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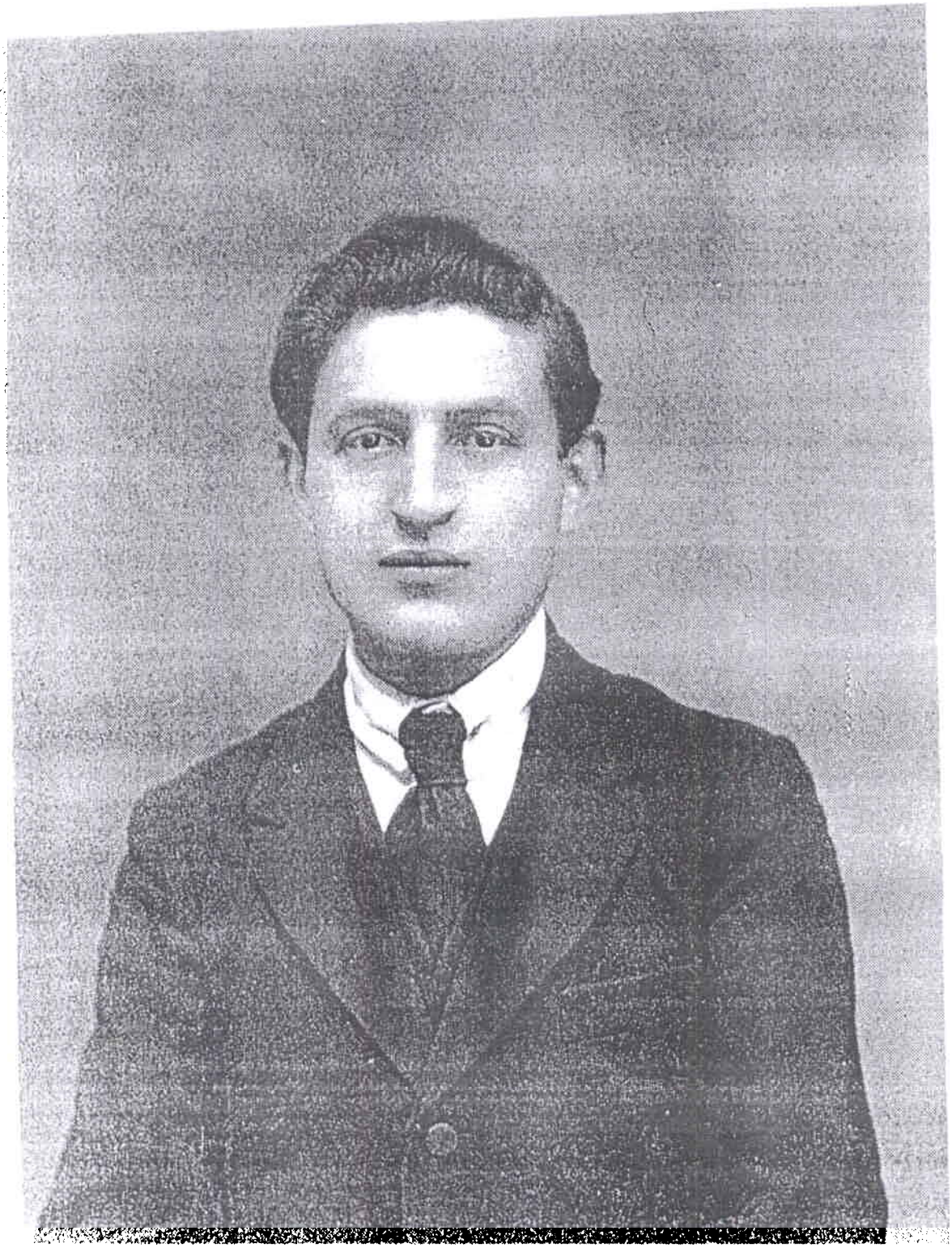

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THE REDUCTION OF THE SINGULARITIES OF AN ALGEBRAIC SURFACE

BY OSCAR ZARISKI

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INTRODUCTION

The reduction of the singularities of an algebraic surface has been studied by several authors, in particular by B. Levi [5], Chisini [2], Albanese [1] and R. J. Walker [9].[1] All these proofs, except the one by Walker, are essentially geometric. The proof by Walker, based in part on a paper by Jung [3], is analytic and is entirely rigorous. We give in this paper an *arithmetic* proof of the reduction theorem, based on the theory of valuations and the theory of ideals in algebraic function fields. From the standpoint of rigor, the arithmetic proof leaves nothing to be desired. We should say that the requirement of rigor is to be regarded as trivially satisfied in the present proof. More significant, however, is the clarification of the problem brought about by the use of the methods of modern algebra. The formal apparatus of ideal theory and the concepts of valuation theory enable one to gain full control of the intricate aspects of the problem, and to see clearly through the logical necessity of the various steps of the reduction process. What is gained concretely thereby is that the center of gravity of the proof is shifted from minute details to underlying concepts. As for these, we insist above all on clear-cut definitions, in order that at no time should it be less than perfectly clear what are the objects we are dealing with.