MULTI-PARTY ENTANGLEMENT

- D. ROBERTS, S. POPESCU, V. SCARANI
- N. LINDEN, S. POPESCU, W. WOOTTERS
- C. H. BENNETT, S. POPESCU, D. ROHRLICH, J. SMOLIN, A. THAPLIAL

QUANTUM ENTANGLEMENT

(PURE STATES)

- DIRECT PRODUCT (NON-ENTANGLED)

$$|\psi\rangle = |\phi\rangle_A |\chi\rangle_B |\lambda\rangle_C$$

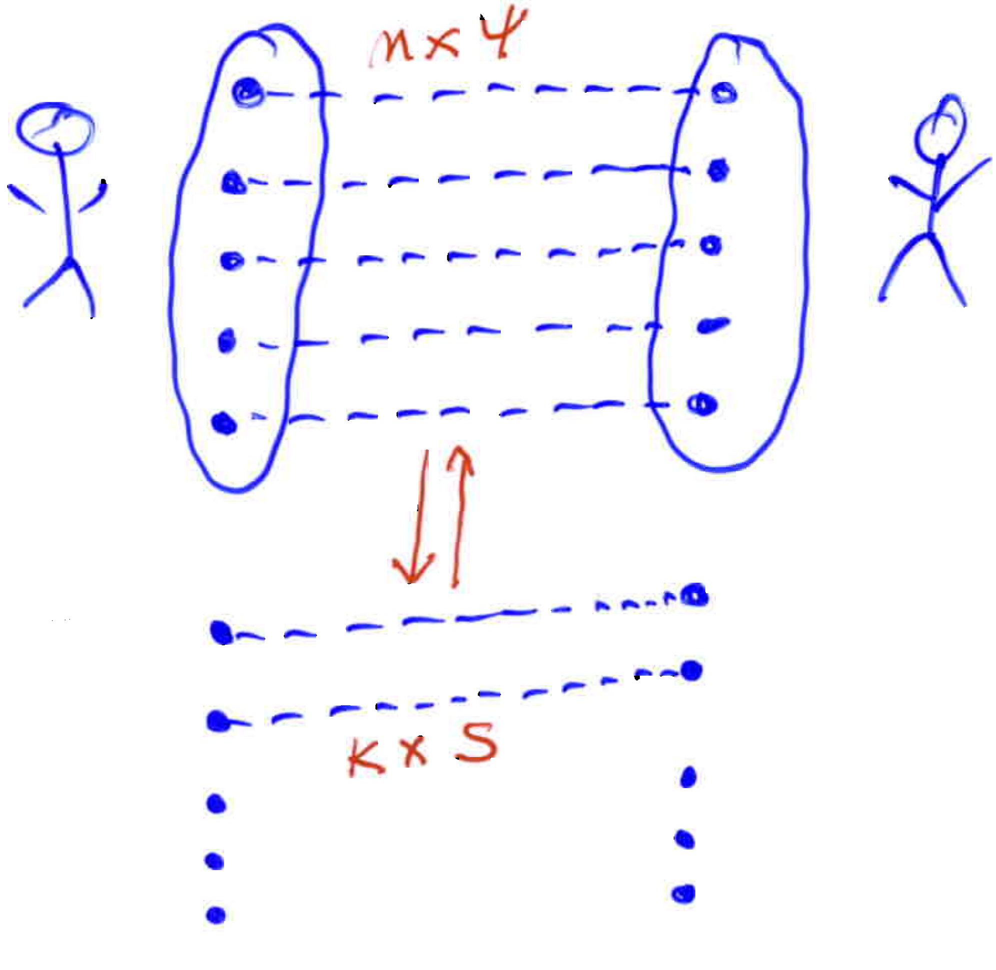
- ENTANGLED : NOT A DIRECT PRODUCT

EXP: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$

TWO-PARTY ENTANGLEMENT

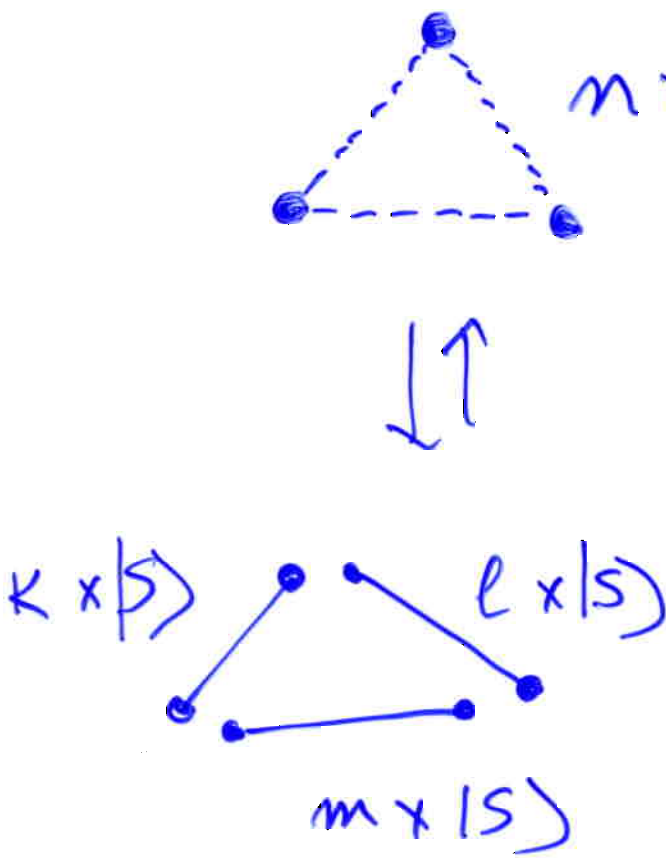
(PURE STATES)

· COMPLETELY SOLVED



$$S = \frac{1}{\sqrt{2}} \left(|0\rangle|0\rangle + |1\rangle|1\rangle \right)$$

CONCENTRATING 3-PARTY ENTANGLEMENT



$$|S\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

IMPOSSIBLE

MAXIMAL REVERSIBLE ENTANGLEMENT GENERATING SETS

MREGS

$$n \times |\psi\rangle \overset{?}{\rightleftarrows} \left\{ \begin{array}{ll} K_{AB} & |S\rangle_{AB} \\ K_{BC} & |S\rangle_{BC} \\ K_{CA} & |S\rangle_{CA} \\ K_{GHZ} & |GHZ\rangle_{ABC} \\ \vdots & \end{array}\right.$$

$$\alpha\, \uparrow\uparrow\uparrow + \beta\, \downarrow\downarrow\downarrow \qquad\qquad \alpha\, \uparrow\uparrow + \beta\, \downarrow\downarrow$$

$$n H(\alpha^2) \quad \frac{\uparrow\uparrow\uparrow + \downarrow\downarrow\downarrow}{\sqrt{2}} \qquad\qquad n H(\alpha^2) \quad \frac{\uparrow\uparrow + \downarrow\downarrow}{\sqrt{2}}$$

$$\alpha\, \uparrow_A \psi^+_{BC} + \beta\, \downarrow_A \psi^-_{BC} \qquad\qquad \frac{\uparrow\downarrow \pm \downarrow\uparrow}{\sqrt{2}}$$

$$P_S \le \frac{3}{4} \qquad\qquad P_S \le \frac{6}{8}$$

$$P_S^{QM} \le \frac{2+\sqrt{2}}{4} \qquad\qquad P_S^{QM} \le \frac{4+2\sqrt{2}}{8}$$

$$e^{X(x,y) + Y(y,z) + \cdots}$$

$|1\rangle \to |0\rangle$
$|0\rangle \to |1\rangle$

$$\psi = \psi_1 + \psi_2 e^{i\varphi}$$

$$|\psi\rangle = |1\rangle_L |0\rangle_R + e^{i\varphi} |0\rangle_L |1\rangle_R$$

$$|\psi\rangle = |1\rangle_L |0\rangle_R \pm |0\rangle_L |1\rangle_R$$

QUANTUM NON-LOCALITY

BELL'S INEQUALITY

$A = B$

$A = B'$

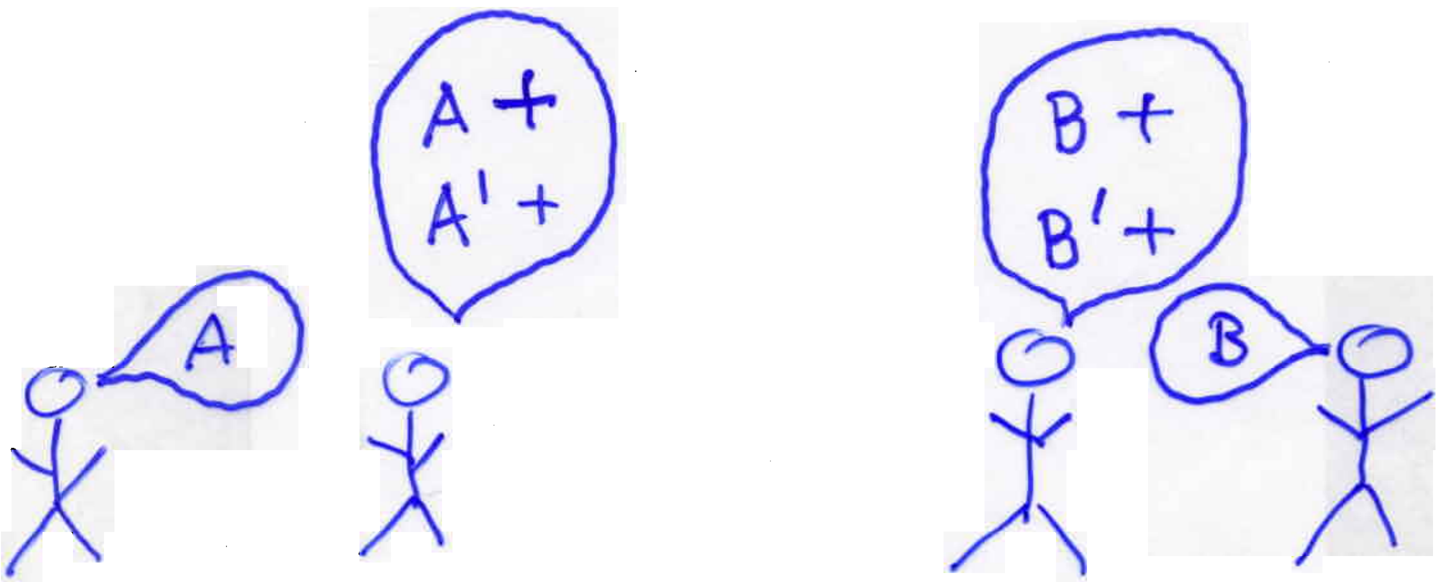
$A' = B$

$A' \neq B'$

$A, A', B, B' = \pm 1$

QUANTUM NON-LOCALITY

BELL'S INEQUALITY



$A = B$

$A = B'$

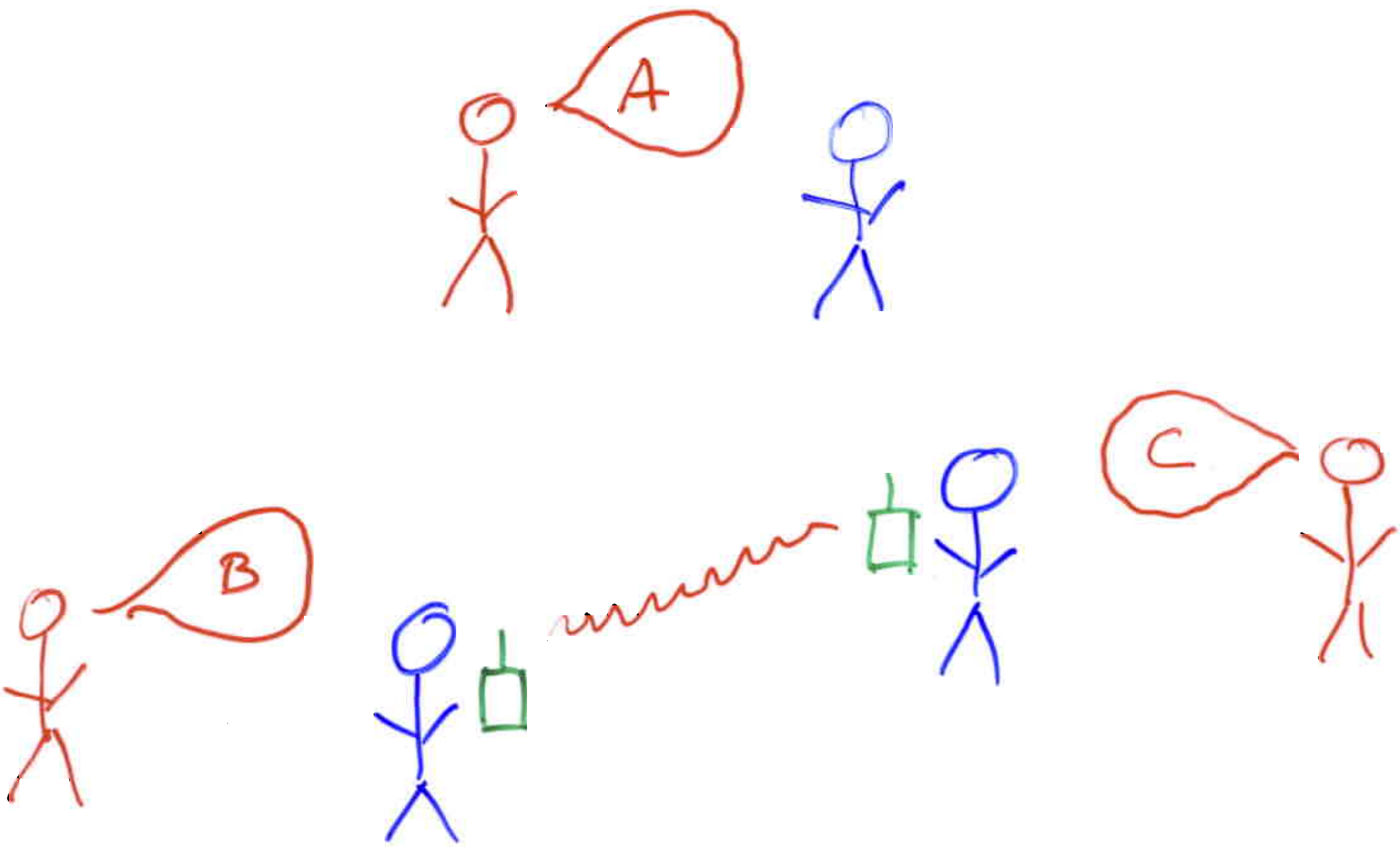
$A' = B$

$A' \neq B'$

$A, A', B, B' = \pm 1$

GENUINE 3-PARTY NON-LOCALITY

SVETLICHNY'S INEQUALITY



$A, A', B, B', C, C' = \pm 1$

GOAL: $A \cdot B \cdot C = -1$

$$A' \cdot B' \cdot C' = -1$$

ALL OTHER PRODUCTS $= +1$

AB $+$

AB' $-$ $\qquad$ C $-$

$A'B$ $-$ $\qquad$ C' $+$

$A'B'$ $-$

CORRELATIONS IN CLASSICAL PROBABILITY

X $\qquad\qquad$ Y

$P(X)$, $P(Y)$

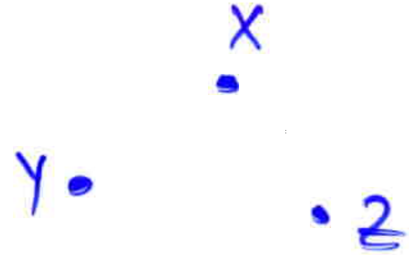
FIND $\tilde{P}(XY)$ WHICH HAS <u>NO MORE</u> INFORMATION THAN $P(X)$ AND $P(Y)$

<u>SOLUTION</u>: FIND $P(XY)$ WHICH HAS MAXIMAL ENTROPY

$$\tilde{P}(XY) = P(X) \cdot P(Y)$$

$\tilde{P}(XY)$ HAS NO TWO PARTY CORRELATIONS

PROBLEM:



$$P(X,Y)\;,\quad P(Y,Z)\;,\quad P(Z,X)$$

FIND $\tilde{P}(X,Y,Z)$ WHICH HAS NO MORE INFORMATION

QUANTUM PROBLEMS

A $\cdot$ $\quad\quad$ $\cdot$ B

ρ_A AND ρ_B GIVEN

FIND $\tilde{\rho}_{AB}$ WHICH HAS NO MORE INFORMATION

$$\tilde{\rho}_{AB} = \rho_A \otimes \rho_B$$

MEASURE OF 2-PARTY CORRELATIONS

$$\rho_{AB} \longrightarrow \begin{array}{c} \rho_A = \mathrm{Tr}_B\, \rho_{AB} \\ \\ \rho_B = \mathrm{Tr}_A\, \rho_{AB} \end{array} \longrightarrow \tilde{\rho}_{AB}$$

DISTANCE $(\rho_{AB}, \tilde{\rho}_{AB})$

MEASURE FOR GENUINE

3-PARTY CORRELATIONS

$$\rho_{ABC} \longrightarrow \begin{array}{l} \rho_{AB} = \mathrm{Tr}_C\, \rho_{ABC} \\ \rho_{BC} = \mathrm{Tr}_A\, \rho_{ABC} \\ \rho_{CA} = \mathrm{Tr}_B\, \rho_{ABC} \end{array} \longrightarrow \tilde{\rho}_{ABC}$$

$$\text{DISTANCE}\left(\rho_{ABC}, \tilde{\rho}_{ABC}\right)$$

SIMPLE EXAMPLE

$$|\psi\rangle = \alpha\,|0\rangle_A|0\rangle_B|0\rangle_C + \beta\,|1\rangle_A|1\rangle_B|1\rangle_C$$

$$\rho_{ABC} = |\psi\rangle\langle\psi|$$

$$\tilde{\rho}_{ABC} = \alpha^2\,|000\rangle\langle 000| + \beta^2\,|111\rangle\langle 111|$$

SURPRISE

$$|\psi\rangle \neq \alpha|0\rangle_A|0\rangle_B|0\rangle_C + \beta|1\rangle_A|1\rangle_B|1\rangle_C$$

$$\rho_{ABC} = |\psi\rangle\langle\psi|$$

$$\tilde{\rho}_{ABC} = |\psi\rangle\langle\psi|$$

$|\psi\rangle$ HAS NO GENUINE 3-PARTY CORRELATIONS

$$\boxed{|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right)}$$

VIOLATES SVETLICANY'S INEQ.