Defining a Mathematical Research School:

The Case of Algebra at the University of Chicago 1892–1950

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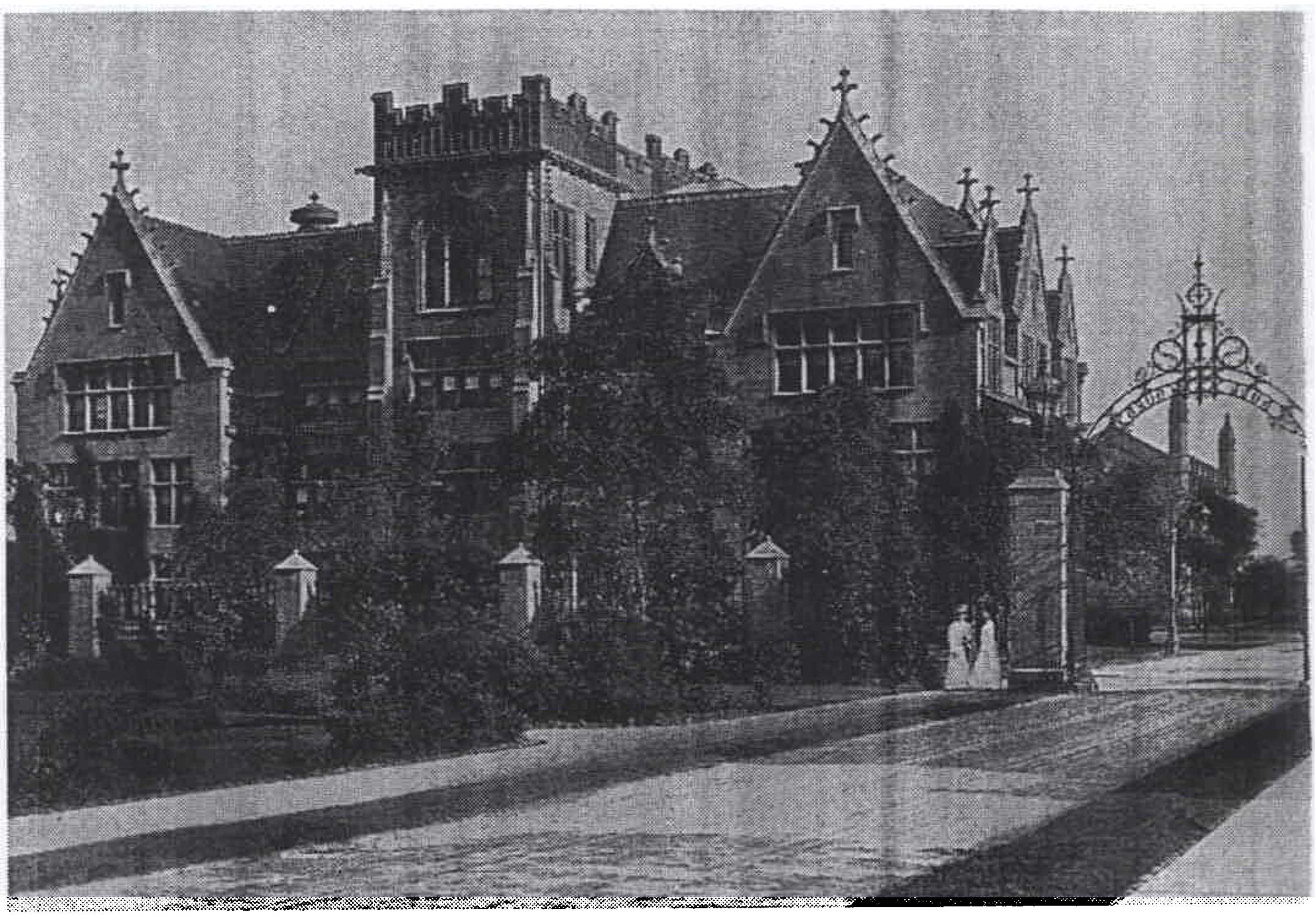

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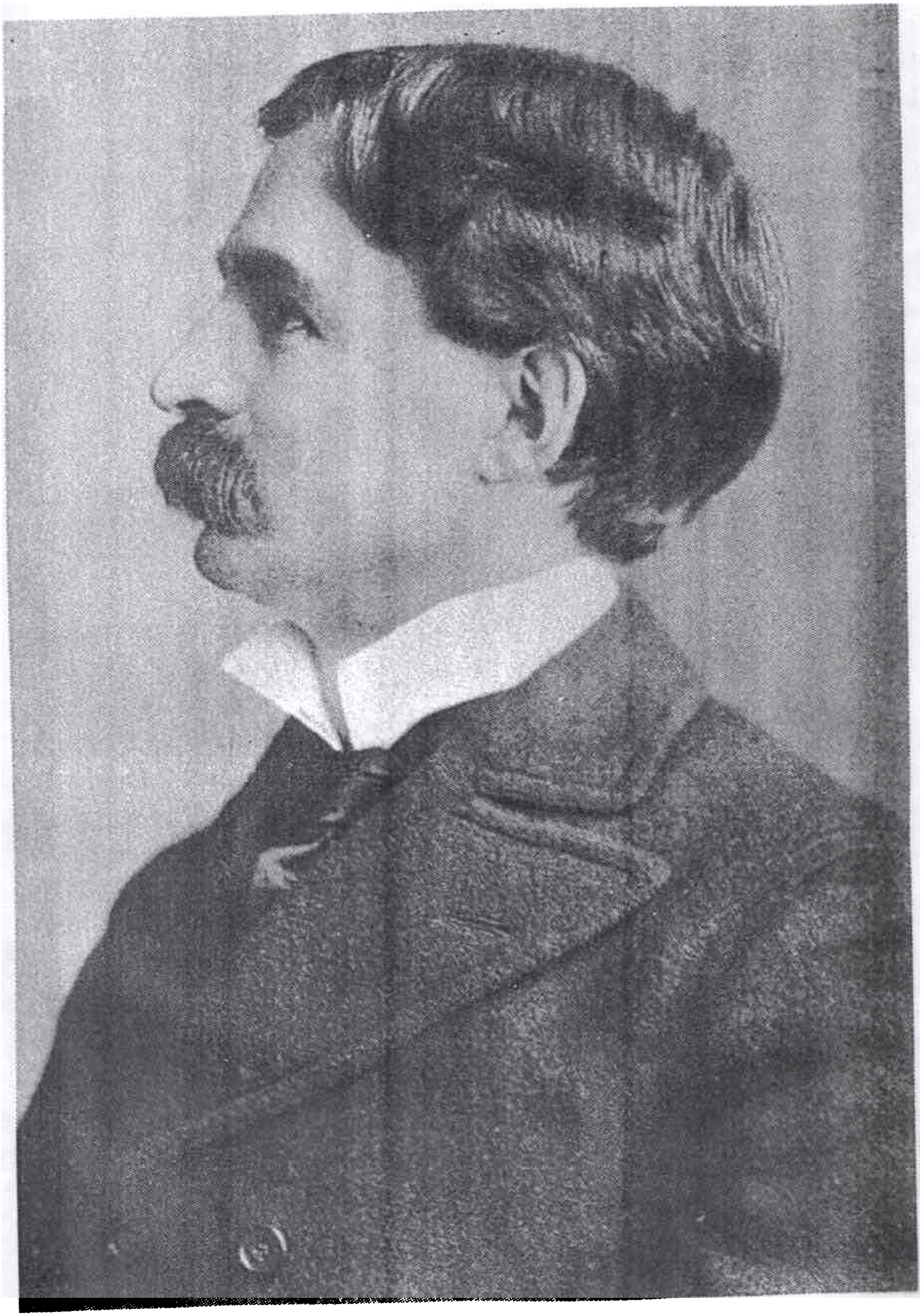


Eliakim Hastings Moore

Moore's Chicago Congress Paper 1893

There were six known types of finite simple groups in 1893. One of them was $PSL_m(p)$. The groups in this class have order

$$\frac{(p^m-1)p^{m-1}(p^{m-1}-1)p^{m-2}\ldots(p^2-1)p}{\delta},$$

where $(p,m) \neq (2,2), (3,2)$ and $\delta = \gcd(p-1,m)$.

In 1892, Moore had discovered a group of order 360–and in 1893, Cole had discovered one of order 504–that was not in one of the six known classes.

In his 1893 Congress paper, Moore found a new class of finite simple groups, the $PSL_2(p^n)$, of order

$$\frac{p^n(p^{2n}-1)}{\delta},$$

for $(p,n) \neq (2,1), (3,1)$. His group of order 360 is $PSL_2(3^2)$; Cole's group of order 504 is $PSL_2(2^3)$. Moore also proved:

Theorem: Every existent field F[s] is the abstract form of a Galois field $GF[p^n]$ where $s=p^n$.

Dickson's 1896 Thesis

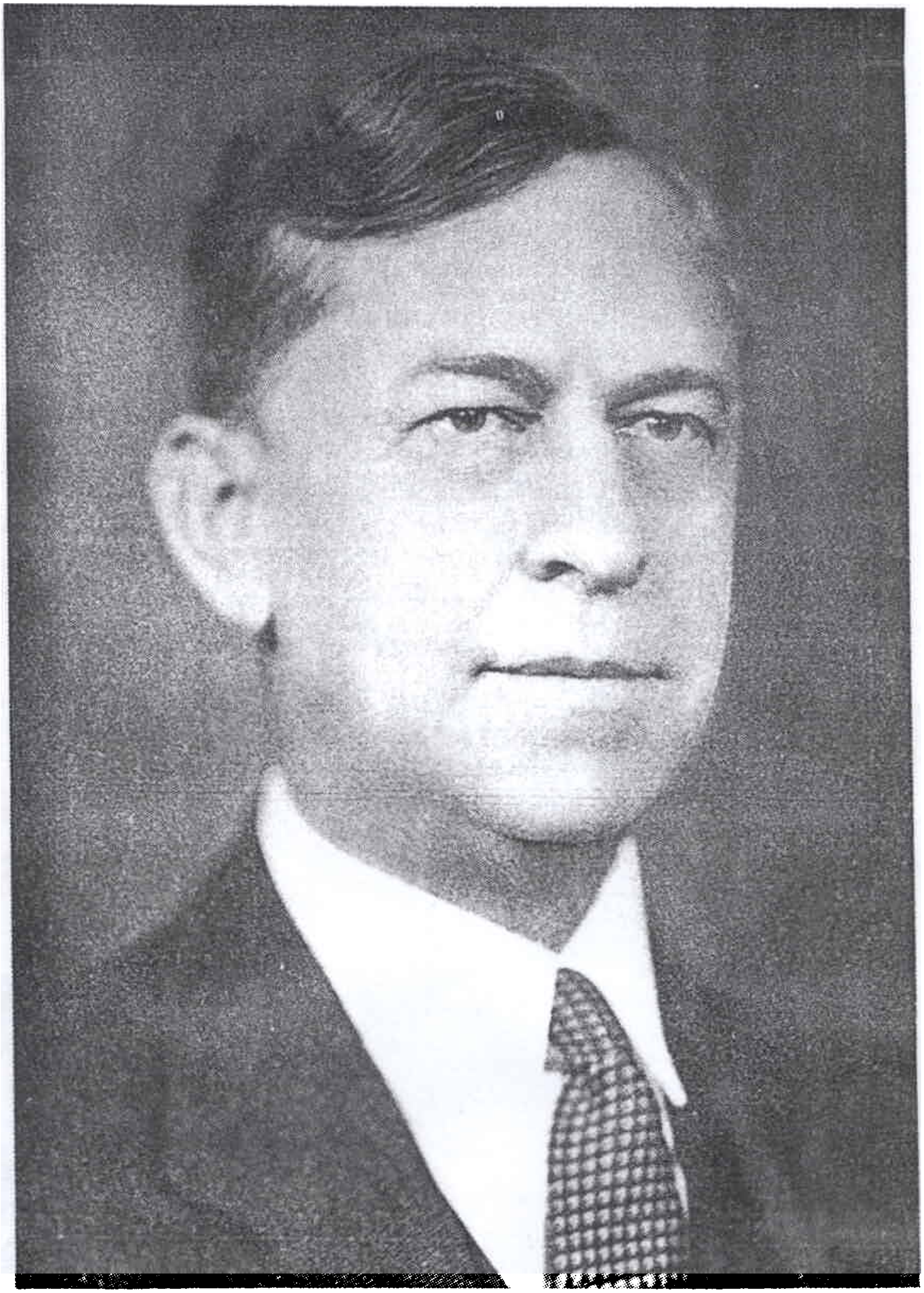
Let $F = GF[p^n]$ for p a prime and n in Z^+.

In Part 1 of his thesis, Dickson considered a polynomial $\varphi(X)$ of degree $k \leq p^n$ with coefficients in F and defined an associated map $\varphi : F \to F, \xi \to \varphi(\xi)$ to be a *substitution quantic* $SQ[k;p^n]$ of degree k on p^n letters provided it was bijective. He aimed to completely determine "all quantics up to as high a degree as practicable which are suitable to represent substitutions on p^n letters" and obtained complete results for degrees $k < 7$ and partial results for $k = 7, 11$.

In Part 2, he considered the general linear group $GL_n(F)$. He aimed to generalize Jordan's results on $GL_n(p)$ to $GL_n(F)$. One of his main results was:

Theorem: Let Z denote the center of $SL_m(F)$. $SL_m(F)/Z$ is simple provided $(m,n,p) \neq (2,1,2)$ or $(2,1,3)$.

He also found a new class of finite simple groups, viz., $SL_m(F)/Z$ for $m \geq 3$ and $n > 1$.



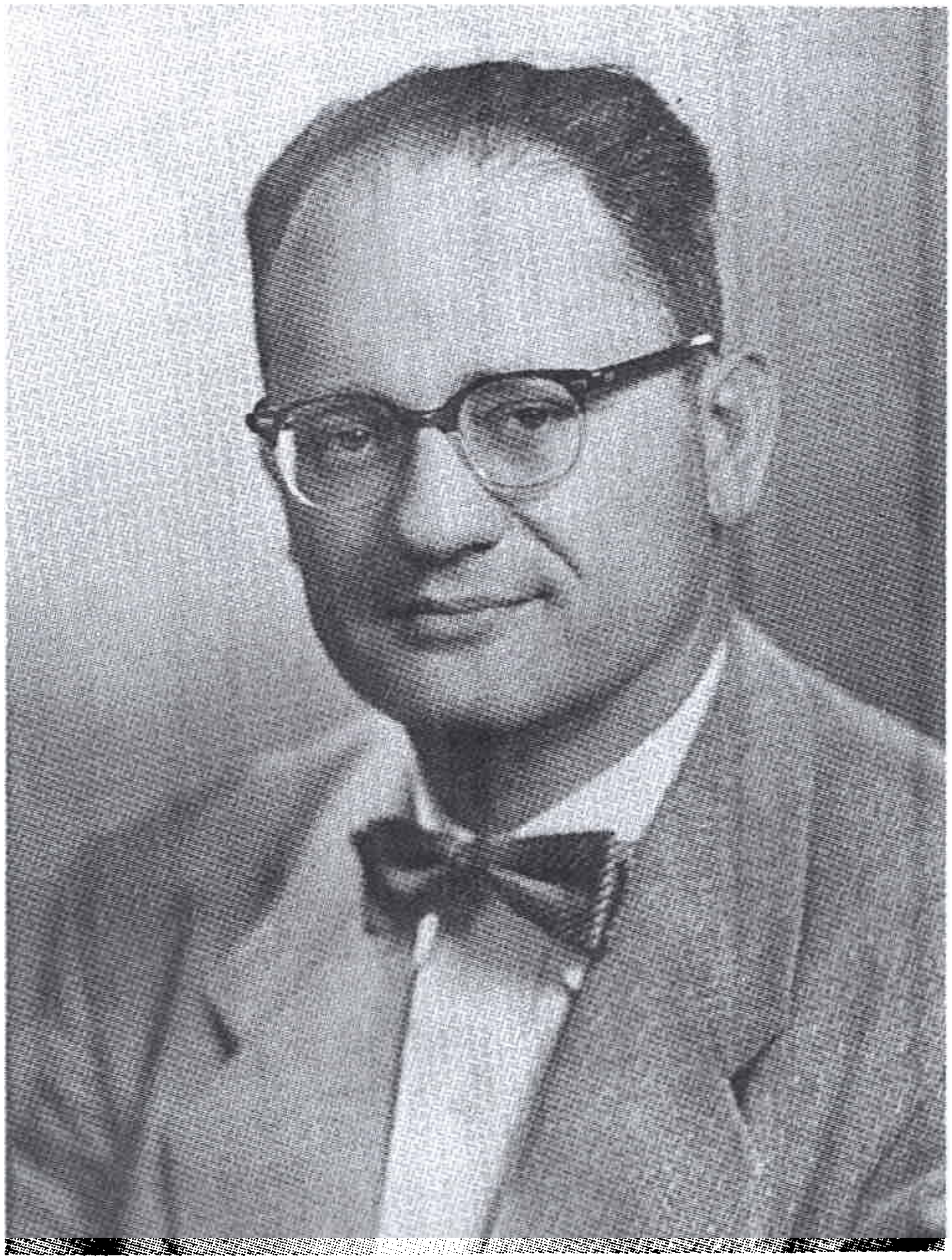
Leonard E. Dickson.

J. H. M. WEDDERBURN, F.R.S., D.Sc. (Edinburgh)

Leonard E. Dickson.

Bernardino Gaetano Scorza

Olive C. Hazlett



A. Adrian Albert

Nathan Jacobson

Jacobson's Work of 1945

Jacobson drew from the ring-theoretic work of Emmy Noether in the 1920s and 1930s as well as from Emil Artin's extension in 1927 of Wedderburn's structure theory to rings satisfying the descending chain condition on right ideals.

In a series of six papers in 1945, he laid the groundwork for a structure theory of rings without finiteness condition.

In particular, he defined the so-called *Jacobson radical* of a ring, defined as the set of all elements of a ring *R* which annihilate all the irreducible *R*-modules.

Jacobson's Ph.D. Students at Yale 1950—1960

Eugene Schenkman (1950), "A Theory of Subinvariant Lie Algebras"

Charles Curtis (1951), "Additive Ideal Theory in General Rings"

William Lister (1951), "A Structure Theory of Lie Triple Systems"

Henry Jacob (1953), "A Theorem on Kronecker Products"

George Seligman (1954), "Lie Algebras of Prime Characteristic"

Morris Weisfeld (1954), "Derivations in Division Rings"

Bruno Harris (1956), "Galois Theory of Jordan Algebras"

Earl Taft (1956), "Invariant Wedderburn Factors"

Dallas Sasser (1957), "On Jordan Matrix Algebras"

Maria Wonenburger (1957), "On the Group of Similitudes and Its Projective Group"

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1939

Albert's Students at Chicago 1945–1960

- Daniel Zelinsky (1946), "Integral Sets of Quasi-quaternion Algebras"
- Nathan Divinsky (1950), "Power Associativity and Crossed Extension Algebras"
- Charles Price (1950), "Jordan Division Algebras and Their Arithmetics"
- Murray Gerstenhaber (1951), "Rings of Derivations"
- David Merriel (1951), "An Almost Alternate Flexible Algebra"
- Louis Weiner (1951), "Lie Admissible Algebras"
- Louis Kokoris (1952), "New Results on Power-Associative Algebras"
- John Moore (1952), "Primary Central Division Algebras"
- Robert Oehmke (1954), "A Class on Non-Commutative Power-Associative Algebras"
- Eugene Paige (1954), "Jordan Algebras of Char 2"
- Richard Block (1956), "New Simple Lie Algebras of Prime Characteristic"
- James Osborn (1957), "Commutative Disassociative Loops"
- Laurence Harper (1959), "Some Properties of Partially Stable Algebra"

Morrell's Criteria for a "Research School"

- Existence of a leader
- Existence of manpower or students
- An area of inquiry with relatively simple experimental techniques that can be applied by brilliant and ordinary students to solve significant problems
- Publications
- Institutional power for the leader to allow for the realization of research goals
- Charisma
- Institutional support for the leader to assure the stability of the laboratory

Geison's Definition of "Research School"

A "research school" is "a small group of mature scientists pursuing a reasonably coherent programme of research side-by-side with advanced students in the same institutional context and engaging in direct, continuous social and intellectual interaction."

(See Gerald Geison, "Scientific Change, Emerging Specialties, and Research Schools," ***History of Science*** **19 (1981):20-40 on p. 23.)**

Mathematics vs. the Laboratory Sciences

- **Mathematics lends itself much more easily than the laboratory sciences to the individual investigator or to small groups of two or three investigators.**
- **Mathematics does *not* require close physical proximity.**
- **Mathematics *is* learned through a kind of apprenticeship, but the goal is the production less of a apprentice to serve the adviser's research agenda and more of an evolving, independent researcher.**
- **Mathematics *is* communicated through publications, and publications *do* serve to establish priority and reputation.**

First Approximation of Criteria for a "Mathematical Research School"

- Existence of a leader (who may or not be charismatic but who actively pursues research in a particular area of mathematics)
- Existence of a fundamental idea or approach to some set of inherently related research interests or research interests that become related by virtue of the idea or approach
- Students trained in the area and in the approach who then go out and pursue research in that area and according to that approach
- Publications