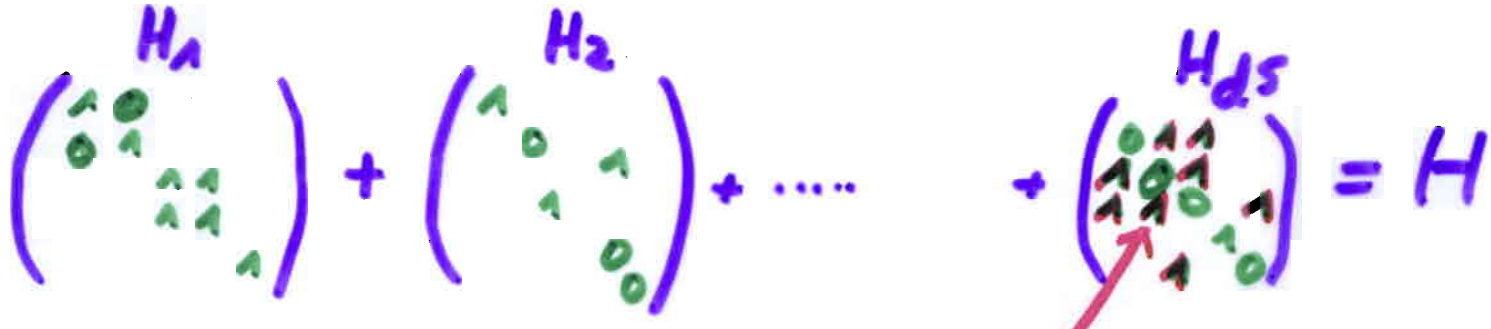


$deg = d$

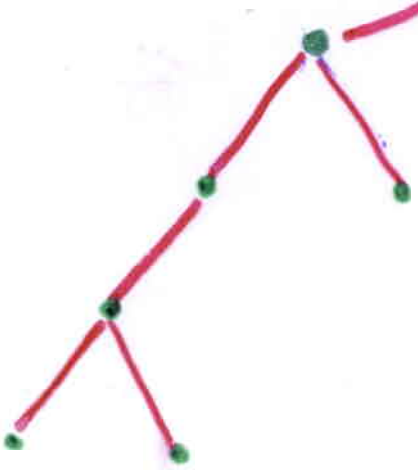
$\#\ colors = d^5$

EACH ELEMENT $H_{i,l}$ PICKS A RANDOM COLOR $b \in [1, \dots d^5]$

$H_1 + H_2 + \dots + H_{d^5} = H$



WHAT IS THE SIZE OF A BLOCK?



BRANCHING PROCESS ; $Pr(\#\ SONS = j) < \frac{1}{c^j}$

LAST DETAIL : HASH FUNCTIONS

PROBLEM: COLORS CANNOT BE CHOSEN RANDOMLY

INCONSISTENCY, DECOHERENCE, EXP MANY...

SOLUTION: CHOOSE A RANDOM HASH

$$F_{q^r} \ni X \longrightarrow Tr\left(\sum_{i=1}^{d^{10}} c_i X^i\right) \in [1, \cdots q]$$

CONCLUSIONS & OPEN PROBLEMS

- ADIABATIC UNIVERSAL

  NEW TOOLS ?

  PHYSICISTS METHODS...

  LOCAL UNIVERSALITY ?

  (FAULT TOLERANCE.
  K-COLORABILITY EQMA)

- IMPORTANCE OF Q SAMPLING

  NEW ALGORITHMS ?

  OLD ALGORITHMS ?

- TOOLS

  OTHER TOOLS ?

  PHYSICS.

  MARKOV CHAINS: CONDUCTANCE,
  LOG SOBOLEV,
  FLOWS...