ORIENTIFOLDS

AN INTRODUCTION

MARCH 24, 2004 MSRI

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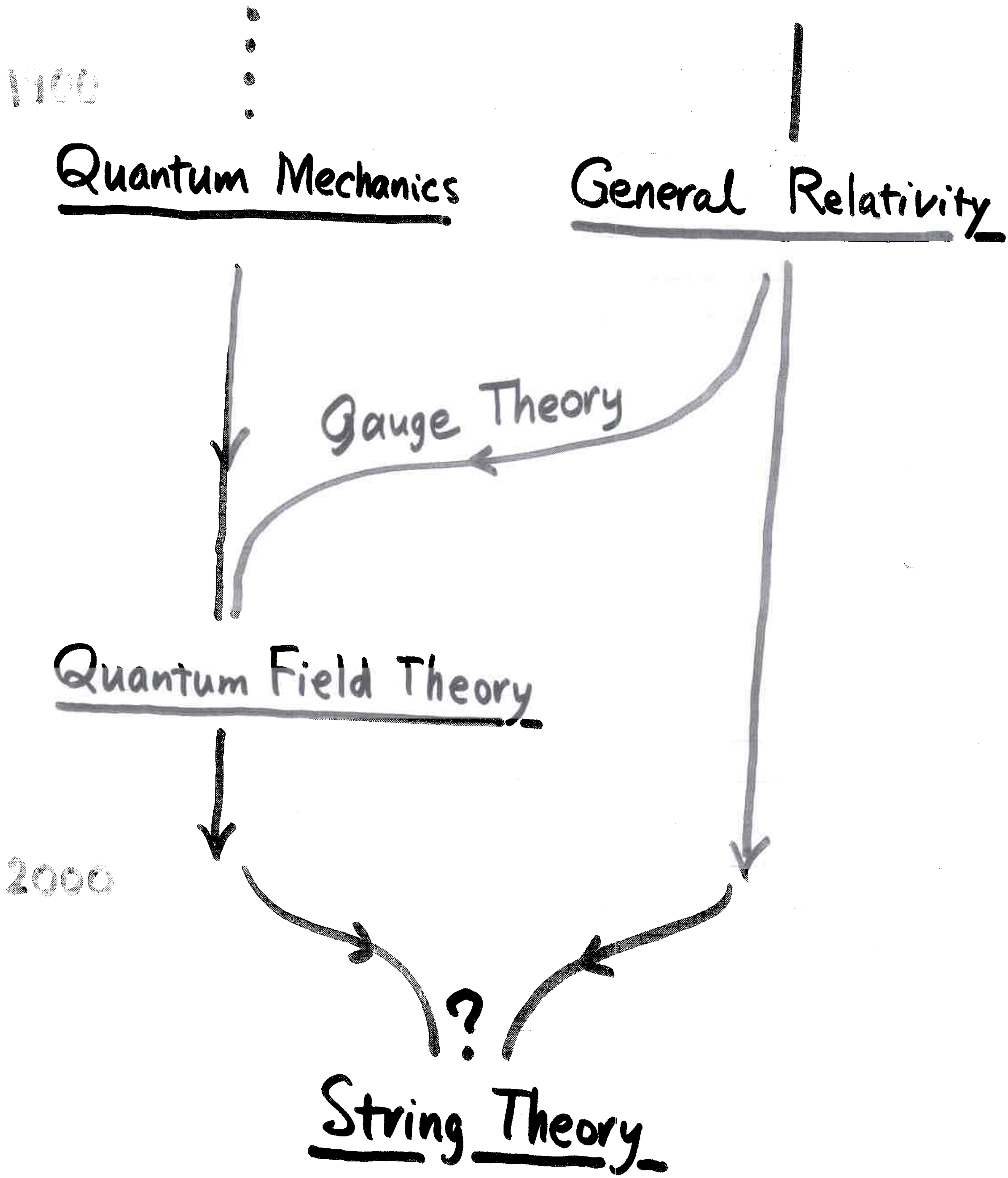
I. Brunner, K. Hosomichi, J. Walcher, KH :
"Orientifolds of Gepner Models" 04

I. Brunner, KH : "Orientifolds & Mirror Symmetry" 03

B. Acharya, M. Aganagic, C. Vafa, KH
"Orientifolds, Mirror Symmetry, Superpotentials" 0

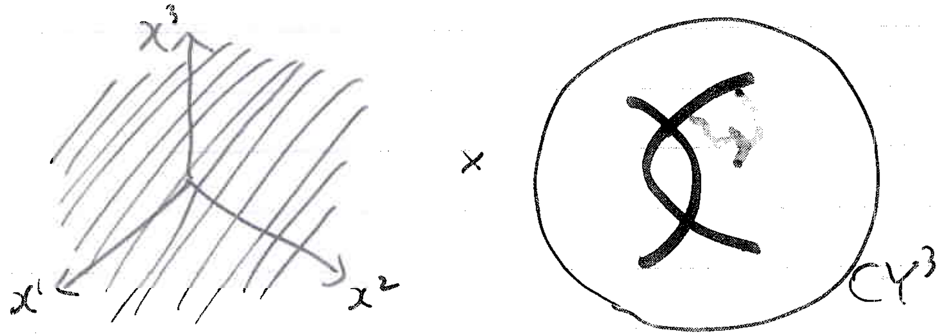

Quantum Mechanics
General Relativity
Gauge Theory
Quantum Field Theory
2000
?
String Theory

Type II String Theory on $\mathbb{R}^{3+1} \times CY^3$

… $\mathcal{N}=2$ Supersymmetry in $\mathbb{R}^{3+1}$

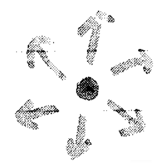
$\Downarrow$

+ D-branes at $\mathbb{R}^{3+1} \times W \subset CY^3$



… $\mathcal{N}=1$ Supersymmetry in $\mathbb{R}^{3+1}$

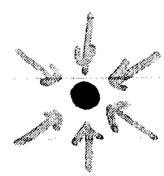
But D-branes generate RR flux



… requires something that absorbes it

⋮

Orientifolds



(Non-compact CY^3 avoids the problem
but $\nexists$ 4d Gravity)

$$Z_i\left(t(\mu); \frac{\mu_1}{\mu}\right)$$

$$\therefore\ t(\mu) = t_1(\mu_1) - b_1 \log\left(\frac{\mu_1}{\mu}\right) - \sum_i T_{R_i} \log Z_i\left(\frac{\mu_1}{\mu}\right)$$

Math:

Mirror Symmetry

Symplectic Geometry $\longleftrightarrow$ Algebraic Geometry

:	GW invariants	VHS
	⋮	⋮
:	Lagrangian submfds	Coherent Sheaves
	⋮	⋮

Orientifolds: "Reality"

needs to be introduced

Orientifold is to gauge a parity symmetry of the worldsheet

$$P = \tau\Omega : X(t,\sigma) \to \tau X(t,-\sigma)$$

4d $\mathcal{N}=2$ SUSY $\Rightarrow$ WS $(2,2)$ SUSY

$\cup$

4d $\mathcal{N}=1$ SUSY $\Rightarrow$ diagonal $\mathcal{N}=2$ SUSY

2 kinds of $\mathcal{N}=2_{\text{diag}}$.

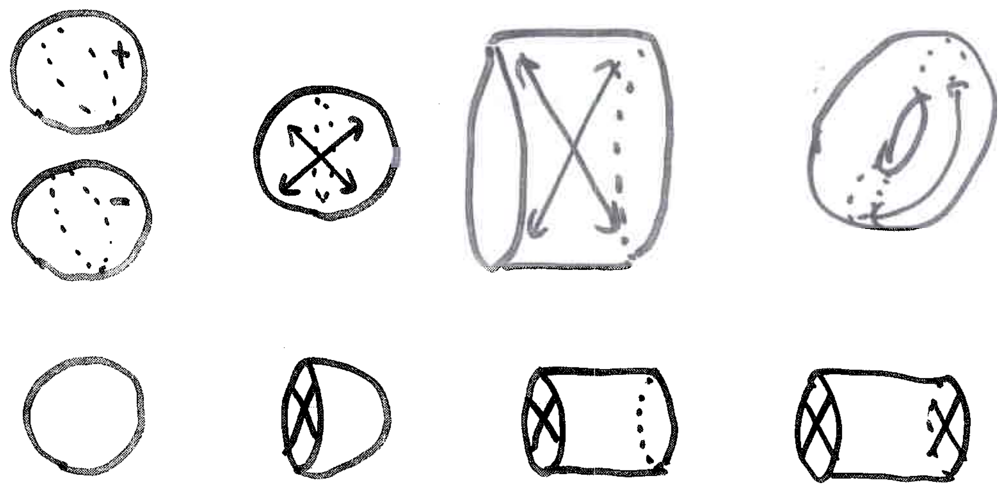
A-type (Type IIA) : $\tau : M \to M$ antiholomorphic

B-type (Type IIB) $\tau : M \to M$ holomorphic

Σ unoriented $\rightsquigarrow$ $\tilde{\Sigma}$ oriented double cover

$\tilde{\Sigma} \circlearrowleft \Omega$

Σ



Sigma model with target X, involution τ

field ϕ : section of

$$\begin{array}{c} \tilde{\Sigma} \times_{\mathbb{Z}_2} X \\ \downarrow \\ \Sigma \end{array}$$

or

$$\begin{tikzcd} \tilde{\Sigma} \arrow[r, "\tilde{\phi}"] \arrow[d, "\Omega"'] & X \arrow[d, "\tau"] \\ \tilde{\Sigma} \arrow[r] & X \end{tikzcd}$$

$\mathbb{Z}_2$-equiv. map

Linear Sigma Model : $U(1)$ gauge theory

matters	P	X_1	X_2	X_3	X_4	X_5	
charges	-1	$1/h_1$	$1/h_2$	$1/h_3$	$1/h_4$	$1/h_5$	$\sum_{i=1}^{5}\frac{1}{h_i}=1$

superpotential $W = P(X_1^{h_1}+X_2^{h_2}+X_3^{h_3}+X_4^{h_4}+X_5^{h_5})$

FI $\gg 0$: NLσM on a Calabi-Yau

$$X_1^{h_1}+\cdots+X_5^{h_5}=0 \quad \text{in a } W\mathbb{CP}^4$$

FI $\ll 0$: $\langle P\rangle \neq 0$ Landau Ginzburg Orbifold

- $W = X_1^{h_1}+\cdots+X_5^{h_5}$
- Γ = generated by $\gamma : X_i \to e^{\frac{2\pi i}{h_i}} X_i \quad \forall i$

$\cong \mathbb{Z}_H \qquad H = \text{l.c.m.}\{h_1,\dots,h_5\}$

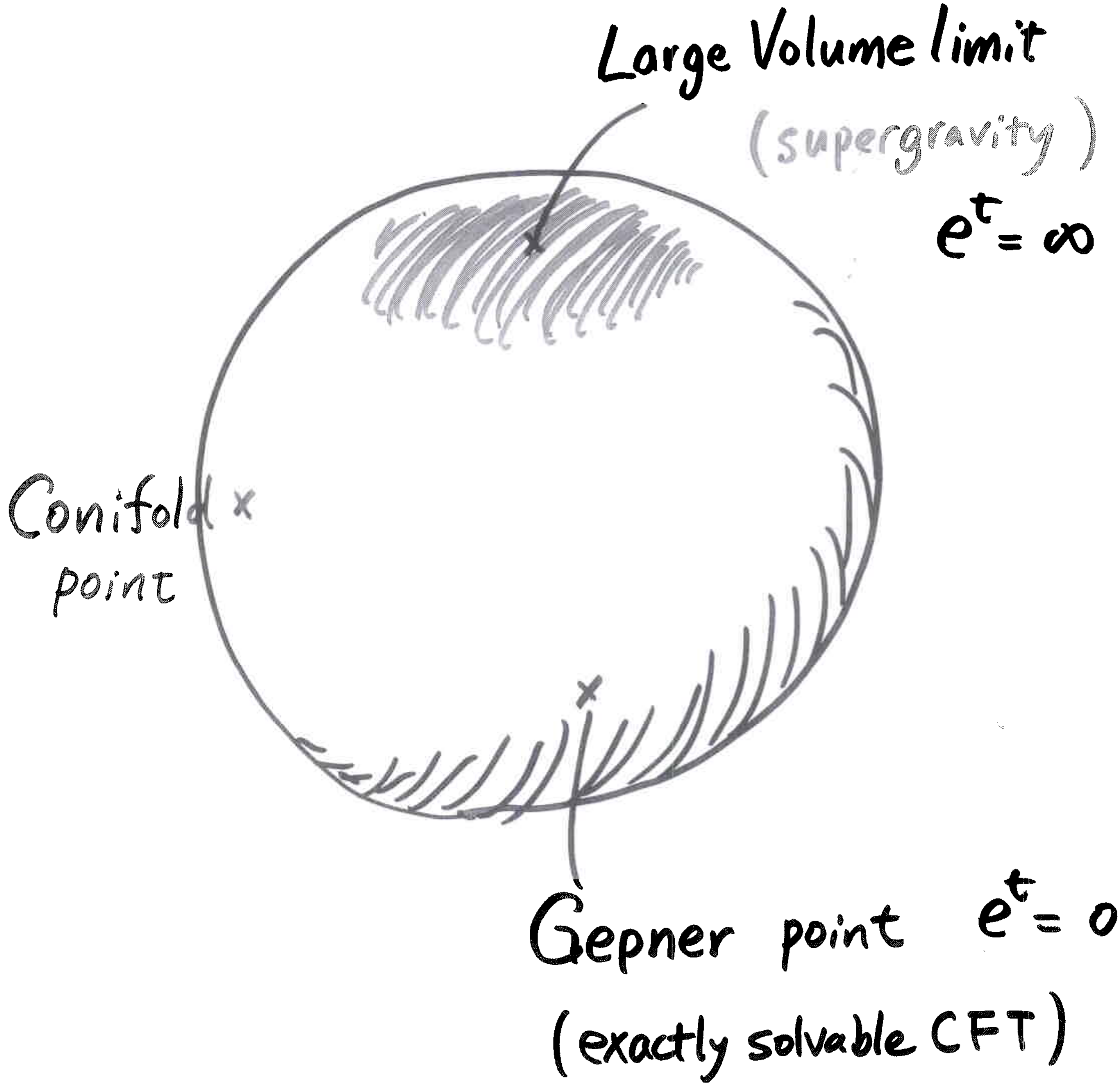
= Gepner Model

$$M_{h_1}\times M_{h_2}\times M_{h_3}\times M_{h_4}\times M_{h_5} / \mathbb{Z}_H$$

M_h : IR fixed point of LG $W=X^h$

$= SU(2)_{k=h-2}/U(1)$ supercoset

"$\mathcal{N}=2$ minimal model"



$$t = \underset{\text{FI}}{r} - i\theta$$

$= \text{Kähler class} - i\ B\text{-field}$

Parity Symmetries

basic parity $\times \begin{cases} \vec{m} : X_i \text{ phase rotation } & \dfrac{\prod_i \mathbb{Z}_{h_i}}{\mathbb{Z}_H} \\ \omega : \text{quantum symmetry} & \mathbb{Z}_H \end{cases}$

σ : permutation $\quad h_{\sigma(i)} = h_i \, , \; \sigma^2 = 1$

A - Parity $\quad P^A_{\omega;\vec{m},\sigma}$

$$P \to \bar{P}$$

$$X_i \to e^{2\pi i \frac{m_i}{h_i}} \overline{X}_{\sigma i}$$

dual variables $\quad \tilde{X}_i \to e^{\frac{\pi i}{h_i}} \omega_i \tilde{X}_{\sigma i} \qquad \omega_i \omega_{\sigma i} \equiv 1$

B - Parity $\quad P^B_{\omega;\vec{m},\sigma}$

$$P \to -P$$

$$X_i \to e^{2\pi i \frac{m_i}{h_i}} X_{\sigma i} \qquad m_i + m_{\sigma(i)} \equiv 0$$

$$\tilde{X}_i \to \omega_i \overline{\tilde{X}_{\sigma i}}$$

Quintic $X_1^5+X_2^5+X_3^5+X_4^5+X_5^5=0$ in $\mathbb{C}P^4$ $\begin{cases} K=1_{\mathbb{C}} \\ C=101_{\mathbb{C}} \end{cases}$

<u>A: $X_i \to \overline{X}_{\sigma(i)}$</u> $\forall\sigma$: $K=1_{\mathbb{C}}$, $C=101_{\mathbb{R}}$

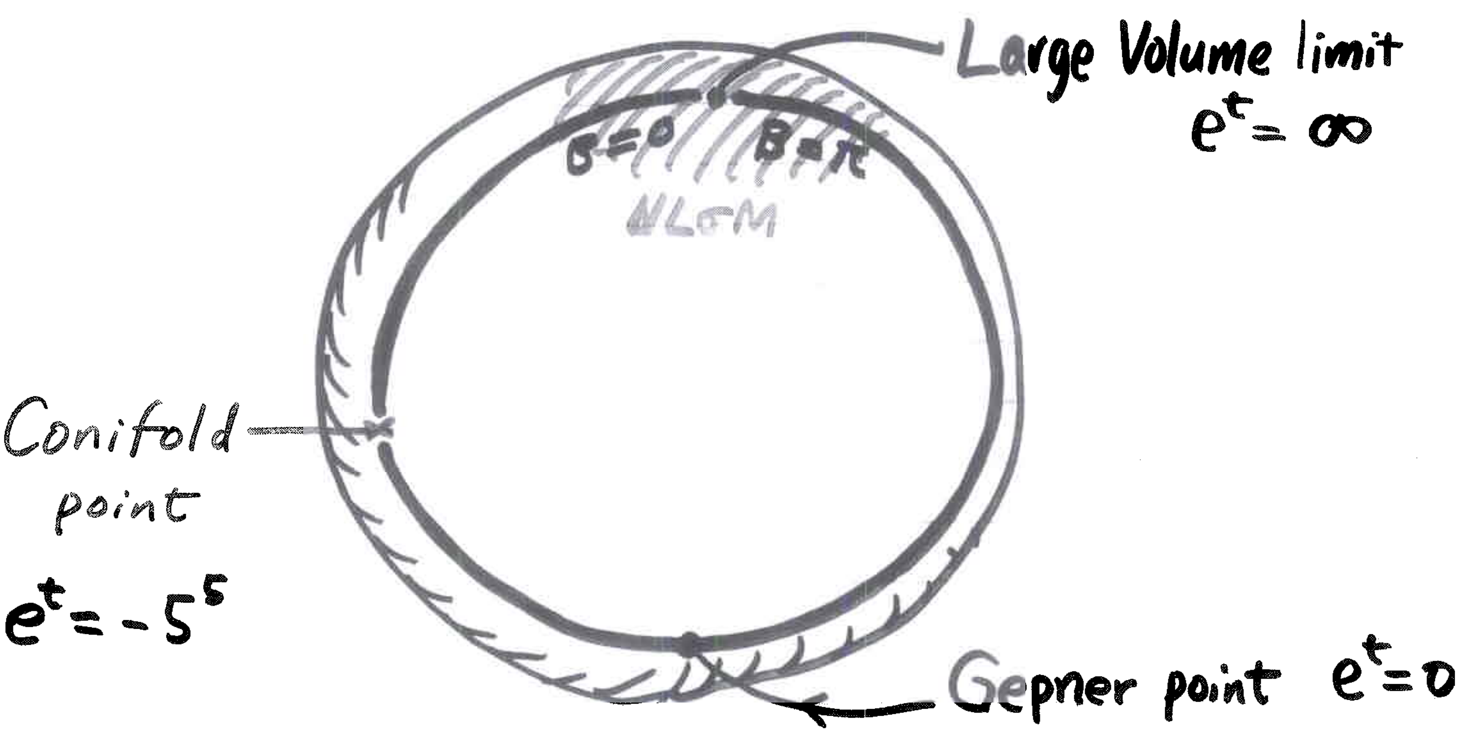
<u>O6</u> at an $\mathbb{RP}^3$

<u>B : $X_i \to X_{\sigma(i)}$</u> $\sigma=id$: $(K,C)=(1_{\mathbb{R}}, 101_{\mathbb{C}})$ <u>O9</u>

$\sigma=(12)$: $(1_{\mathbb{R}}, 63_{\mathbb{C}})$ <u>O3</u> + <u>O7</u>

$\sigma=(12)(34)$: $(1_{\mathbb{R}}, 53_{\mathbb{C}})$ $\underbrace{\text{O5}}_{g=0}$ + $\underbrace{\text{O5}}_{g=6}$

<u>Kähler moduli (B)</u> <u>e^t : real</u>



A "Two parameter" Model

$$X_1^8 + X_2^8 + X_3^4 + X_4^4 + X_5^4 = 0 \quad \text{in } W\mathbb{CP}^4_{11222}$$

$$K = 2_c \quad C = 86_c$$

$\sigma = 1$

(A) $X_i \to e^{\frac{2\pi i}{h_i} m_i} \overline{X_i}$

fixed point : $X_i = e^{\frac{\pi i}{h_i} m_i} x_i \qquad x_i \in \mathbb{R}$

$$(-1)^{m_1} x_1^8 + (-1)^{m_2} x_2^8 + (-1)^{m_3} x_3^4 + (-1)^{m_4} x_4^4 + (-1)^{m_5} x_5^4 =$$

(i) $(-1)^{m_i} \equiv 1$: No solution

NO O-plane

(ii) $(-1)^{m_i} = (-1, 1, 1, 1, 1)$

O6 at S^3

(iii) $(-1)^{m_i} = (1, 1, 1, 1, -1)$

O6 at $S^3 \vee S^3$ (meeting at S^1) $\xrightarrow{\text{resolve}}$ a single $S^2 \times S^1$

(iv) $(-1)^{m_i} = (1, 1, 1, -1, -1)$

O6 at T^3

Roiban-Romelsberger-Walcher

B $X_i \to \epsilon_i X_i$ $\epsilon_i = \pm 1$

$(+++++)$: O9

$(++-++)$: O7 at a hypersurface

$(++--+)$: O5's at $\Sigma_{g=9}$ & four S^2's.

$(++---)$: O7 at a hypersurface
& O3's at eight points

$(+-+++)$: O7's at two K3 hypersurfaces
homologous

$(+--++)$: O5's at two $\Sigma_{g=3}$'s
homologous

$(+---+)$: O3's at Twelve points

$(+----)$: O5's at two $\Sigma_{g=3}$'s
homologous

Moduli Space

$K:\ t \begin{cases} t_1 \\ t_2 \end{cases}$ $\quad t = 2t_1 + t_2 \quad$ (resolution of $\mathbb{C}^2/\mathbb{Z}_2$ s.i in $W\mathbb{CP}^4_{11222}$

	P	x_1'	x_2'	x_3	x_4	x_5	x_6
$U(1)_1$	-4	0	0	1	1	1	1
$U(1)_2$	0	1	1	0	0	0	-2

$U(1) \to U(1)_1,\ U(1)_2$

$x_i'^2 = x_i^2$, $i=1,2$

mirror: $$\tilde{W} = \tilde{X}_1^8 + \tilde{X}_2^8 + \tilde{X}_3^4 + \tilde{X}_4^4 + \tilde{X}_5^4 + e^{\frac{2t_1+t_2}{8}} \tilde{X}_1\tilde{X}_2\tilde{X}_3\tilde{X}_4\tilde{X}_5 + e^{\frac{t_2}{2}} \tilde{X}_1^4 \tilde{X}_2^4$$

Singularity at $e^{t_2} = 4$ & $e^{t_2}(1-4^{-4}e^{t_1})^2 = 4$

Ⓐ involutive $\Rightarrow$ $\omega = \pm 1$: $\tilde{X}_1 \to \pm\tilde{X}_1$, $\tilde{X}_i \to \tilde{X}_i$ $\quad i = 2 \sim 5$

$\omega = 1$ $\quad$ K : (t_1, t_2) unconstrained $\quad 2_{\mathbb{C}}$

C : real constraint $\quad 86_{\mathbb{R}}$

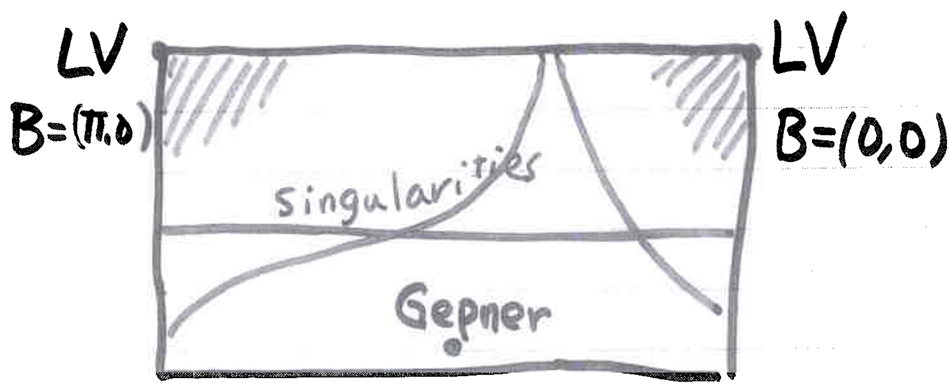
$\omega = -1$ $\quad$ K : $e^{\frac{t}{8}} = 0$, $e^{\frac{t_2}{2}}$ unconstrained $\quad 1_{\mathbb{C}}$

Completely non-geometric

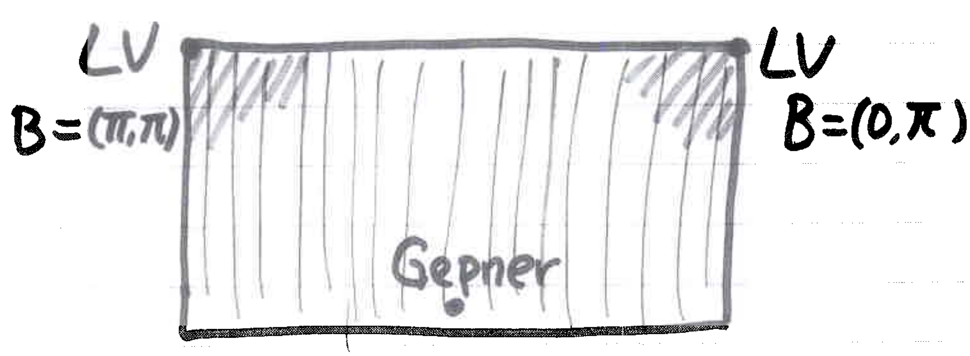
C : real constraint $\quad 86_{\mathbb{R}}$

B $\omega \equiv 1$ or $e^{2\pi i/8}$

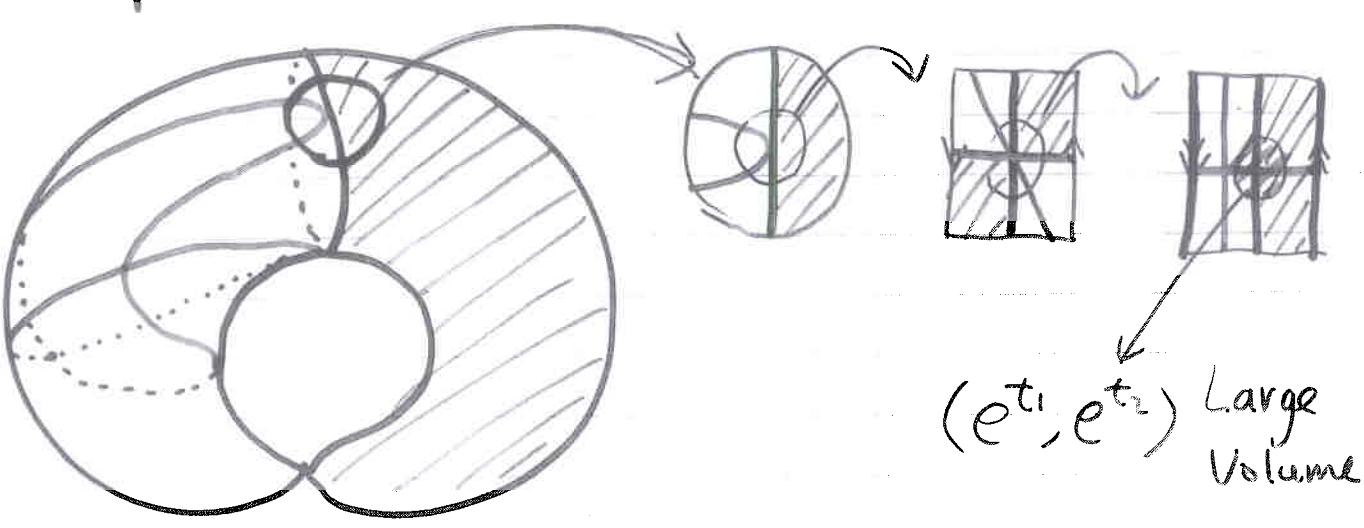
$\underline{\omega \equiv 1}$ K: $e^{t_1} \in \mathbb{R}$ $e^{t_2} \in \mathbb{R}_{\geq 0}$



$\underline{\omega \equiv e^{2\pi i/8}}$ K: $e^{t_1} \in \mathbb{R}$ $e^{t_2} \in \mathbb{R}_{\leq 0}$



Global picture:



C : recall: 86 = 83 polynomial (untwisted) + 3 non-polynomial (twisted)

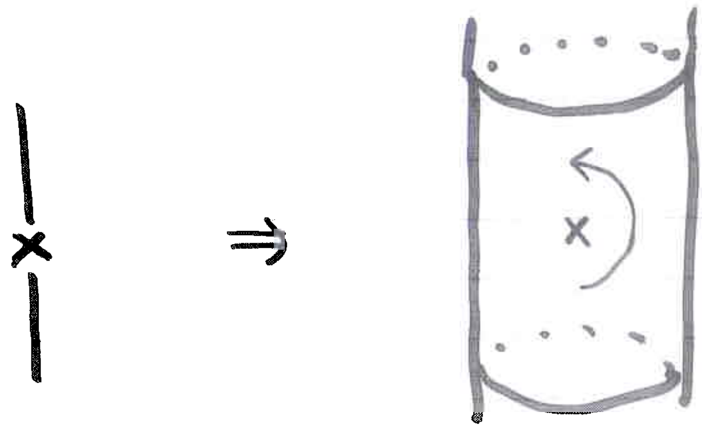
		Gepner pt		Large Volume
(+++++)	$\omega = 1$	83	$\neq$	86
	$\omega = e^{2\pi i/8}$	86	continuity	86
(++−++)	$\omega = 1$	56	$\neq$	57
	$\omega = e^{2\pi i/8}$	57	$\longrightarrow$	57
(++−−+)	$\omega = 1$	47	$\neq$	46
	$\omega = e^{2\pi i/8}$	46	$\longrightarrow$	46
(++−−−)	$\omega = 1$	44	$\neq$	41
	$\omega = e^{2\pi i/8}$	41	$\longrightarrow$	41
(+−+++)	$\omega = 1$	56	$\neq$	53
	$\omega = e^{2\pi i/8}$	53	$\longrightarrow$	53
(+−−++)	$\omega = 1$	47	$\neq$	46
	$\omega = e^{2\pi i/8}$	46	$\longrightarrow$	46
(+−−−+)	$\omega = 1$	44	$\neq$	45
	$\omega = e^{2\pi i/8}$	45	$\longrightarrow$	45
(+−−−−)	$\omega = 1$	43	$\neq$	46
	$\omega = e^{2\pi i/8}$	46	$\longrightarrow$	46

Start here

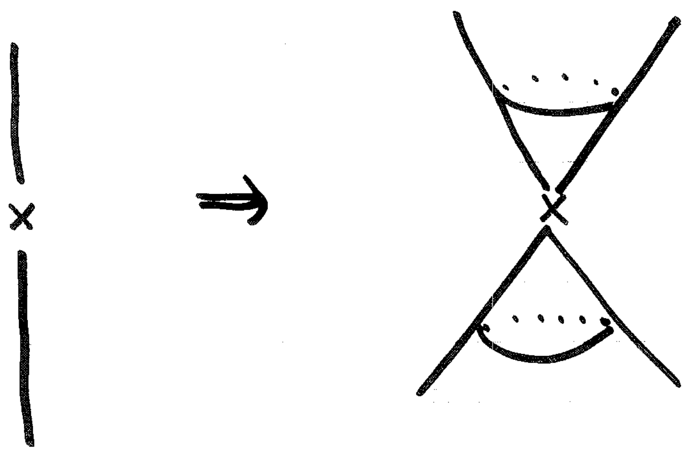
Full String Theory

$\mathbb{R}$-Kähler moduli
$\mathbb{R}$ - RR periods $\Big\} \to$ $\mathbb{C}$ - field $\mathcal{N}=1$ chiral

Often



But here $\exists$ discontinuity. in $\dim \mathcal{C}$



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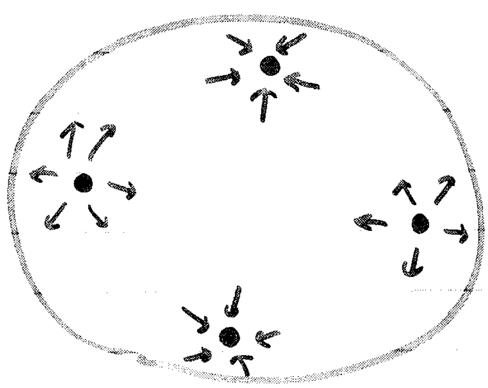
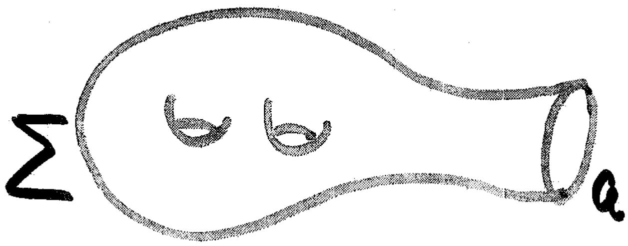
Construction of <u>consistent</u> & supersymmetric models

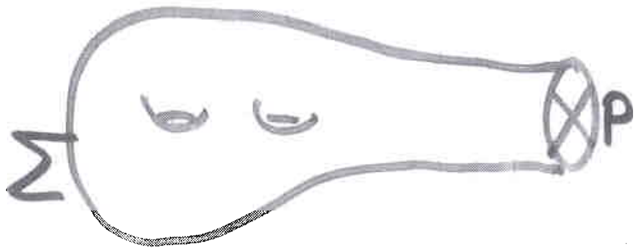
Todpole Cancellation



Boundary & Crosscap states



$$= \langle \Sigma | B_a \rangle$$



$$= \langle \Sigma | C_P \rangle$$

$|B\rangle$ & $|C\rangle$ knows about

★ Tension of D-brane/O-plane ${}_{NSNS}\langle 0 | B \text{ or } C \rangle$

★ RR charge of D/O ${}_{RR}\langle i | B \text{ or } C \rangle$

★ Spacetime SUSY preserved by D/O

$${}_{RR}\langle 0 | B \text{ or } C \rangle = e^{i\alpha} {}_{NSNS}\langle 0 | B \text{ or } C \rangle$$

$\Rightarrow$ $Q_R + e^{i\alpha} Q_L$ is preserved.

We are interested in **Consistent** & **supersymm**

brane Configurations in Type II Orientifolds

Consider parity P & branes $\sum_a n_a B_a$

Consistency Conditions

① Parity invariance $\boxed{n_{P(a)} = n_a}$

② Tadpole cancellation

$$\boxed{\sum_a n_a [B_a] + [O_p] = 0}$$

③ Rank condition : If $P(a) = a$

$n_a B_a$ supports $O(n_a)$ or $Sp(n_a/2)$

↳ n_a must be even

Condition of Spacetime SUSY

④ All $\underset{n_a>0}{B_a}$ & O_p preserve the same SUSY

$$\boxed{e^{i\alpha_a} = e^{i\alpha_p}}$$