# Quantum Algorithms
# and
# Feynman Integrals

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- **This work also supported by National Institute for Standards and Technology (NIST).**

**The quantum algorithmic schema to be discussed in this talk were developed to be tools and aids for the creation of future quantum algorithms.**

# This talk is based on:

- Lomonaco & Kauffman, Quantum Hidden Subgroup Algorithms: A Mathematical Perspective, AMS, CONM/305, (2002). http://xxx.lanl.gov/abs/quant-ph/0201095

- Lomonaco & Kauffman, A Continuous Variable Shor Algorithm, http://xxx.lanl.gov/abs/quant-ph/0210141

- Lomonaco & Kauffman, Continuous Variable Quantum Algorithm Schema, (in preparation).

# *Outline*

**Part 1.** A quantum algorithm on the reals $\mathbb{R}$

**Part 2a.** A quantum algorithm on the circle $\mathbb{R}/\mathbb{Z}$

**Part 2b.** The dual quantum algorithm

**Part 3.** Quantum algorithms based on Feynman path integrals.

# Part 1

**Lomonaco & Kauffman, A Continuous Variable Shor Algorithm, http://xxx.lanl.gov/abs/quant-ph/0210141**

# A Continuous Variable Shor Algorithm

# Continuous Variable Quantum Algorithms

- **Continuous Variable Grover Algorithm**

  **Pati, Braunstein, & Lloyd**

  **Quant-ph/0002082**

- **Continuous Variable Deutsch-Jozsa Algorithm**

  **Pati & Braunstein**

  **Quant-ph/0207108**

# This Talk is Related to

- **Hallgren, Sean, Polynomial-Time Quantum Algorithm for Pell's equation and the Principal Ideal Problem**

- **Hales, Lisa, thesis**

The Quantum Hidden Subgroup Paper Shows how to create a **Meta Algorithm**

We now create a

## Continuous Variable Shor Algorithm

Recall that Shor's algorithm reduces to the task of finding the period $P$ of a function

$$\varphi : \mathbb{Z} \to \mathbb{Z} \bmod N$$

So a CV Shor algorithm should be a HSG algorithm that finds the period $P$ of a function of the form

$$\varphi : \mathbb{R} \to \mathbb{C}$$

# Needed

# Mathematical Machinery

On the reals $\mathbb{R}$ , we need

- The Dirac Delta function $\delta(x)$

- We will also need the generalized function

$$\delta_P(x) = \frac{1}{|P|}\sum_{n=-\infty}^{\infty}\delta\left(x - \frac{n}{P}\right)$$

## Rigged Hilbert Spaces, a.k.a., Gelfand Triplets

- $H_{\mathbb{R}}$ **denotes the rigged Hilbert space with orthonormal basis**

$$\{|x\rangle : x \in \mathbb{R}\} \text{ , i.e., } \langle x|y\rangle = \delta(x-y)$$

- **The elements of** $H_{\mathbb{R}}$ **are formal integrals of the form**

$$\int_{-\infty}^{\infty} dx\, f(x)|x\rangle$$

- $H_{\mathbb{C}}$ **denotes the rigged Hilbert space with orthonormal basis**

$$\{|y\rangle : y \in \mathbb{C}\}$$

- $H_{\mathbb{R}} \otimes H_{\mathbb{C}}$ **denotes the rigged Hilbert space with orthonormal basis**

$$\{|x\rangle|y\rangle : x \in \mathbb{R} \;\&\; y \in \mathbb{C}\}$$

If $x_0$ is a constant, we define

$$|x_0\rangle = \int_{-\infty}^{\infty} dx\, \delta(x - x_0)|x\rangle$$

Hence,

$$\langle y_0|x_0\rangle = \begin{cases} 1 & if \quad x_0 = y_0 \\ 0 & otherwise \end{cases}$$

$$\langle x_0 | y_0 \rangle$$

$$= \left(\int dx\, \delta(x - x_0) \langle x|\right)\left(\int dy\, \delta(y - y_0) |y\rangle\right)$$

$$= \int dx\, \delta(x - x_0) \left(\int dy\, \delta(y - y_0) \langle y | x \rangle\right)$$

$$= \int dx\, \delta(x - x_0) \left(\int dy\, \delta(y - y_0) \delta(y - x)\right)$$

$$= \int dx\, \delta(x - x_0) \left(\int dy\, \delta(y - y_0) \delta(x - y_0)\right)$$

$$= \int dx\, \delta(x - x_0) \delta(x - y_0) \int dy\, \delta(y - y_0)$$

$$= \int dx\, \delta_{x_0 y_0} \delta(x - x_0) \int dy\, \delta(y - y_0) = \delta_{x_0 y_0}$$

# Fourier Analysis on $\mathbb{R}$

Let $\varphi : \mathbb{R} \to \mathbb{C}$ be an admissible periodic function of minimum period $P$ .

**Problem:** In general, $\varphi$ is neither $L^1$ nor $L^2$ nor of compact support. So the usual literature on Fourier analysis does not apply .

**Remark:** One definition of admissible function is one that is Lebesgue integrable on every closed subinterval of $\mathbb{R}$ . But there are other definitions that also will work.

To circumvent this problem we extend the definition of the Fourier transform to period $P$ admissible functions $\varphi$ as follows:

$$\hat{\varphi}(y) = \int_{-\infty}^{\infty} dx\, e^{-2\pi ixy}\varphi(x)$$

$$:= \delta_P(y)\int_0^P dx\, e^{-2\pi ixy}\varphi(x)$$

where

$$\delta_P(y) = \frac{1}{|P|}\sum_{n=-\infty}^{\infty}\delta\left(y - \frac{n}{P}\right)$$

It can be verified that $\varphi(x) = \int_{-\infty}^{\infty} dx\, e^{2\pi ixy}\hat{\varphi}(y)$

# Motivation for above Definition

$$\int_{-\infty}^{\infty} dx\, e^{-2\pi ixy}\varphi(x) = \sum_{n=-\infty}^{\infty} \int_{nP}^{(n+1)P} dx\, e^{-2\pi ixy}\varphi(x)$$

$$= \sum_{n=-\infty}^{\infty} \int_{0}^{P} dx\, e^{-2\pi i(x+nP)y}\varphi(x+nP) = \sum_{n=-\infty}^{\infty} e^{-2\pi inPy} \int_{0}^{P} dx\, e^{-2\pi ixy}\varphi(x)$$

$$= \sum_{m=-\infty}^{\infty} \frac{1}{|P|}\delta\left(y - \frac{m}{P}\right) \int_{0}^{P} dx\, e^{-2\pi ixy}\varphi(x) = \delta_P(y) \int_{0}^{P} dx\, e^{-2\pi ixy}\varphi(x)$$

# Motivation (Cont.)

where, in the context of distributions, we have used the fact that

$$\sum_{n=-\infty}^{\infty} e^{-2\pi i n P y}$$

is, for every $m$, the Fourier series expansion of

$$\frac{1}{|P|}\delta\left(y-\frac{m}{P}\right)$$

on the interval

$$\left[\frac{m}{P},\frac{m+1}{P}\right)=\left\{y\in\mathbb{R}\ :\ \frac{m}{P}\le y<\frac{m+1}{P}\right\}$$

**Let $\varphi : \mathbb{R} \to \mathbb{C}$ be an admissible function of minimum period $P$ .**

**We will now create a continuous variable Shor algorithm to find the period $P$ when $P$ is an integer.**

**After that, we will extend the algorithm to one that can determine the period $P$ when $P$ is rational. Finally, we will extend the algorithm to one that finds irrational periods.**

Let $\varphi : \mathbb{R} \to \mathbb{C}$ be an admissible function of minimum period $P$ .

We will assume that $\varphi$ is one-to one on its fundamental domain $[0, P)$

•Step 0. Initialize $$|\psi_0\rangle = |0\rangle|0\rangle \in H_{\mathbb{R}} \otimes H_{\mathbb{C}}$$

•Step 1. Apply $F^{-1} \otimes 1$

$$|\psi_1\rangle = \int_{-\infty}^{\infty} dx e^{2\pi i x \cdot 0} |x\rangle|0\rangle = \int_{-\infty}^{\infty} dx |x\rangle|0\rangle$$

•Step 2. Apply $U_{\varphi} : |x\rangle|u\rangle \mapsto |x\rangle|u + \varphi(x)\rangle$

$$|\psi_2\rangle = \int_{-\infty}^{\infty} dx |x\rangle|\varphi(x)\rangle$$

- **Step 3. Apply** $F \otimes 1$

$$
\begin{aligned}
|\psi_3\rangle &= \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\, e^{-2\pi i x y} |y\rangle |\varphi(x)\rangle \\
&= \int_{-\infty}^{\infty} dy\, |y\rangle\, \delta_P(y) \int_{0}^{P-} dx\, e^{-2\pi i x y} |\varphi(x)\rangle \\
&= \sum_{n=-\infty}^{\infty} \left|\frac{n}{P}\right\rangle \left( \frac{1}{P} \int_{0}^{P-} dx\, e^{-2\pi i x \frac{n}{P}} |\varphi(x)\rangle \right) \\
&= \sum_{n=-\infty}^{\infty} \left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle
\end{aligned}
$$

•Step 4. Measure

$$\left|\psi_3\right\rangle = \sum_{n=-\infty}^{\infty} \left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle$$

with respect to the observable

$$A = \int_{-\infty}^{\infty} dy \frac{\lfloor Qy \rfloor}{Q} \left|y\right\rangle\left\langle y\right|$$

to produce a random eigenvalue $m / Q$ ,
where $\lfloor Qy \rfloor$ is the greatest integer
$\leq Qy$

# Spectral Decomposition of Observable $A$

$$A = \int_{-\infty}^{\infty} dy \, \frac{\lfloor Qy \rfloor}{Q} |y\rangle\langle y| = \sum_{m=-\infty}^{\infty} \left(\frac{m}{Q}\right) \mathrm{P}_m$$

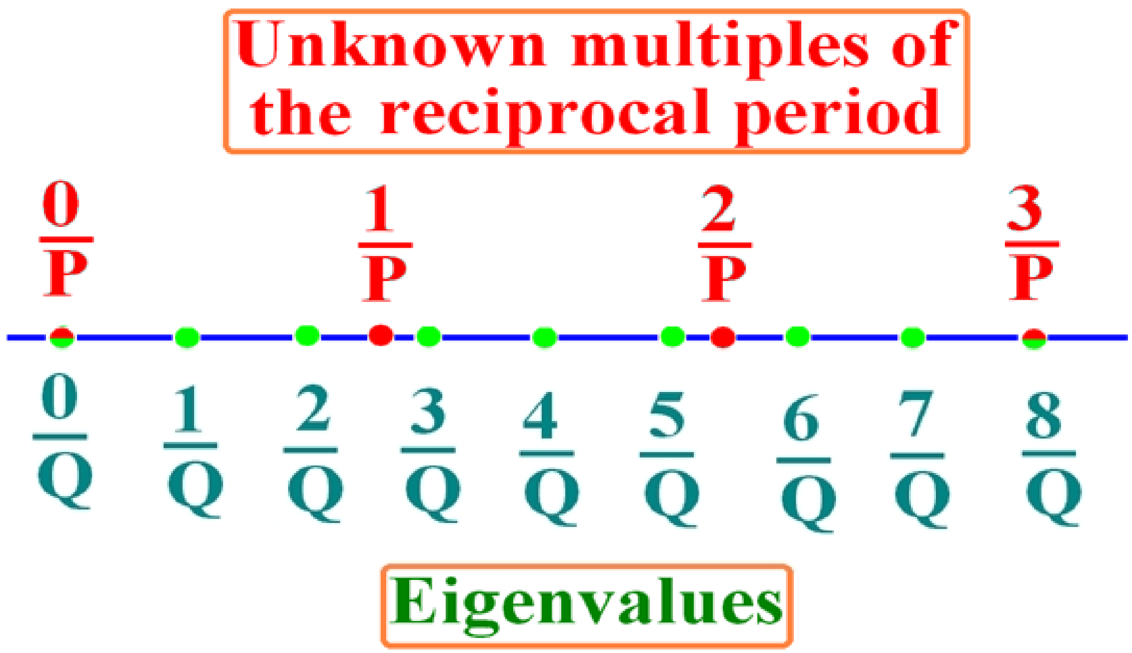
where

$$\mathrm{P}_m = \int_{\frac{m}{Q}}^{\frac{m+1}{Q}} dy \, |y\rangle\langle y| \quad - \quad \left|\frac{m+1}{Q}\right\rangle\left\langle\frac{m+1}{Q}\right|$$

Meas. always produces an eigenvalue $m/Q$
for which there exists an integer $n$ such that

$$\frac{m}{Q} \le \frac{n}{P} < \frac{m+1}{Q}$$

We seek the unknown $n/P$



If $Q \geq 2P^2$, then $n / P$ is a convergent of the continued fraction expansion of the known eigenvalue $m / Q$.

Thus, the continued fraction recursion can be used to determine the period $P$.

# Finding Rational Periods

The above algorithm can be extended to an algorithm for finding rational periods

$$P = a/b, \quad \gcd(a,b) = 1$$

Key Ideas:

- Run the quantum part of the above algorithm (**Steps 0 – 4**) twice to produce two eigenvalues

$$m_1/Q \quad \text{and} \quad m_2/Q$$

- If $Q \geq 2a^2$, then these eigenvalues respectively will have unique convergents of the form

$$\frac{n_1 b}{a} \quad \text{and} \quad \frac{n_2 b}{a}$$

Key Ideas (Cont.):

- If the following Condition A is satisfied

$$\gcd(n_1, n_2) = 1, \gcd(n_1, a) = 1, \gcd(n_2, a) = 1$$

then the reciprocal period is

$$\frac{1}{P} = \frac{\gcd\left(n_1 b, n_2 b\right)}{a}$$

This expression can be computed as follows:

- Step A. For each convergent $p_{1k} / q_{qk}$ of $m_1 / Q$, select (if it exists) a convergent $p_{2\ell} / q_{2\ell}$ of $m_2 / Q$ which has the same denominator

$$\left(q = q_{1k} = q_{2\ell}\right)$$

## Key Ideas (Cont.):

- Step B. **After making this selection, construct the corresponding rational**

$$\frac{\gcd\left(p_{1k}, p_{2\ell}\right)}{q}$$

- Step C. **Test to see if it is a reciprocal period.**

- **If not, repeat** Steps A **through** C **until the reciprocal period is found, output the reciprocal period, and STOP.**

Key Ideas (Cont.):

- The probability that Condition A is satisfied is

$$\Omega\left(\left(\frac{1}{\lg\lg Q}\right)^2\right)$$

- Hence, on average, all of the above need to be repeated

$$O\left(\left(\lg\lg Q\right)^2\right)$$

times to find the reciprocal period.

# Finding Irrational Periods

**If we assume that the map $\varphi$ is continuous, then the same procedure can be used fo find an irrational period to any degree of desired precision.**

Continuity **is needed for determining whether or not a rational is sufficiently close to the unknown irrational period.**

# Implementation

???

# Double Dare!!

## Implement This

# Part 2a

Lomonaco & Kauffman, Continuous Variable Quantum Algorithm Schema, (in preparation).

# Schema for Continuous Variable Quantum Algorithms

# on the

# Circle

# Rigged Hilbert Space

- $H_{\mathbb{R}/\mathbb{Z}}$ **denotes the rigged Hilbert space on** $\mathbb{R}/\mathbb{Z}$ **with orthonormal basis**

$$\{|x\rangle : x \in \mathbb{R}/\mathbb{Z}\}$$ **, i.e.,** $$\langle x|y\rangle = \delta(x-y)$$

- **The elements of** $H_{\mathbb{R}/\mathbb{Z}}$ **are formal integrals of the form**

$$\oint dx\, f(x)|x\rangle$$

Finally, let $H_{\mathbb{Z}}$ denote the space of formal sums

$$\left\{ \sum_{n=-\infty}^{\infty} a_n |n\rangle : a_n \in \mathbb{C} \quad \forall n \in \mathbb{Z} \right\}$$

with orthonormal basis

$$\{ |n\rangle : n \in \mathbb{Z} \}$$

## Periodic Admissible Functions on $\mathbb{R}/\mathbb{Z}$

Let $f:\mathbb{R}/\mathbb{Z}\to\mathbb{C}$ be an admissible periodic function of minimum rational period $\alpha\in\mathbb{Q}/\mathbb{Z}$

**Proposition:** If $\alpha = a_1/a_2$ with $\gcd(a_1,a_2)=1$, then $1/a_2$ is also a period of $f$ .

**Remark:** Hence, the minimum rational period is the reciprocal of an integer modulo 1 .

- **Step 0. Initialize** $$|\psi_0\rangle = |0\rangle|0\rangle \in H_{\mathbb{Z}} \otimes H_{\mathbb{C}}$$

- **Step 1. Apply** $F^{-1} \otimes 1$

$$|\psi_1\rangle = \oint dx\, e^{2\pi i x \cdot 0} |x\rangle|0\rangle = \oint dx |x\rangle|0\rangle \in H_{\mathbb{R}/\mathbb{Z}} \otimes H_{\mathbb{C}}$$

- **Step 2. Apply** $U_\varphi : |x\rangle|u\rangle \mapsto |x\rangle|u + \varphi(x)\rangle$

$$|\psi_2\rangle = \oint dx |x\rangle|\varphi(x)\rangle$$

- **Step 3. Apply** $F \otimes \mathbf{1}$

$$
\begin{aligned}
|\psi_3\rangle &= \sum_{n\in\mathbb{Z}} \oint dx\, e^{-2\pi i n x} |n\rangle |\varphi(x)\rangle \\
&= \sum_{n\in\mathbb{Z}} |n\rangle \oint dx\, e^{-2\pi i n x} |\varphi(x)\rangle \in H_{\mathbb{Z}} \otimes H_{\mathbb{C}}
\end{aligned}
$$

Letting $x_m = x - \frac{m}{a}$, we have

$$\oint dx\, e^{-2\pi inx} \left|\varphi(x)\right\rangle = \sum_{m=0}^{a-1} \int_{\frac{m}{a}}^{\frac{m+1}{a}} dx\, e^{-2\pi inx} \left|\varphi(x)\right\rangle$$

$$= \sum_{m=0}^{a-1} \int_0^{\frac{1}{a}} dx_m\, e^{-2\pi in\left(x_m + \frac{m}{a}\right)} \left|\varphi\left(x_m + \frac{m}{a}\right)\right\rangle$$

$$= \left(\sum_{m=0}^{a-1} e^{-\frac{2\pi inm}{a}}\right) \int_0^{\frac{1}{a}} dx\, e^{-2\pi inx} \left|\varphi(x)\right\rangle$$

**But** $$\sum_{m=0}^{a-1} e^{-\frac{2\pi i n m}{a}} = a\delta_{n=0 \bmod a} = \begin{cases} a & \text{if } n = 0 \bmod a \\ 0 & \textit{otherwise} \end{cases}$$

**Thus,**

$$\begin{aligned} |\psi_3\rangle &= \sum_{n\in\mathbb{Z}} |n\rangle \oint dx\, e^{-2\pi i n x} |\varphi(x)\rangle \\ &= \sum_{n\in\mathbb{Z}} |n\rangle \delta_{n=0 \bmod a} \int_0^{1/a} dx\, e^{-2\pi i n x} |\varphi(x)\rangle \\ &= \sum_{\ell\in\mathbb{Z}} |\ell a\rangle \left( \int_0^{1/a} dx\, e^{-2\pi i \ell a x} |\varphi(x)\rangle \right) \\ &= \sum_{\ell\in\mathbb{Z}} |\ell a\rangle |\Omega(\ell a)\rangle \end{aligned}$$

•Step 4. Measure

$$\left|\psi_3\right\rangle = \sum_{\ell\in\mathbb{Z}} \left|\ell a\right\rangle \left|\Omega(\ell a)\right\rangle$$

with respect to the observable

$$A = \sum_{n\in\mathbb{Z}} n \left|n\right\rangle\left\langle n\right|$$

to produce a random eigenvalue $\ell a$

# Part 2b

## The

## Dual

## Algorithm

# Needed

# Mathematical Machinery

- **Dirac Delta function** $\delta(x)$ **on** $\mathbb{R}/\mathbb{Z}$

- **For** $P$ **a non-zero integer, we will also need on** $\mathbb{R}/\mathbb{Z}$ **the generalized function**

$$\delta_P(x) = \frac{1}{|P|}\sum_{n=0}^{P-1}\delta\left(x - \frac{n}{P}\right)$$

# Periodic Functions on $\mathbb{Z}$

Let $\varphi : \mathbb{Z} \to \mathbb{C}$ be periodic function of minimum period $P$ .

•Step 0. Initialize $\left|\psi_0\right\rangle = \left|0\right\rangle\left|0\right\rangle \in H_{\mathbb{R}/\mathbb{Z}} \otimes H_{\mathbb{C}}$

•Step 1. Apply $F^{-1} \otimes 1$

$$\left|\psi_1\right\rangle = \sum_{n\in\mathbb{Z}} e^{2\pi i n\bullet 0}\left|n\right\rangle\left|0\right\rangle = \sum_{n\in\mathbb{Z}}\left|n\right\rangle\left|0\right\rangle \in H_{\mathbb{Z}} \otimes H_{\mathbb{C}}$$

•Step 2. Apply $U_\varphi : \left|n\right\rangle\left|u\right\rangle \mapsto \left|n\right\rangle\left|u+\varphi(n)\right\rangle$

$$\left|\psi_2\right\rangle = \sum_{n\in\mathbb{Z}}\left|n\right\rangle\left|\varphi(n)\right\rangle$$

- **Step 3. Apply** $F \otimes 1$

$$
\begin{aligned}
|\psi_3\rangle &= \oint dx\, |x\rangle \sum_{n\in\mathbb{Z}} e^{-2\pi i n x} |\varphi(n)\rangle \in H_{\mathbb{R}/\mathbb{Z}} \otimes H_{\mathbb{C}} \\
&= \oint dx\, |x\rangle \sum_{n_1\in\mathbb{Z}} \sum_{n_0=0}^{P-1} e^{-2\pi i (n_1 P + n_0) x} |\varphi(n_1 P + n_0)\rangle \\
&= \oint dx\, |x\rangle \left( \sum_{n_1\in\mathbb{Z}} e^{-2\pi i n_1 P x} \right) \sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x} |\varphi(n_0)\rangle \\
&= \oint dx\, |x\rangle \delta_P(x) \sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x} |\varphi(n_0)\rangle \\
&= \sum_{n=0}^{P-1} \left|\frac{n}{P}\right\rangle \left( \frac{1}{P} \sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x} |\varphi(n_0)\rangle \right) \\
&= \sum_{n=0}^{P-1} \left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle
\end{aligned}
$$

•Step 4. Measure

$$\left|\psi_3\right\rangle = \sum_{n=0}^{P-1} \left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle$$

with respect to the observable

$$A = \oint dy \frac{\lfloor Qy \rfloor}{Q} \left|y\right\rangle\left\langle y\right|$$

to produce a random eigenvalue $m/Q$ and then proceed to find the corresponding $n/P$ using the continued fraction recursion.

(We assume $Q \geq 2P^2$ )

# Part 3

???

**Quantum Algorithms based on Feynman path integrals**

???

# Caveat Emptor

The functional integral quantum algorithm given in the following slides was developed in the spirit of Feynman's non-mathematically rigorous description of functional integrals. Many of the steps given below are yet to be justified with the cutting edge of mathematical rigor.

# The Space Paths

**Paths** = all continuous paths $x:[0,1]\to\mathbb{R}^n$
which are $L^2$ with respect to the inner product

$$x \bullet y = \int_0^1 ds\, x(s)\, y(s)$$

**Paths** is a vector space over $\mathbb{R}$ with respect to

$$\begin{cases} (\lambda x)(s) = \lambda x(s) \\ (x+y)(s) = x(s)+y(s) \end{cases}$$

# The Problem to be Solved

**Let $\varphi : Paths \to \mathbb{C}$ be a functional with a hidden subspace $V$ of $Paths$ such that**

$$\varphi(x+v) = \varphi(x) \quad \forall v \in V$$

**Objective.** **Create a quantum algorithm that finds the hidden subspace $V$ .**

# The Ambient Rigged Hilbert Space

Let $\boldsymbol{H}_{Paths}$ be the rigged Hilbert space with orthonormal basis ,

$$\left\{ \left| x \right\rangle : x \in Paths \right\}$$

and with bracket product

$$\left\langle x \mid y \right\rangle = \delta\left( x - y \right)$$

# Parenthetical Remark

Please note that $Paths$ can be written as the following disjoint union:

$$Paths = \bigcup_{v \in V} \left( v + V^{\perp} \right)$$

•Step 0. Initialize $|\psi_0\rangle = |0\rangle|0\rangle \in H_{Paths} \otimes H_{\mathbb{C}}$

•Step 1. Apply $F^{-1} \otimes 1$

$$|\psi_1\rangle = \int_{Paths} \mathcal{D}x \; e^{2\pi i x \cdot 0} |x\rangle|0\rangle = \int_{Paths} \mathcal{D}x \, |x\rangle|0\rangle$$

•Step 2. Apply $U_{\varphi} : |x\rangle|u\rangle \mapsto |x\rangle|u + \varphi(x)\rangle$

$$|\psi_2\rangle = \int_{Paths} \mathcal{D}x \, |x\rangle|\varphi(x)\rangle$$

- **Step 3. Apply** $F \otimes 1$

$$
\begin{aligned}
|\psi_3\rangle &= \int_{Paths} \mathcal{D}y \int_{Paths} \mathcal{D}x \, e^{-2\pi i x \cdot y} |y\rangle |\varphi(x)\rangle \\
&= \int_{Paths} \mathcal{D}y |y\rangle \int_{Paths} \mathcal{D}x \, e^{-2\pi i x \cdot y} |\varphi(x)\rangle
\end{aligned}
$$

## But

$$\int_{Paths} \mathcal{D}x\, e^{-2\pi i x\cdot y} \left|\varphi(x)\right\rangle = \int_V \mathcal{D}v \int_{v+V^\perp} \mathcal{D}x\, e^{-2\pi i x\cdot y} \left|\varphi(x)\right\rangle$$

$$= \int_V \mathcal{D}v \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i (v+x)\cdot y} \left|\varphi(v+x)\right\rangle$$

$$= \int_V \mathcal{D}v\, e^{-2\pi i v\cdot y} \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\cdot y} \left|\varphi(x)\right\rangle$$

However, $$\int_V \mathcal{D}v\, e^{-2\pi i v\cdot y} = \int_{V^\perp} \mathcal{D}u\, \delta(y-u)$$

So,

$$|\psi_3\rangle = \int_{Paths_n} \mathcal{D}y\, |y\rangle \int_V \mathcal{D}v\, e^{-2\pi i v\cdot y} \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\cdot y} |\varphi(x)\rangle$$

$$= \int_{Paths_n} \mathcal{D}y\, |y\rangle \int_{V^\perp} \mathcal{D}u\, \delta(y-u) \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\cdot y} |\varphi(x)\rangle$$

$$= \int_{V^\perp} \mathcal{D}u\, |u\rangle \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\cdot u} |\varphi(x)\rangle$$

$$= \int_{V^\perp} \mathcal{D}u\, |u\rangle |\Omega(u)\rangle$$

•Step 4. Measure

$$\left|\psi_3\right\rangle = \int_{V^\perp} \mathcal{D}u \left|u\right\rangle\left|\Omega(u)\right\rangle$$

with respect to the observable

$$A = \int_{Paths} \mathcal{D}w\, w\left|w\right\rangle\left\langle w\right|$$

to produce a random element of $V^\perp$

# Question

Can the above path integral quantum algorithm be modified in such a way as to create a quantum algorithm for the Jones polynomial ?

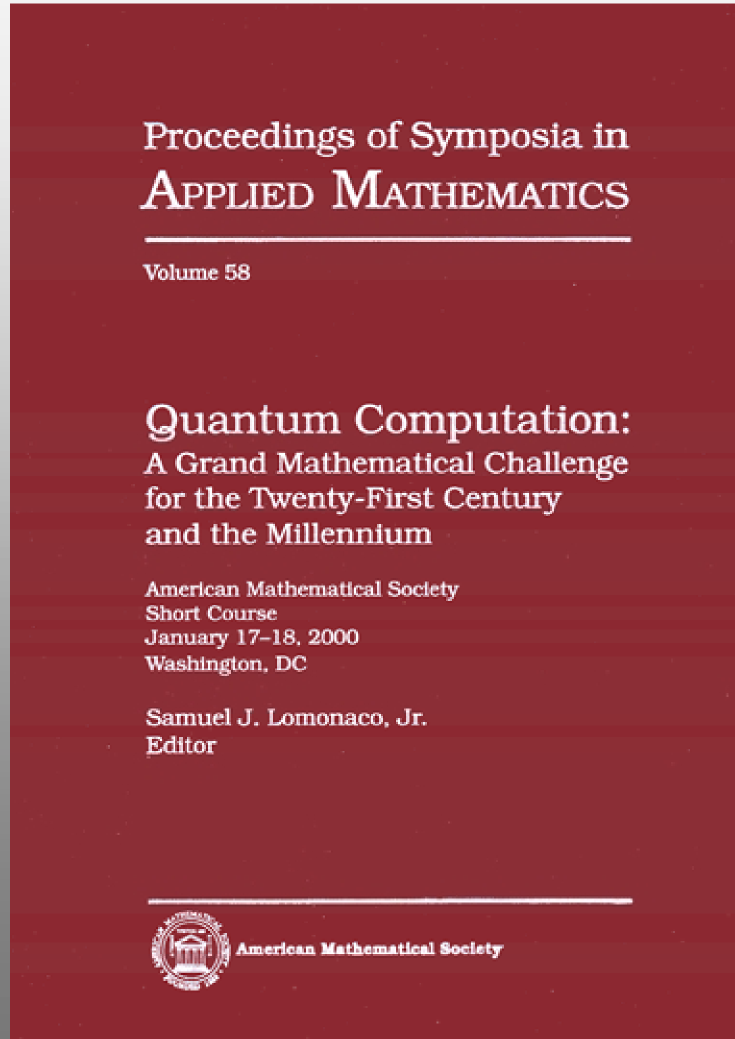
I.e., can it be modified by replacing $Paths$ by the space of gauge connections, and by making suitable modifications?

$$\int DA\,\psi(A)\,W_K(A)$$

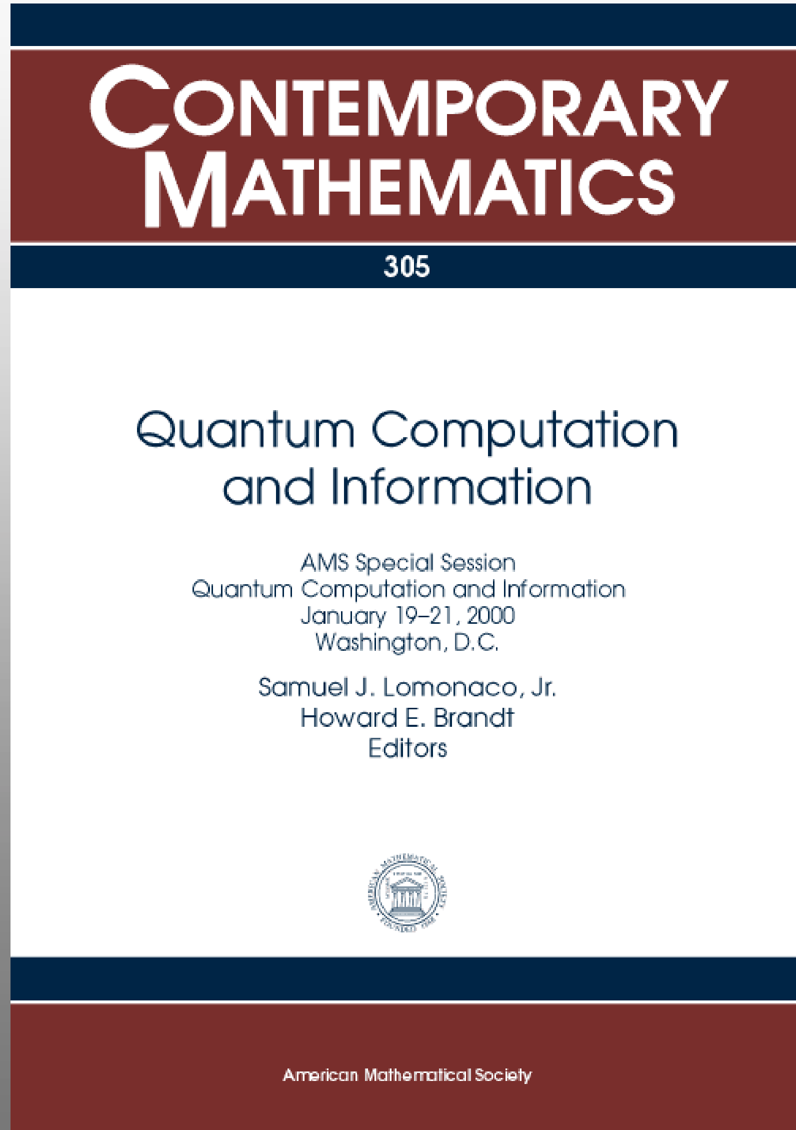
where $W_K(A)$ is the Wilson loop

$$W_K(A) = tr\left(P\exp\left(\oint_K A\right)\right)$$

**Quantum Computation:** A Grand Mathematical Mathematical Challenge for the Twenty-First Century Century and the Millennium, **Samuel J. Lomonaco, Jr. (editor),** **AMS PSAPM/58, (2002).**



**Quantum Computation and Information, Samuel J. Lomonaco, Jr. and Howard E. Brandt (editors), AMS CONM/305, (2002).**



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Samuel J. Lomonaco

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