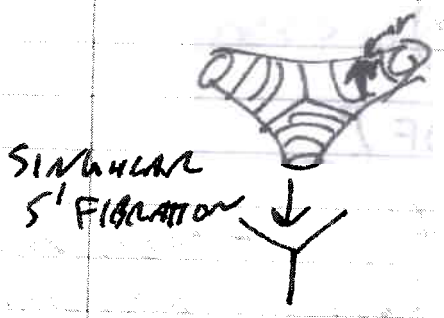
# MIKHALKIN: DIVISORS & TROPICAL JACOBIANS II

STARTING INGREDIENTS
- TROPICAL SEMIFIELD $\mathbb{R}$ $(\max,+)$ INDUCED ON $\mathbb{R}$ $(+,\times)$ BY $\log_t:\mathbb{R}_+\to\mathbb{R}$ (t → ∞)

WHAT'S INDUCED DEGENERATION OF CX STRUCTURE ON ALGEBRAS?




START FROM $\mathcal{C}$ ON $\Gamma$ TANGENT TO FIBER

$J_t$-STRUCTURE $\mathcal{V}\mapsto t i\mathcal{V}$

$T^{1,0}\subset TX\otimes\mathbb{C}$

$T_t=TX\otimes\mathbb{R}$

WE CAN ASSOCIATE A LIMITING OBJECT TO A FAMILY OF POLYNOMIALS

$f_t(x)=\sum_{j\in\mathbb{Z}^n}\alpha_j(t)x^j$, $x\in(\mathbb{C}^*)^n$

$V_t=\{f_t=0\}\subset(\mathbb{C}^*)^n$ — $\alpha_t(t)$ — (nisent series)

$\lim_{t\to\infty}\log_t(V_t)$

$\log_t(x_1,\dots,x_n)=(\log_t|x_1|,\dots,\log_t|x_n|)$

THIS IS (VIRO'S) PATCHWORKING

(2) TROPICAL CURVES (ABSTRACT)

(IN GENERAL TROPICAL VARIETIES ARE POLYHEDRAL COMPLEXES W/ $\mathbb{Z}$-AFFINE STRUCTURE

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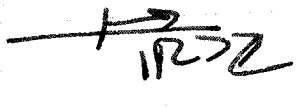
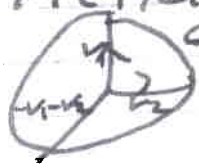
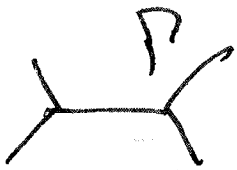
IN DIM 1 A TROPICAL CURVE IS A COMPLETE METRIC GRAPH:





$n$ VECTORS - AND NEED $\mathbb{R}^{n-1}$

Genus



$g(\Gamma) = b_1(\Gamma)$

$n$ = # ENDS OF $\Gamma$

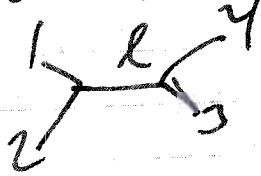
$\dim \mathcal{M}_{g,n} = 3g-3+n$

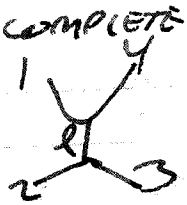
$M_{0,n}$ – TROP. VARIETY

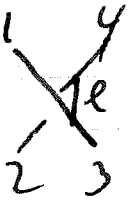
$\mathcal{M}_{g,n}$ – TROP. STACK

IT'S POSSIBLE TO DO INTERSECTION HERE - NOT COMPACT, BUT COMPLETE

Ex $M_{0,4} =$





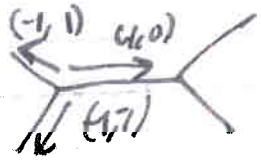


$M_{0,5} =$ cone on

$M_{1,1} =$

$\cong$ PUNCTURED $\mathbb{R}^2/\mathbb{Z}$ acts $\mathbb{Z}$ trivially

TROPICAL MAP: $\Gamma \to \mathbb{R}^n$ RESPECTING AFFINE STRUCTURE (i.e. TANGENT INTEGER VECTORS $\to \mathbb{Z}$)




ALSO MULTIPLICITIES ON 3-VALENT VERTICES

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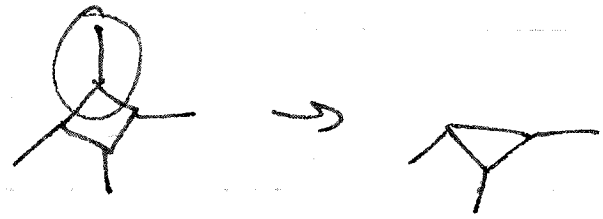
$M_{g,n+1} \to M_{g,n}$ Forget



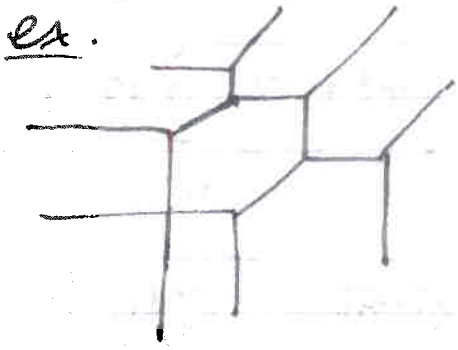
$M^{\beta}_{g,n}(\mathbb{R}^n)$ $\beta$ = "homology class" in $\mathbb{R}^n$ of degree

"set of directions of unbounded edges"

Agrees w/ degree for toric varieties

Ex $\mathbb{P}^2$, DEGREE $d$ CURVES

ex.

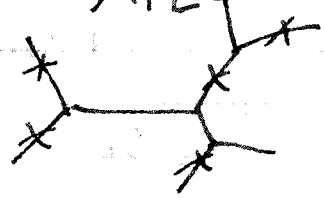


RATIONAL $\hookrightarrow$ PROJ. CUBIC

$ev: M^{\beta}_{g,n}(\mathbb{R}^n) \to \mathbb{R}$

INTERSECTIONS OF $ev^*$ (TROPICAL CYCLES) IN $M_{g,n}(\mathbb{R}^n)$ GIVE NUMBERS THAT COINCIDE WITH CORRESPONDING CX. #'s FOR $\mathbb{R}^2$ OR $\mathbb{R}^n$ (ANY GENUS) FOR $(\mathbb{C}^*)^n$

Ex 1 $\mathbb{R}^2$, ALL CYCLES POINTS.



x = MARKED POINTS

EACH IS PRESCRIBED A MULTIPLICITY $= \prod_{\text{VERTICES}}$ MULT

MULT. IS MIXED (NORMAL) PRODUCT $(v_1 \times v_2) v_3$

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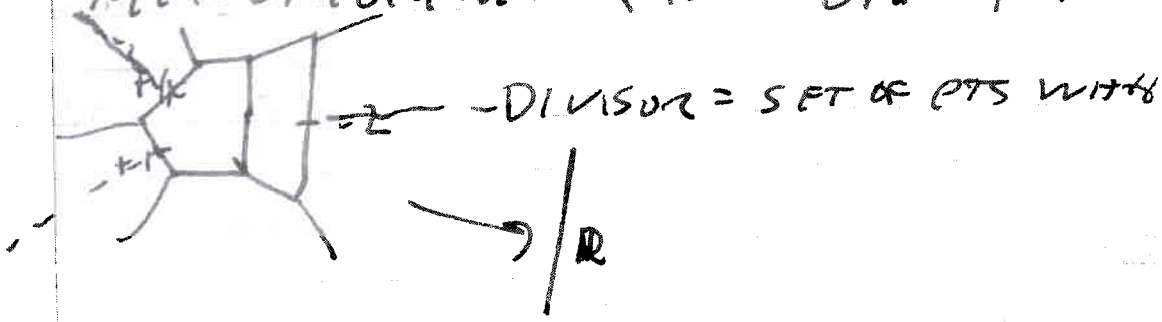
(4) DIVISORS & LINEAR EQUIVALENCE

MEROMORPHIC FN ON $\Gamma$.




Mittag-Leffler PROBLEM:
DOES D come from a meromorphic function

ASSOCIATE CLASSICALLY JACOBIAN TO $\Gamma$ $\mathbb{C}^g/\mathbb{Z}^g$ $w_1, \dots, w_g$

$(H^{1,0})^*/H_1(S;\mathbb{Z})$

$\mathbb{R}^g/\mathbb{Z}^g$ $H^1(\Gamma)^*/H_1(\Gamma)$

$\mathbb{C}^2/\begin{pmatrix} I & 0 \\ -B & I \end{pmatrix}$

TROPICAL $\theta$ FN (ALEXEEV) QUADRATIC FORM

" $\sum_n n^2 x^n$ " OR " $\sum B(n,n) x^n$ "

THETA DIVISOR: Sym$^g(\Gamma) \to$ Jac

QUESTIONS