

LEHRBUCH
DER
A L G E B R A
VON
HEINRICH WEBER
PROFESSOR DER MATHEMATIK AN DER UNIVERSITÄT STRASSBURG
ZWEITE AUFLAGE
ERSTER BAND
BRAUNSCHWEIG
DRUCK UND VERLAG VON FRIEDRICH VIEWEG UND SOHN
1898
W

From *Algebra* (1895) to
Moderne Algebra (1930):
Changing Conceptions of
a Discipline

A Guided Tour Using the *Jahrbuch
über die Fortschritte der Mathematik*



Changing Conceptions of Algebra

Heinrich Weber

Lehrbuch der Algebra

Vol. 1 (1895)

Book 1: The Foundations

Book 2: The Roots

Book 3: Algebraic Magnitudes

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DER
ALGEBRA

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ZWEITE AUFLAGE

ERSTER BAND

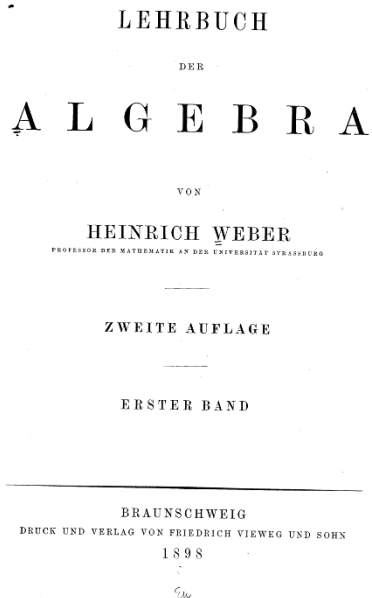
BRAUNSCHWEIG
DRUCK UND VERLAG VON FRIEDRICH VIEWEG UND SOHN
1895
W



Changing Conceptions of Algebra

Heinrich Weber

Lehrbuch der Algebra



Vol. 2 (1896)

Book 1: Groups

Book 2: Linear Groups

Book 3: Applications of Groups

Book 4: Algebraic Numbers



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Lehrbuch der Algebra

Vol. 3 (1891/1908)

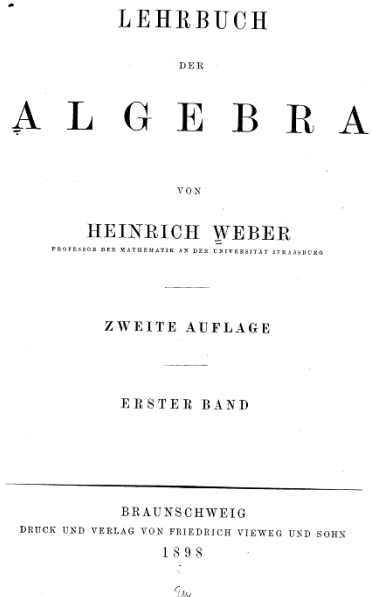
Book 1: Analytical Part

Book 2: Quadratic Fields

Book 3: Complex Multiplication

Book 4: Class Fields

Book 5: Algebraic Functions

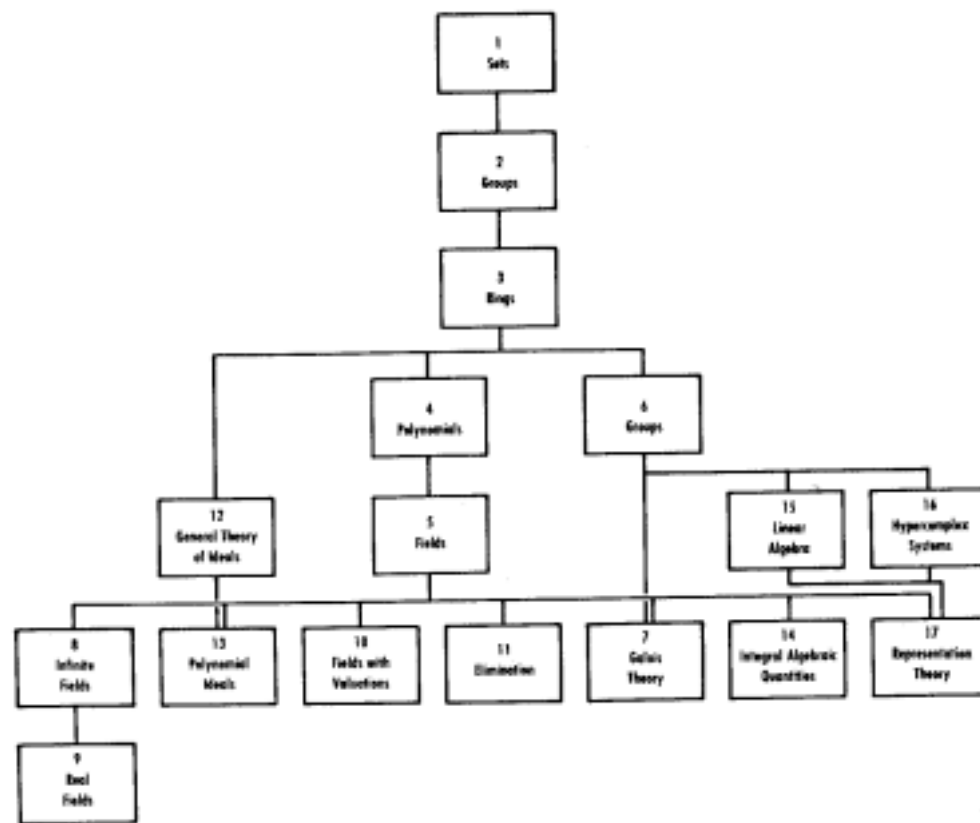




Changing Conceptions of Algebra

B.L. van der Waerden

Moderne Algebra (1930)



J a h r b u c h
über die
Fortschritte der Mathematik

im Verein mit anderen Mathematikern

herausgegeben

von

Dr. Carl Ohrtmann und Dr. Felix Müller.

Erster Band.

Jahrgang 1868.

Berlin.

Druck und Verlag von Georg Reimer.

1871.

Changing Conceptions of Algebra

*Jahrbuch über
die Fortschritte
der Mathematik*

Vol. 1 - 1868

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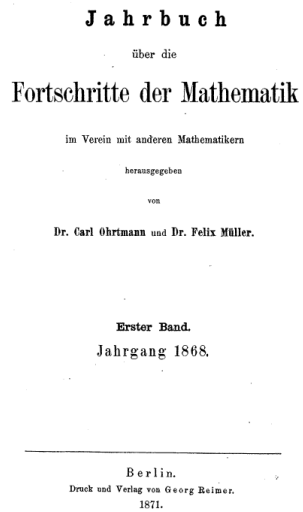
H. Weber

“Die allgemeinen Grundlagen der
Galois'schen Gleichungstheorie”
Math. Ann. **63** (1893).

In Section II: Algebra.

Ch. 1: Equations (General Theory . Special
Algebraic and Transcendental Equations)

Vol. 25 - 1896



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 31, 1900 (Publ. 1902)

Eds. Emil Lampe, Georg Wallenberg

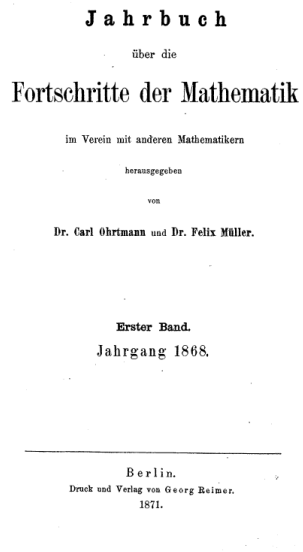
Section II - Algebra

Ch. 1: Equations

Ch. 2: Theory of Forms

Ch. 3: Substitutions and Group Theory

Section III - Lower and Higher Arithmetic



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Jahrbuch über die Fortschritte der Mathematik
Vol. 31, 1900 (Publ. 1902)

Section II - Algebra

Ch. 1: Equations

- * **General Theory**
- * **Special Algebraic and Transcendental Equations**
(Includes articles on “Galois Theory”)

Ch. 2: Theory of Forms

Ch. 3: Substitutions and Group Theory

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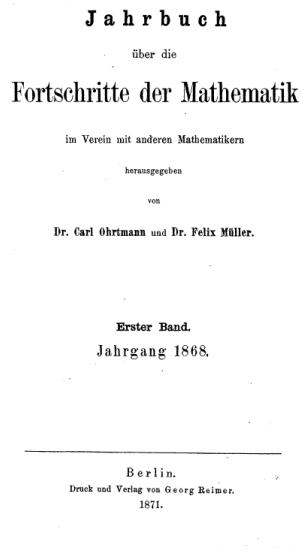
Section II - Algebra

Ch. 1: Equations

Ch. 2: Theory of Forms

* Theory of Invariants

Ch. 3: Substitutions and Group Theory



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Vol. 31, 1900 (Publ. 1902)

Section II - Algebra

Ch. 1: Equations

Ch. 2: Theory of Forms

Ch. 3: Substitutions and Group Theory

* **Determinants**

* **Elimination**

* **Symmetric Functions**

In Vol. 33 (1902):

E. Pascal

“Introduzione alla teoria invariantiva delle equazioni di tipo generale ai differenziali totali di secondo ordine (Memoria I)” *Annali di Mat.* (3) 7 (1902).

Classified as:

Differential and Integral Calculus →

Partial differential Equations

A. Theory of Algebraic Forms

B. Differential Invariants

Various articles by E. Pascal. For instance:

“Introduzione alla teoria delle forme differenziale di ordine qualunque” *Rom. Acc. L. Rend.* (5) 12.

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Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 36, 1905

Section II - Algebra

Ch. 1: Equations, Universal Algebra, Vector Analysis

A. Equations (as before)

B. Universal Algebra, Vector Analysis

... quaternions, vectors ...

Ch. 2: Theory of Forms

Ch. 3: Substitutions and Group Theory

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Jahrbuch über die Fortschritte der Mathematik Vol. 36, 1905

B. Universal Algebra and Vector Analysis

J.H.M. Wedderburn

“A theorem on finite
algebras”, *AM S Trans.* 6,
349-352 (1905).



In earlier vols. :

- “Note on the linear matrix equation” *Edinb. M. S. Proc.* 23 (1902), 49-53. **Classified as: Algebra → Substitutions and Group Theory → Determinants**
- “On the general scalar function of a vector” *Edinb. M. S. Proc.* 24 (1903), 409-412. **Classified as: Algebra → Equations**

J.H.M. Wedderburn “A theorem on finite algebras”, *AMS Trans.* 6, 349-352 (1905).

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Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 36, 1905

B. Universal Algebra and Vector Analysis

L.E. Dickson

“On finite algebras”, *Gött. Nachr.*

“On hypercomplex number
systems”, *AMS. Trans.* 6 (1905)



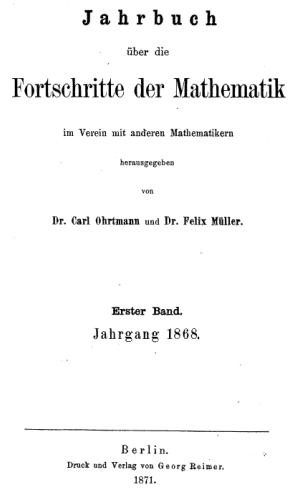
In earlier vols. :

- A huge amount of articles on groups
- “Fields whose elements are linear differential expressions”, *AMS Bull.* 10 (1903).
- “Definitions of a field by independent postulates”, *AMS Trans.* 4 (1903).
- **All classified as: Algebra → Substitutions and Group Theory → Determinants, Elimination, Symmetric Functions → Substitutions and Group Theory**
- “Definitions of a linear associative algebra by independent postulates”, *AMS Trans.*, 4 (1903). **Classified as: Algebra → Equations (rather than “Determinants” !!!)**

L.E. Dickson

“On finite algebras”

“On hypercomplex number systems”.



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik Vol. 36, 1905

B. Universal Algebra and Vector Analysis

I. Schur

“Zur Theorie der vertauschbaren
Matrizen”, *Crelle* 130 (1905).



In earlier vols. :

- „Über einen Satz aus der Theorie der vertauschbaren Matrizen”, *Berl. Ber.* (1902)
- **Classified as: Algebra → Group Theory, Determinants, Elimination → Determinants**

B. Universal Algebra and Vector Analysis

I. Schur

“Zur Theorie der vertauschbaren Matrizen”,
Crelle 130 (1905).

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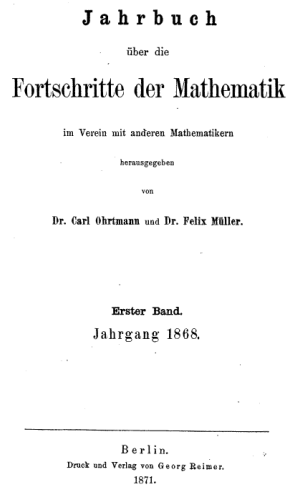
Jahrbuch über die Fortschritte der Mathematik
Vol. 36, 1905

**Section III: Lower and Higher
Arithmetic → Number Theory
→ Theory of Forms**

E. Lasker

“Zur Theorie der Moduln und
Idealen”, *Math. Ann.* 130
(1905).





Changing Conceptions of Algebra

Transactions of the AMS Vol. 10, 1909: Index Vols. 1-10

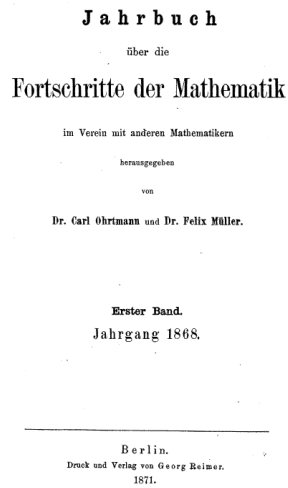
A. Logical Analysis of Mathematical Disciplines

Articles by Dickson, Huntington, E.H.

Moore, R.L. Moore

“Postulational analysis”

(appear also in different sections)



Changing Conceptions of Algebra

Transactions of the AMS Vol. 10, 1909: Index Vols. 1-10

J.H.M. Wedderburn

“A theorem on finite algebras”, *AMS Trans.* 6, 349-352
(1905).

(JFM: Universal Algebra and Vector Analysis)

Determinants. Symmetric Functions.

B2. Algebraic Forms

B3. Linear Associative Algebra. Hypercomplex
Systems. Fields

B4. Algebra of Logic.

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Transactions of the AMS Vol. 10, 1909: Index Vols. 1-10

E. Huntington

“Sets of Independent Postulates for Algebra of Logic”,
AMS Trans. 5, 288-309 (1904).

(JFM: History and Philosophy → Philosophy)

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**+ A. Logical Analysis of
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Determinants. Symmetric Functions.

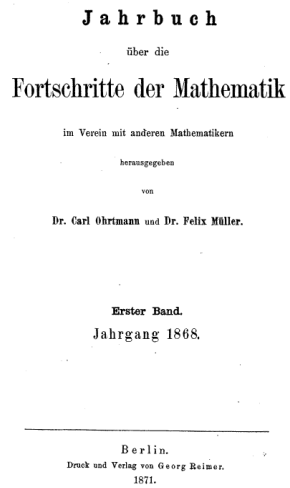
B2. Algebraic Forms

B3. Linear Associative Algebra. Hypercomplex

Systems. Fields

B4. Algebra of Logic.

**+ A. Logical Analysis of
Mathematical Disciplines**



Changing Conceptions of Algebra

Transactions of the AMS Vol. 10, 1909: Index Vols. 1-10

B. Theory of Numbers

E. Groups

E1. Discrete Groups in General

E2. Linear Groups in Arbitrary or Special Fields

E3. Continuous Groups

F. Geomertry

F2. Algebraic Geometry

JFM: “Analytic
Geometry”

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Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik Vol. 41, 1910



E. Steinitz

“Algebraische Theorie der
Körper“, *Crelle* 137
(1910).

**Classified as: Function
Theory → General**

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über die
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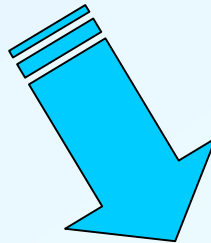
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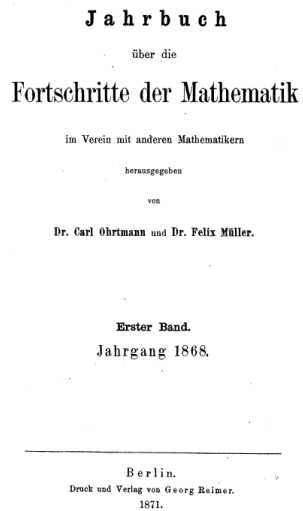
Jahrbuch über die Fortschritte der Mathematik
Vol. 46, 1916-17 (Publ. 1923/24)
Ed. Leon Lichtenstein

Section II - Algebra

Section III - Lower and Higher Arithmetic



Section II – Arithmetic and Algebra



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 46, 1916-17 (Publ. 1923/24)

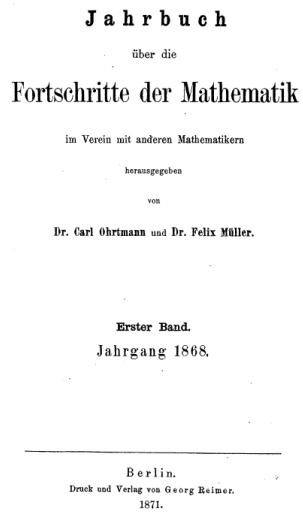
Section II – Arithmetic and Algebra

- Ch.1 Foundations of Arithmetic and Algebra. General
- Ch. 2 Elementary Arithmetic and Algebra. Combinatorics
- Ch. 3 Theory of Polynomials and Algebraic Equations
- Ch. 4 Theory of Forms. Determinants. Theory of Invariants
- Ch.5 Group Theory. Abstract Theory of Fields and Modules
- Ch. 6 Lower Theory of Numbers. Additive Number Theory
- Ch. 7 Arithmetic Theory of Forms
- Ch. 8 Algebraic Number Theory. Analytic Number Theory
- Ch. 9 Transcendental Numbers

Benjamin Abram Bernstein (1881-1964. Emeritus 1951)

In earlier vols. :

- “On an algebra of probability”, *AMS Bull.* (2) 18, 450 (1912).
Classified as: Section IV – Theory of Combinatorics and Probability Calculus
- “A set of four independent postulates for *Boolean* algebras”, *AMS Bull.* (2) 22, 6 (1915).
Classified as: Algebra → Equations, Universal Algebra and Vector Analysis → Universal Algebra and Vector Analysis
- **ALSO Classified as: History and Philosophy → Philosophy, Set Theory and Pedagogy → Philosophy**



Changing Conceptions of Algebra

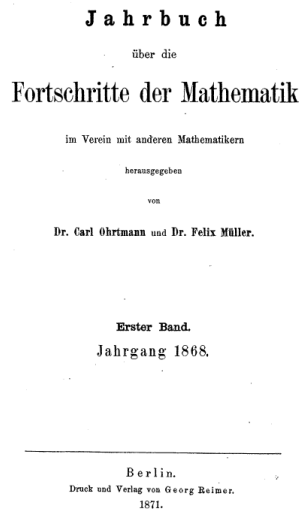
Jahrbuch über die Fortschritte der Mathematik
Vol. 46, 1916-17 (Publ. 1923/24)

Section II – Arithmetic and Algebra

Ch. 2 Elementary Arithmetic and Algebra. Combinatorics

E.g.: examples of elementary methods for calculating
logarithms, roots, etc.

**In earlier vols.: Combinatorics together with
Probability Calculus**



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 46, 1916-17 (Publ. 1923/24)

Section II – Arithmetic and Algebra

Ch. 3 Theory of Polynomials and Algebraic Equations.
Algebraic Properties of Polynomials. Root Division
(Verteilung der Wurzeln). **Galois Theory**

Ch. 4 Theory of Forms. Determinants. Theory of
Invariants. Symmetric Functions and Related Matters.
Bilinear and Quadratic Forms. Linear Substitutions.
Module Systems and Elimination

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Vol. 46, 1916-17 (Publ. 1923/24)

Section II – Arithmetic and Algebra

**Ch.5 Group Theory: Abstract Theory of
Fields and Modules. Group Theory. System
of Hypercomplex Numbers**

Mostly Group Theory!!!!

In earlier vols. :

- “Axiomatische Begründung von Hensels p -adischen Zahlen”, *Crelle* 141, 43-76. (1912)
- “Über die Teiler der Null und die Zerlegung von Ringen”, *Crelle* 145, 139-176 (1915). Diss. Marburg. (1914).
Classified as: Higher and Lower Arithmetic → Number Theory

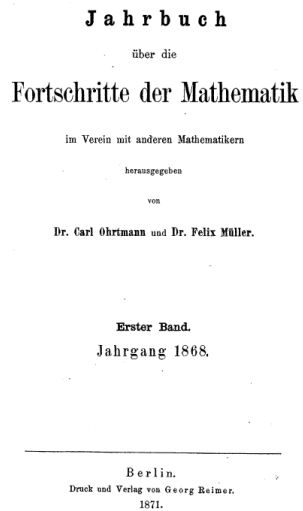
Ch.5 Group Theory: Abstract Theory of Fields and Modules. Group Theory. System of Hypercomplex Numbers

A. Fraenkel, *Über gewisse Teilbereiche und Erweiterungen von Ringen*, Leipzig: B. G. Teubner (1916).

In earlier vols. :

- “On the algebraic theory of modular systems (or modules of polynomials)”, *Brit. Ass. Rep. Australia* 84, 310-311 (1915).
Classified as: Algebra → Equations, Universal Algebra and Vector Analysis → Equations (General Theory)
- “On the resolution of a given modular system into primary systems including some properties of Hilbert numbers”, *Math. Ann.* 74, 66-121 (1913). [Macaulay-Lasker Theorem]
Classified as: Higher and Lower Arithmetic → Number Theory

F.S. Macaulay *The algebraic theory of modular systems*, Cambridge: University Press (1916).



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 46, 1916-17 (Publ. 1923/24)

Section II – Arithmetic and Algebra

Ch. 6 Lower Theory of Numbers. Additive Number Theory. Diophantine Equations.

Ch. 7 Arithmetic Theory of Forms

Ch. 8 Algebraic Number Theory. Analytic Number Theory

Ch. 9 Transcendental Numbers Approximation of Real Numbers by Rationals. Geometry of Numbers



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik

Vol. 48, 1921-22 (Publ. 1925/26)

Section II – Arithmetic and Algebra

Ch.5 Group Theory: Abstract Theory of Fields and Modules. Group Theory. System of Hypercomplex Numbers

W. Krull, “Algebraische Theorie der Ringe. I”, *Math. Ann.* 88, 80-122 (1922).

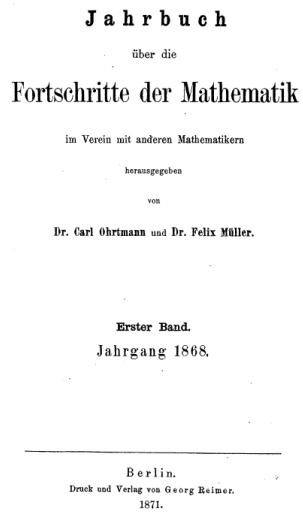
I. Schur “Über Ringbereiche im Gebiete der ganzzahligen linearen Substitutionen”, *Berl. Ber.* 1922, 145-168.

From Fraenkel's self-review:

The characterization of all possible types of fields in Steinitz's article "Algebraische Theorie der Körper" has suggested the task of thoroughly undertaking a similar research for the "rings" ...

Starting from a previous work in which results related with this task were derived, as far as possible without using the theory of ideals, the author undertakes once again this investigation under very general assumptions, and making essential use of the theory of ideals as presented in the path breaking (*einschlägigen*) work of Ms. Noether, in order to achieve a complete parallel between the theory of fields and the theory of rings.

(1921).



Changing Conceptions of Algebra

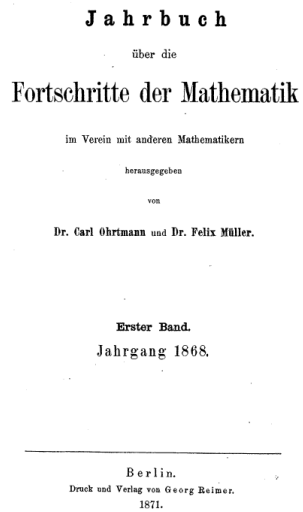
Jahrbuch über die Fortschritte der Mathematik
Vol. 49, 1923 (Publ. 1927/28)

Section II – Arithmetic and Algebra

Ch.5 Group Theory: Abstract Theory of Fields and Modules. Group Theory. System of Hypercomplex Numbers

L. E. Dickson *Algebras and their Arithmetics*,
Chicago: Univ. of Chicago Press. (1923).

L. E. Dickson “General Theory of
Hypercomplex Integers”, *AMS Bull.* 29, 200
(1923).



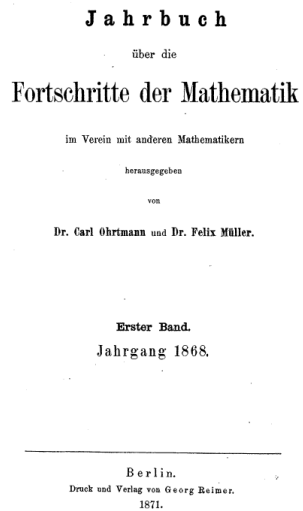
Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 51, 1925

Section II – Arithmetic and Algebra

Ch.5 Group Theory. Abstract Algebra

W. Krull “Über Multiplikationsringe“,
Sitzungsberichte Heidelberg 1925, Nr. 5, 13-
18.



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik **Vol. 51, 1925**

Section II – Arithmetic and Algebra

Ch. 7 Theory of Ideals

M. Sono “On the reduction of ideals”, *Memoirs Kyoto (A)* 7 (1924), 191-204.

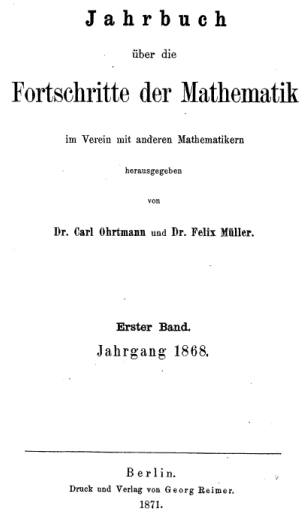
H. Hasse “Über das allgemeine Reziprozitätsgesetz in algebraischen Zahlkörpern”, *Jahresbericht DMV* 33, 97-101 (1925).

Emmy Noether's review :

The article presents a theory of relations among the ideals of super- and sub-rings, generally defined for a system of rings, of which only several isolated facts for the special cases of fields of numbers and domains of polynomials are already known. ...

It also announces a theory of fields of modules and ideals, namely one for domains in which the operations of addition, subtraction, intersection and quotient formation are closed with respect to modules and ideals. The results are derived, wherever possible, also for fields of ideals in rings. (See also Section II, Chap.

7)



Changing Conceptions of Algebra

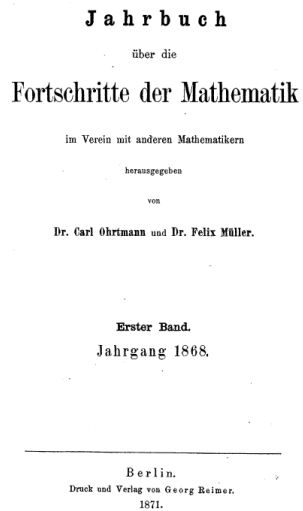
Jahrbuch über die Fortschritte der Mathematik
Vol. 56, 1930

Section II – Arithmetic and Algebra

Ch.5 Group Theory. Abstract Algebra

H. Hasse “Die moderne algebraische Methode”,
Jahresbericht DMV 39, 22-34 (1930).

B.L. van der Waerden *Moderne Algebra* (Unter
Benutzung von Vorlesungen von E. Artin und
E. Noether, Bd. I, Berlin, Springer (1930)



Changing Conceptions of Algebra

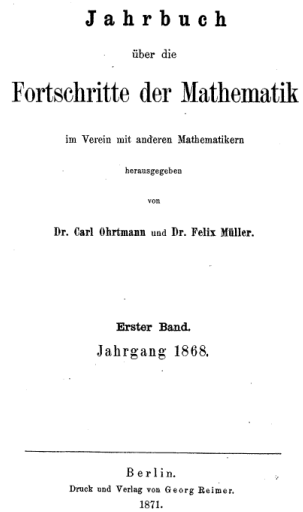
Jahrbuch über die Fortschritte der Mathematik **Vol. 61, 1935**

Section II – Arithmetic and Algebra

Ch.5 Group Theory. Abstract Algebra

A. Groups

B. Rings,
Fields



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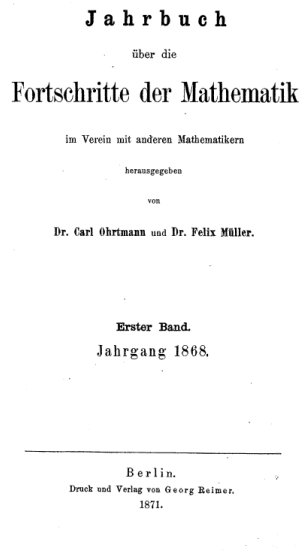
Jahrbuch über die Fortschritte der Mathematik
Vol. 63, 1937

Section II – Arithmetic and Algebra

Ch.5 Group Theory. Abstract Algebra

A. Groups

B. Lattices, Rings,
Fields



Changing Conceptions of Algebra

Jahrbuch über die Fortschritte der Mathematik
Vol. 65, 1939 (Publ. 1941)

C. Arithmetic and Algebra

Ch.1 General and Combinatorics

Ch. 2 Linear Algebra. Theory of Invariants

Ch. 3 Polynomials and Algebraic Equations

Ch. 4 Group Theory

Ch.5 Abstract Theory of Lattices, Rings and Fields

Ch. 6 Fields of Numbers and Functions

Ch. 7 Number Theory

Ch. 8 Diophantine Approximations and Transcendental Problems

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Jahrbuch über die Fortschritte der Mathematik
Until 1942

~~Commutative Algebra?~~

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DER
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VON
HEINRICH WEBER
PROFESSOR DER MATHEMATIK AN DER UNIVERSITÄT STRASSBURG
ZWEITE AUFLAGE
ERSTER BAND
BRAUNSCHWEIG
DRUCK UND VERLAG VON FRIEDRICH VIEWEG UND SOHN
1898
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From *Algebra* (1895) to
Moderne Algebra (1930):
Changing Conceptions of
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