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CHAOTIC FIELD THEORY

$$\phi = [\bar\psi_{a\alpha}(x), \bar\psi^{a\alpha}(x), A_{i\mu}(x)]$$

$$S[\phi] = \bar\psi(\partial\!\!\!/ + A\!\!\!/ + m)\psi + \frac{1}{4}F^2$$

have:

$$Z[J] = \int [d\phi]\, e^{\frac{i}{\hbar}[S + \phi\cdot J]}$$

proposal:

$$e^{\Gamma[\phi]+\phi\cdot J} \simeq \sum_{sc}^{\infty} \frac{e^{\frac{i}{\hbar}[S_{sc} + \frac{\pi}{4}m + \phi_{sc}\cdot J + \sum \hbar^n \Gamma^{(n)}]}}{|\det S''|^{1/2}} +$$

$$\simeq \sum^{\infty} \left(\begin{array}{c}\text{turbulent} \\ \text{classical} \\ \text{solutions}\end{array}\right) + (\text{tunneling}) + (\text{diffraction})$$

NOT: free ,

instantons, ...

"e^T" unstable patterns: numerical

* 1-d field theories, dissipative

* search for spatiotemporal recurrences

* high d Hamiltonian flows ?

$$\sum_{p}^{\text{saddles}} \frac{e^{\frac{i}{\hbar}S_p + \frac{i\pi}{4}m_p + J\phi}}{|\det S''|^{1/2}} \left(1 + \overset{\text{Feynman}}{\sum} \hbar^n \Gamma^{(n)} + \dots\right)$$

* 1-d map + noise
  - Feynman diagrams
  - Feynman + Poincaré
  - Transfer operator

* gauge invariance ?

* renormalization ?

Go with the flow

desiderata for a "simplest" field theory:

- 1 space, 1 time dim.
- scalar field
- 1 "Raynold's" parameter
  "laminar" $\longleftrightarrow$ turbulent
- UV finite (smooth solutions)
- IR finite (compact support)

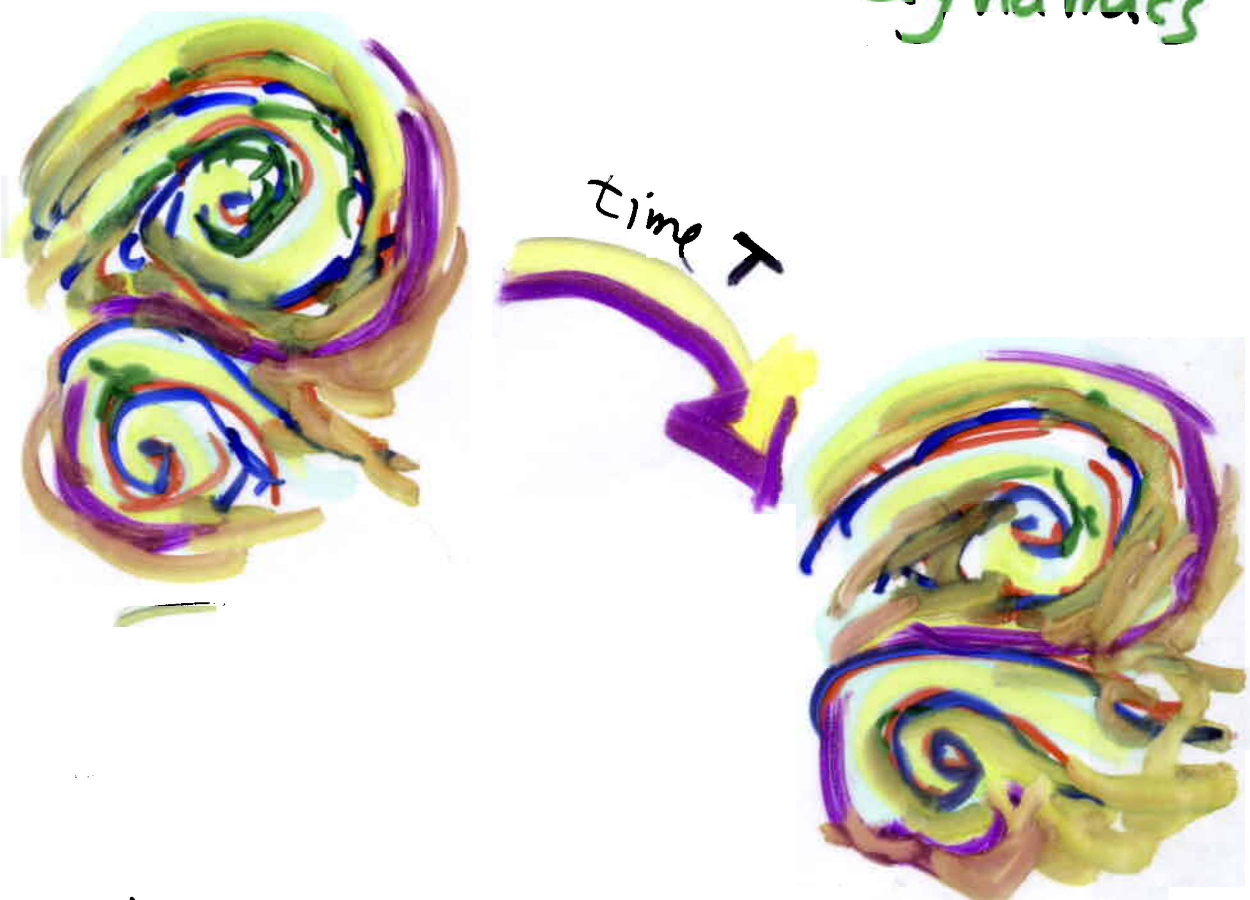
most popular

$$\dot{\phi} = L(\phi, \partial\phi, \cdots) + N(\phi, \partial\phi, \cdots)$$

- Nonlin Schrödinger
- Burgers
- complex Ginzburg-Landau
- Kuramoto-Sivashinski

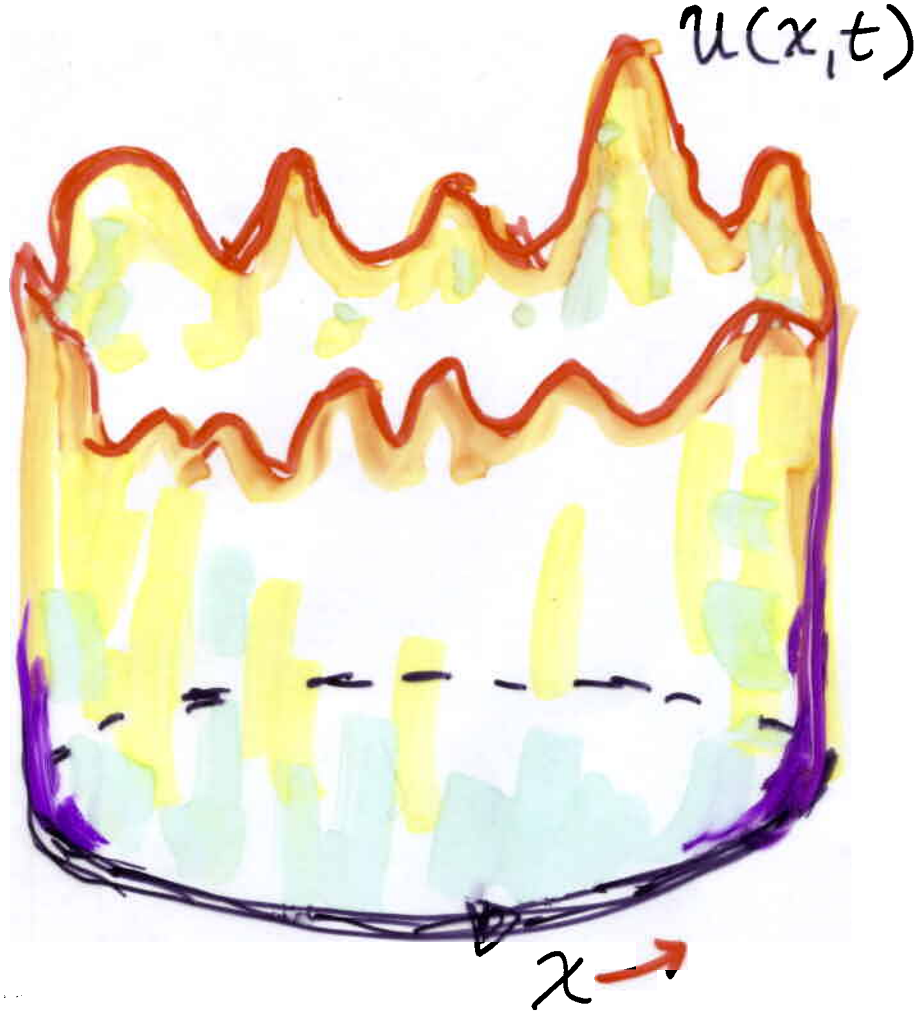
Hopf's last hope

classical theory of turbulent dynamics



in terms of spatiotemporally recurrent patterns

Burning flame front



$$u(x,t) = u(x+2\pi, t)$$

STRATEGY

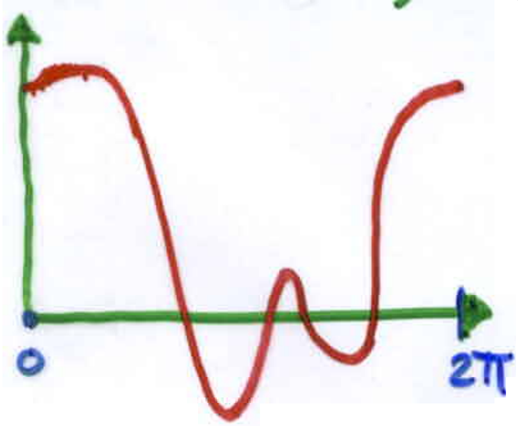
(V. Putkaradze, F. Christiansen, P.C.)
Nonlinearity 10, 1 (1997).

take "simplest" spatiotemporally "turbulent" dynamical system of physical interest ("parabola" of PDE's):

Kuramoto-Sivashinsky (1976):

$$u_t = \underbrace{(u^2)_x}_{\text{nonlinear}} - u_{xx} - \underbrace{\nu}_{\text{damped}} u_{xxxx} \qquad x \in [-\pi, \pi]$$

$u(x,t)$ = velocity of a 1-d flame front



at given instant t

(slide)

Theorem (Temam, ..)

$\nu \neq 0$ attractor finite dimensional

viscosity

$\nu > 1$ death

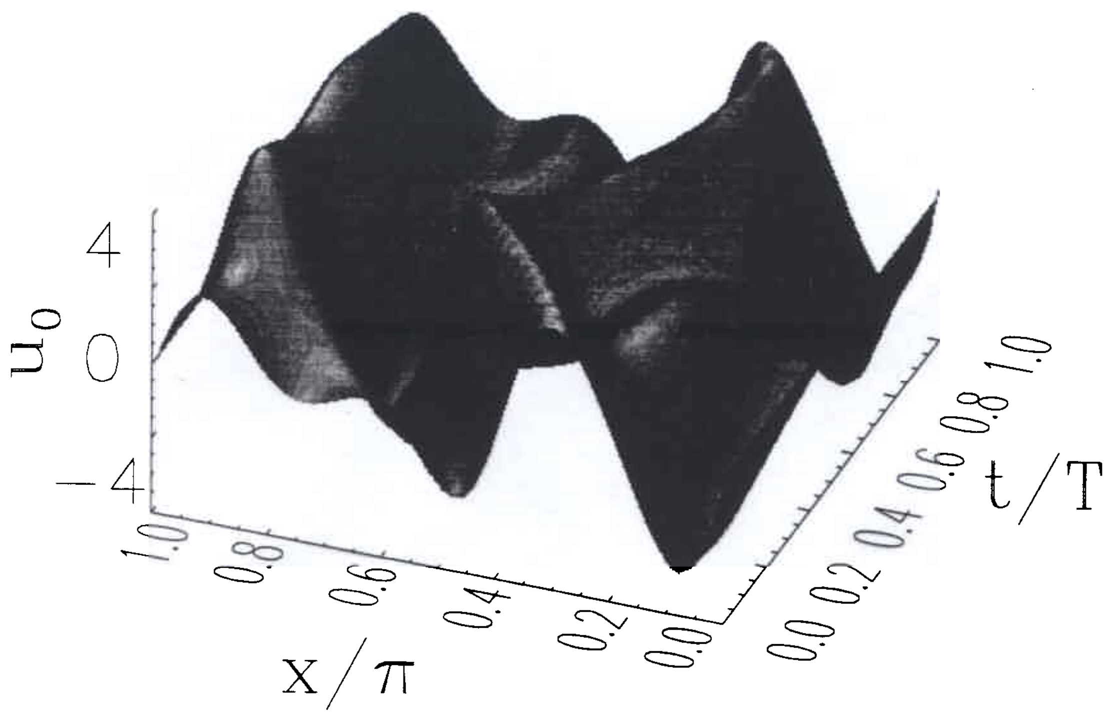
$\nu \leq 1$ spatio-temporal chaos

$\nu \ll 1$ turbulence

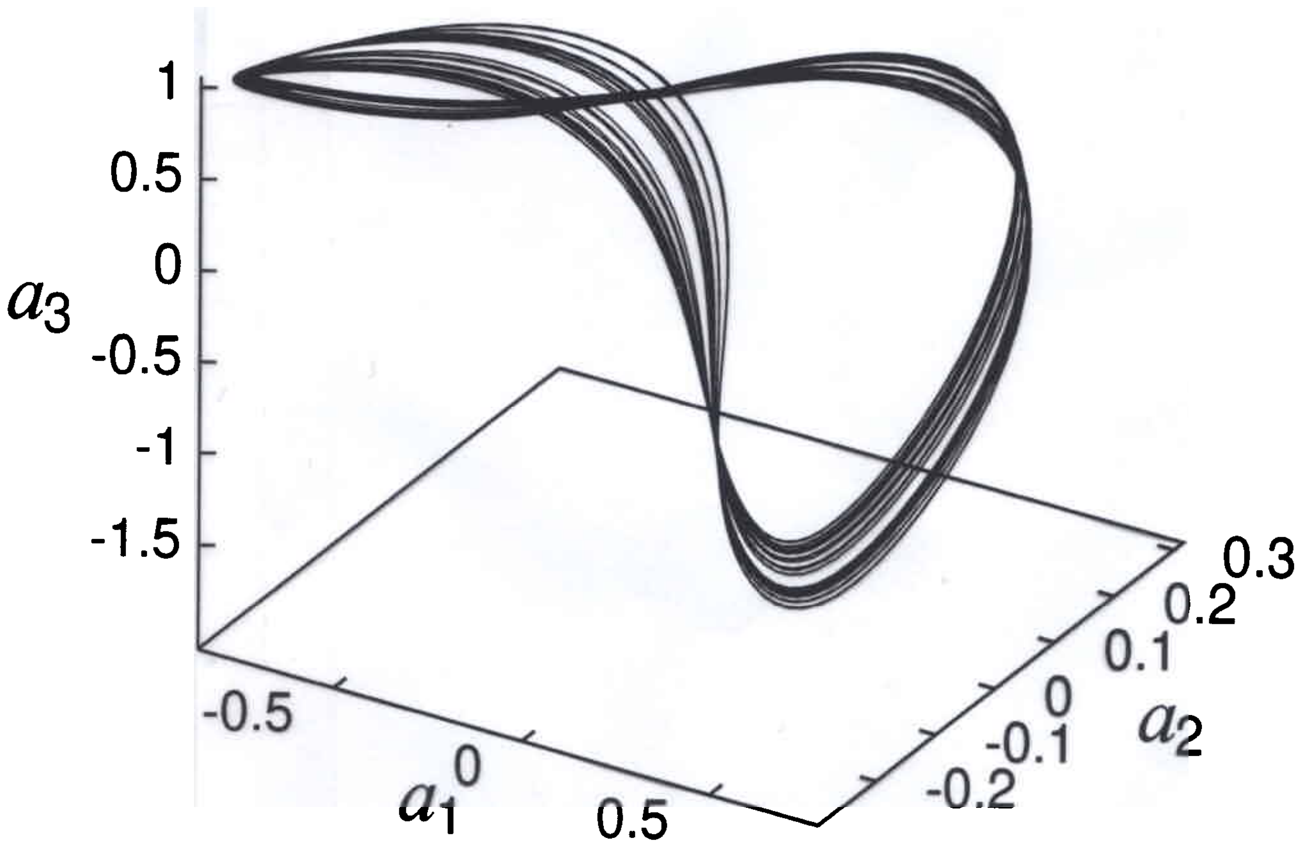
difficult because:

$u(x,t) \to (a_1, \dots a_N)$

$N = 15-1000$



u_0
4
0
−4
1.0
0.8
0.6
0.4
0.2
0.0
x/π
0.0
0.2
0.4
0.6
0.8
1.0
t/T



a_3
1
0.5
0
-0.5
-1
-1.5
a_1
-0.5
0
0.5
a_2
-0.2
-0.1
0
0.1
0.2
0.3

accomplished so far:

1. determined "all" unstable spatiotemporally periodic solutions $T_p < T_{cutoff}$

$$u_p(x,t) = u_p(x,t+T_p) \qquad p=1,2,\cdots,\text{"1000"}$$

2. described how <average>s are computed (<u>not</u> linear superpositions of p's)

open questions

3. unclear if useful as viscosity $\nu \to 0$ ("strong turbulence")

hope: periodic orbit skeleton remains sufficiently dense to explore the ∞ dim space of patterns ?

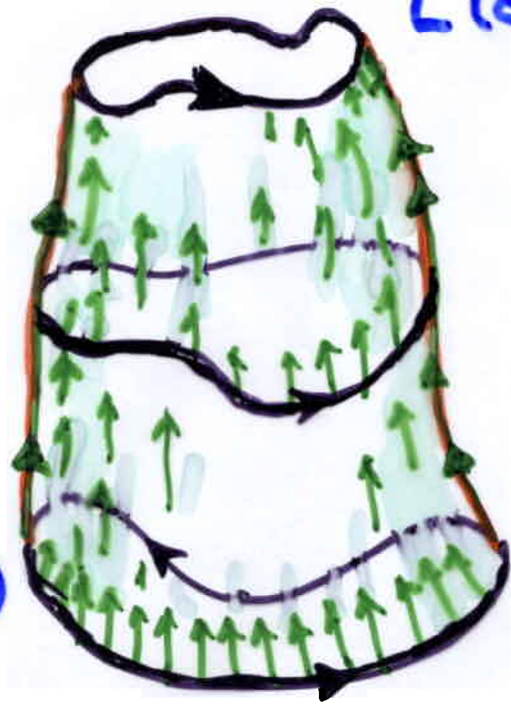
periodic orbit p

$x(t) \in p$ $\qquad$ $x \in \mathbb{R}^{\text{"}\infty\text{"}}$

velocity: $\frac{dx}{dt} = v(x)$, $\qquad$ $x(t) = x(t+T_p)$

t = time

$L(\infty) = p$



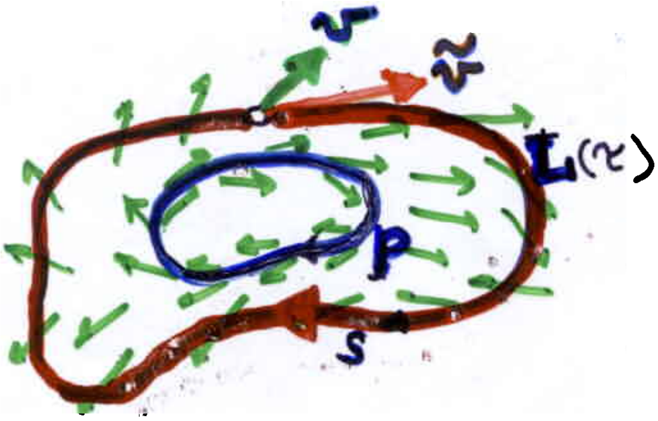
$L(\tau)$ $\qquad$ τ-flow in loop space

$L(0)$

$x(s) \in L$ $\qquad$ smooth

tangent: $\frac{dx}{ds} = \tilde{v}(x)$, $\qquad$ $x(s) = x(s+2\pi)$

s = arbitrary loop parameter



$v(x(s,\tau))$ = velocity

$\tilde{v}(x(s,\tau))$ = tangent

$x(s,\tau) \in L(\tau)$ = loop

p = periodic orbit

$$\boxed{F^2(\tau) = \oint_L ds\,(\tilde{v} - \lambda v)^2} \qquad \lambda = |\tilde{v}|/|v|$$

τ - fictitious time flow:

$$\frac{d}{d\tau}(\tilde{v} - \lambda v) = -(\tilde{v} - \lambda v)$$

"Newton descent"

$$\boxed{\frac{\partial^2 x}{\partial s\,\partial \tau} - \lambda A \cdot \frac{\partial x}{\partial \tau} - \frac{\partial \lambda}{\partial \tau} v = \lambda v - \tilde{v}}$$

drives the initial loop $L(0) \rightarrow p$

with $(\tilde{v} - \lambda v)\big|_{\tau} = e^{-\tau}(\tilde{v} - \lambda v)\big|_{\tau=0}$

robust p.o. search

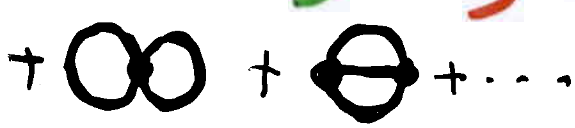
✓ SOLVED TURBULENCE

what do I do next?

- Hamiltonian dynamics in ∞ dimensions (mhm...)
- "Significant corrections"

$$e^{\Gamma[\phi]+\phi\cdot J} = \sum_{sc}^{\infty} \frac{e^{-\frac{i}{\hbar}S_{sc}+\frac{i\pi}{4}m+\phi_{sc}\cdot J+\hbar\Gamma^{(1)}_{sc}+\hbar^2\Gamma^{(2)}_{sc}+\dots}}{|\det S''_{sc}|^{1/2}}$$

- 1-d playpen $(x \to x^4 +$ noise?$)$
- method 1) Feynman



 no transl. invariance

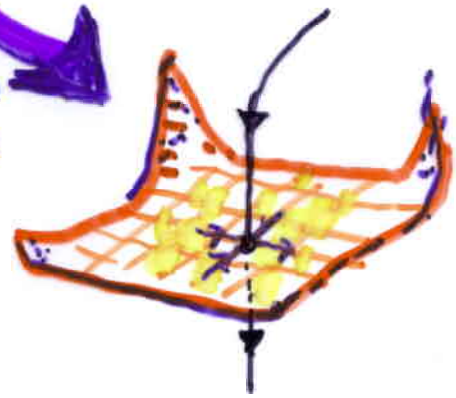
method 2)

Poincaré meets Feynman

$$\int [d\phi]\, e^{S[\phi]}$$

$$\phi = h(\tilde{\phi})$$

$$\int [d\tilde{\phi}] \left\| \frac{\partial \phi}{\partial \tilde{\phi}} \right\| e^{-\frac{1}{2}\tilde{\phi}\Delta^{-1}\tilde{\phi}}$$



penalty "trivialized"

= <u>not</u> Feyman graphs

method 3)

Transfer operators

$\mathcal{L} \to L_{ab}$: $\hbar^{56}$ (!)

"somewhat classical matters"

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