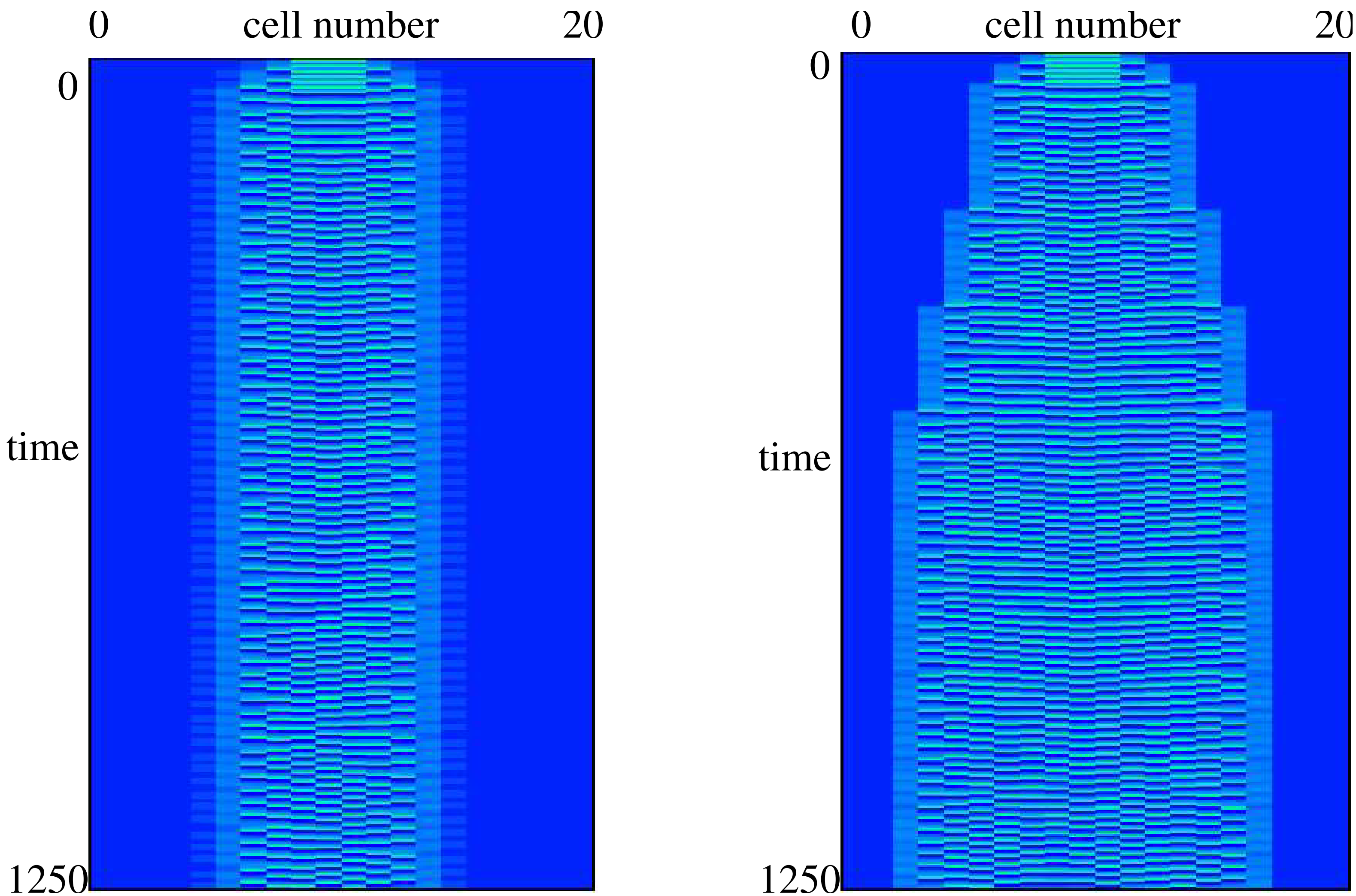
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Activity Patterns in Purely Excitatory Networks

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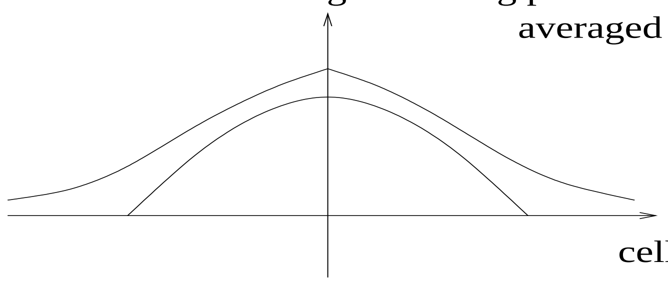
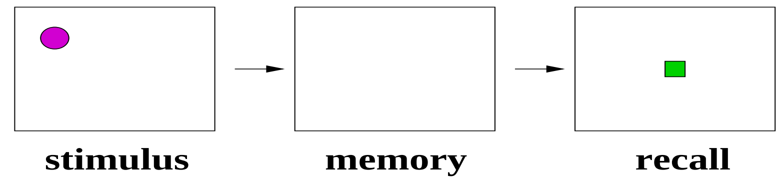
MAIN POINTS:

- spatially localized, temporally sustained activity (bumps) can occur in purely excitatory networks
- the go curve provides a useful construct for understanding the underlying mechanism and properties

Bumps: what are they and why do we care?



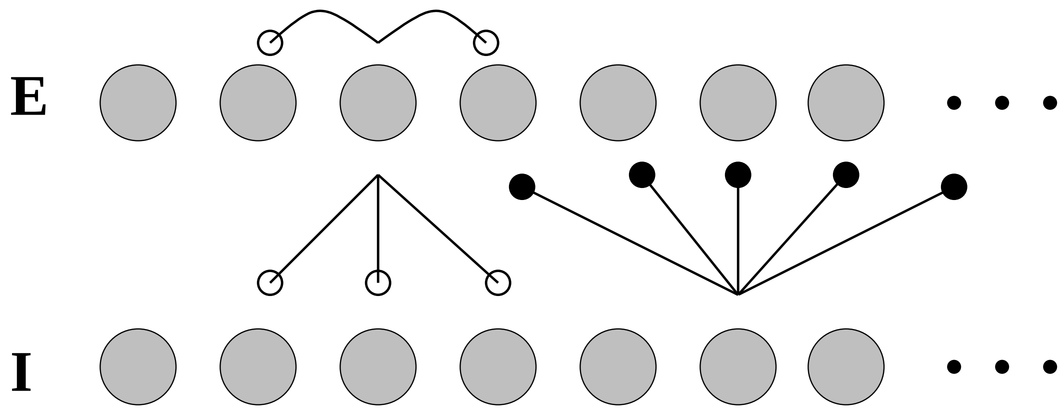


- spatially localized, temporally sustained activity
- which cells are active depends on some external feature
- activity persists without persistent stimulus
- seen in visual system, head direction system, and pre-frontal cortex (working memory):

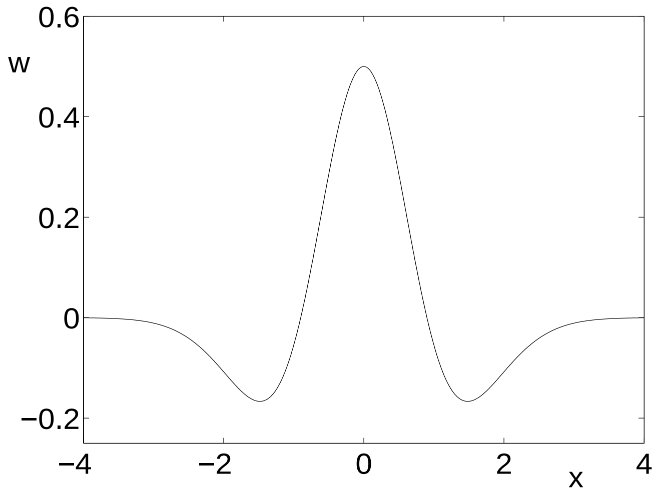
  stimulus shown, subject must later recall position; localized group of cells fire until recall

Recipes for bumps

1.



e.g. Wilson/Cowan/Amari: $u_t(x,t) = h - \sigma u(x,t) + \int_{-\infty}^{\infty} w(x-y) f(u(y,t))\, dy$

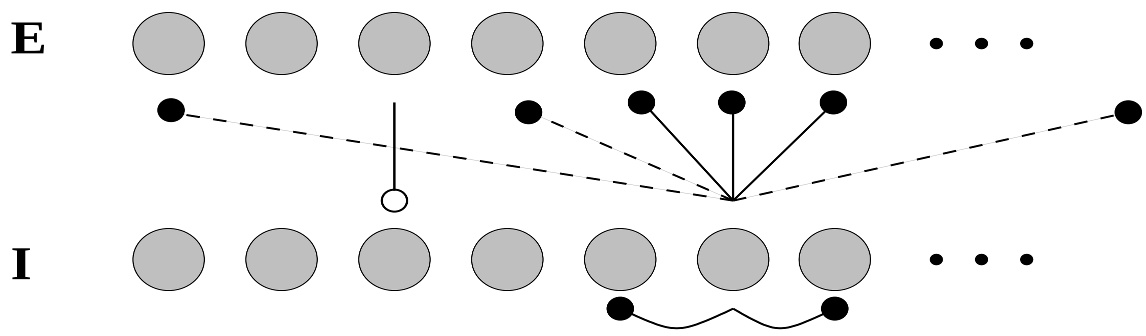


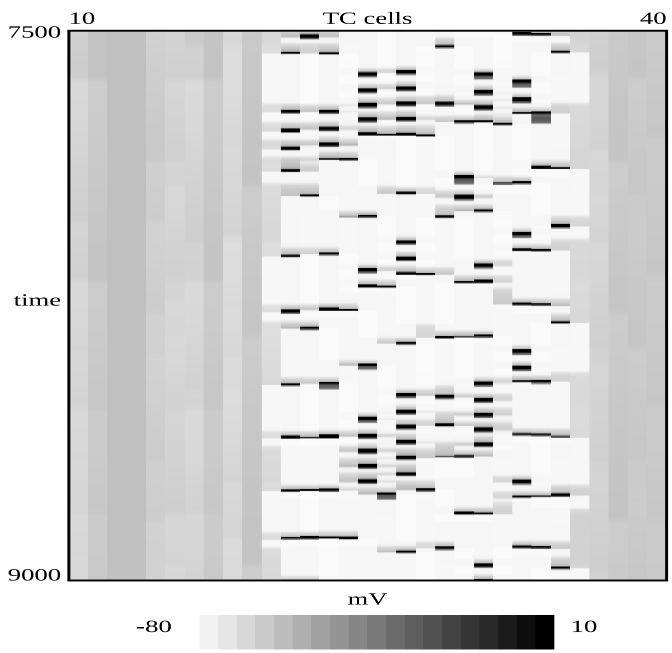
$$\begin{cases} w(x) > 0 \text{ on } (-\bar{x}, \bar{x}) \\ w(-\bar{x}) = w(\bar{x}) = 0 \\ w(x) < 0 \text{ on } (-\infty, -\bar{x}) \cup (\bar{x}, \infty) \end{cases}$$

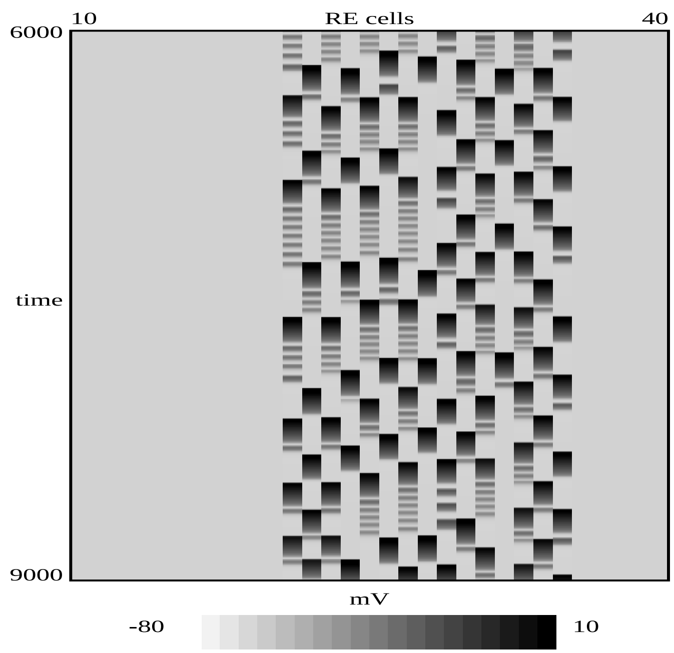
Bumps in neuroscience models with Mexican hat

- Guo and Chow, 2003
- Coombes, Lord, and Owen, 2003
- Renart, Song, and Wang, 2003
- Laing and Troy, 2002 & 2003
- Laing, Troy, Gutkin, and Ermentrout, 2002
- Gutkin, Laing, Colby, Chow, and Ermentrout, 2001
- Laing and Chow, 2001
- Pinto and Ermentrout, 2001
- Werner and Richter, 2001
- Compte, Brunel, Goldman-Rakic, and Wang, 2000
- Taylor, 1999
- Camperi and Wang, 1998
- Hansel and Sompolinsky, 1998
- Amit and Brunel, 1997
- Kishimoto and Amari, 1979
- Amari, 1977
- Wilson and Cowan, 1972 & 1973

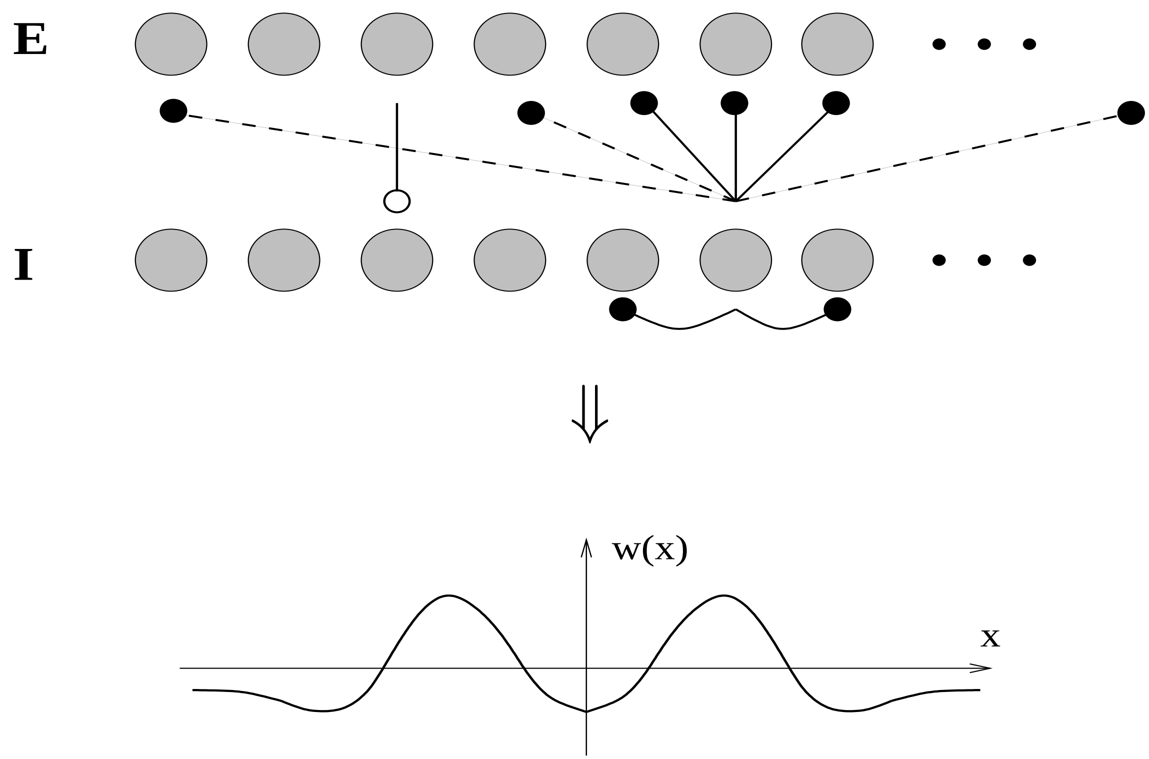
2. Rubin, Terman, and Chow, JCNS, 2001: no E-E; PIR







3. Rubin and Troy, SIAP, to appear:

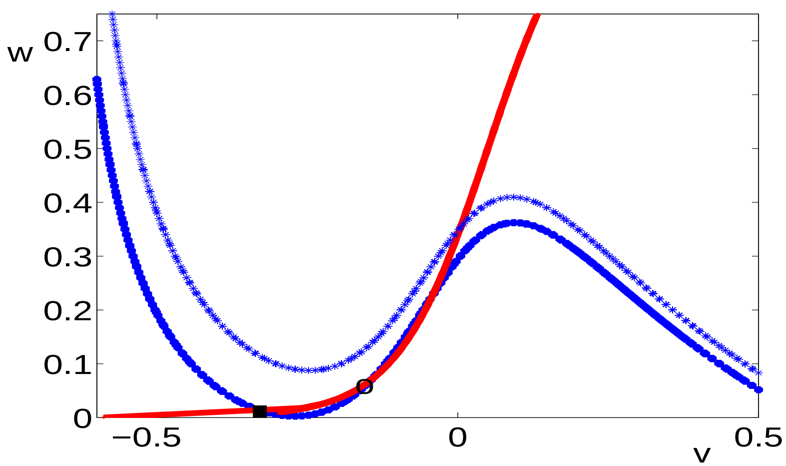


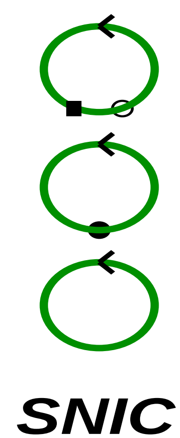
off-center coupling

4. One-layer network with purely excitatory coupling [Drover/Ermentrout, SIAP, 2003; Rubin/Bose, 2004]:

Equations

$$\begin{cases} v_i' = f(v_i, w_i) - \bar{g}_{syn}[v_i - E_{syn}]\left[c_o s_i + \Sigma_{j=1}^{j=3} c_j [s_{i-j} + s_{i+j}]\right] \\ w_i' = [w_\infty(v_i) - w_i]/\tau_w(v_i) \\ s_i' = \alpha[1 - s_i]H(v_i - v_\theta) - \beta s_i \text{ (on a ring)} \end{cases}$$





(movie)

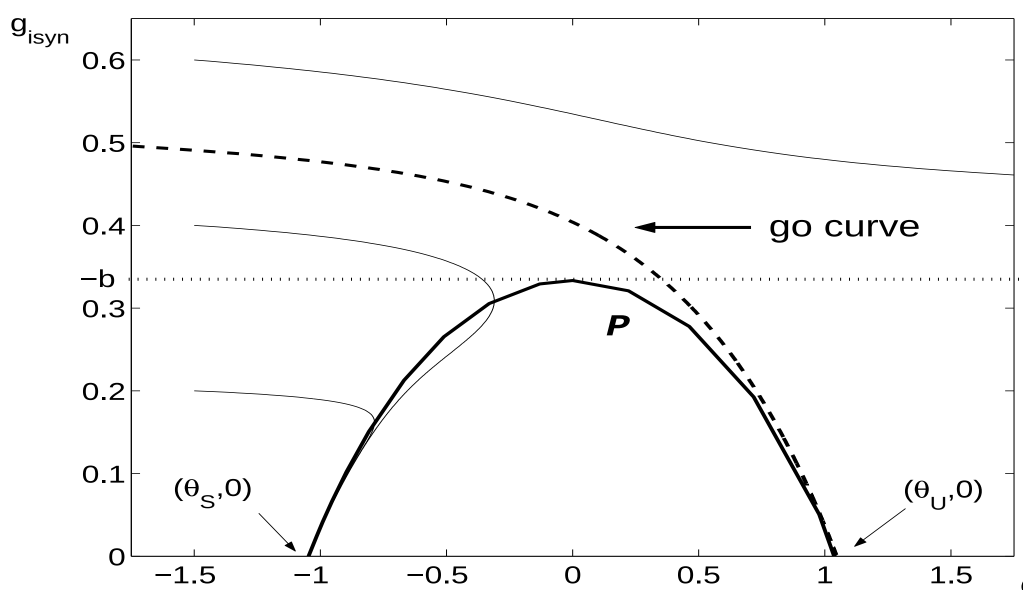
Geometry - θ model:

$$\begin{cases} \theta_i' = 1 - \cos\theta_i + (1+\cos\theta_i)(b + g_{i_{syn}}) \\ g_{i_{syn}} = \bar{g}_{syn}\left[c_0 s_i + \Sigma_{j=1}^{j=3} c_j (s_{i-j} + s_{i+j})\right] \\ s_j' = \alpha[1 - s_j]e^{-\gamma(1+\cos\theta_j)} - \beta s_j,\ j = i-3, \ldots, i+3 \end{cases}$$

with α, γ large

synaptic decay dynamics:

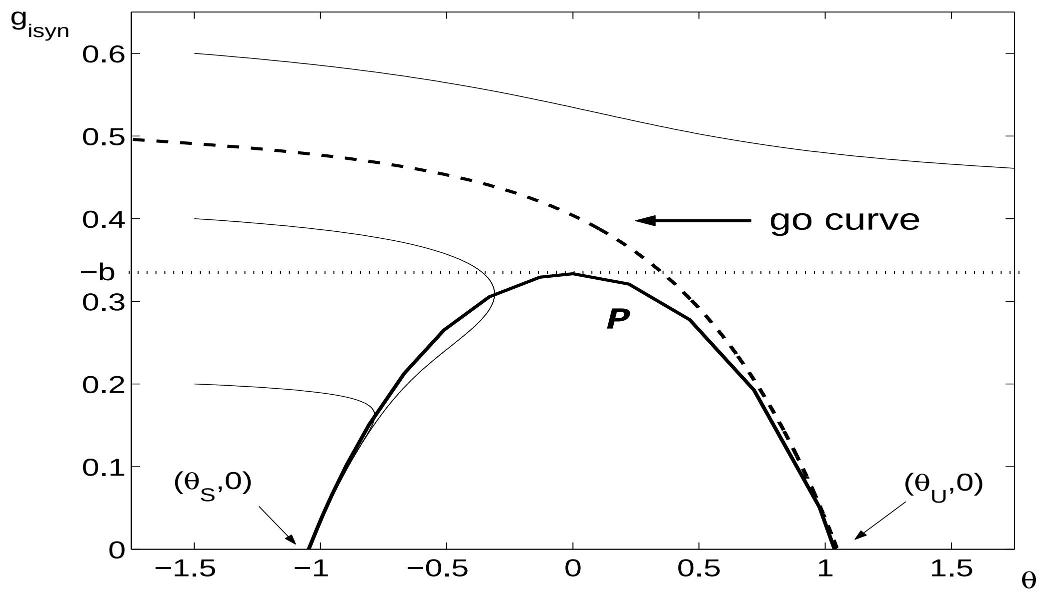
$$\begin{cases} \theta' = f(\theta, g_{i_{syn}}), \\ g_{i_{syn}}' = -\beta g_{i_{syn}} \end{cases}$$



Geometry - θ model - (2):

synaptic decay dynamics:

$$\begin{cases} \theta' &= f(\theta, g_{i_{syn}}), \\ g'_{i_{syn}} &= -\beta g_{i_{syn}} \end{cases}$$



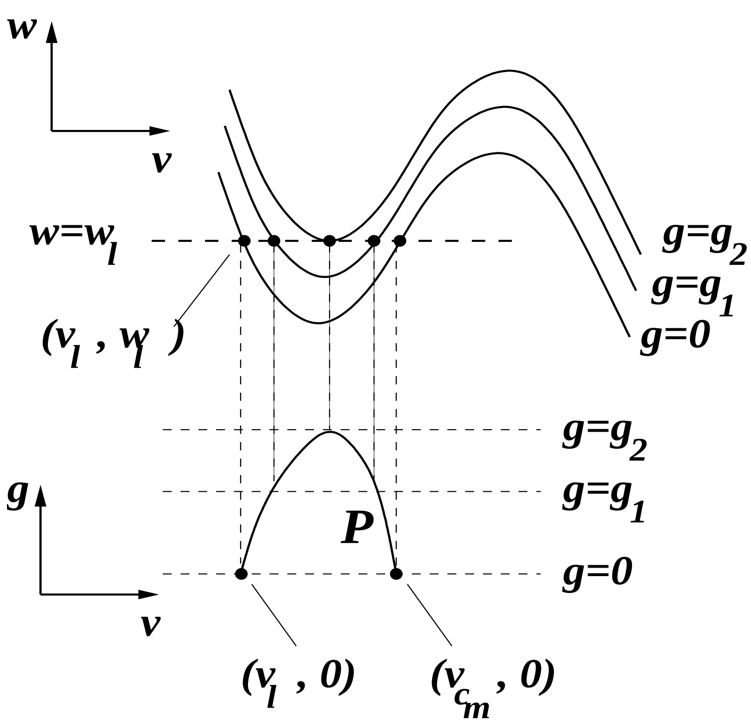
main ingredients:

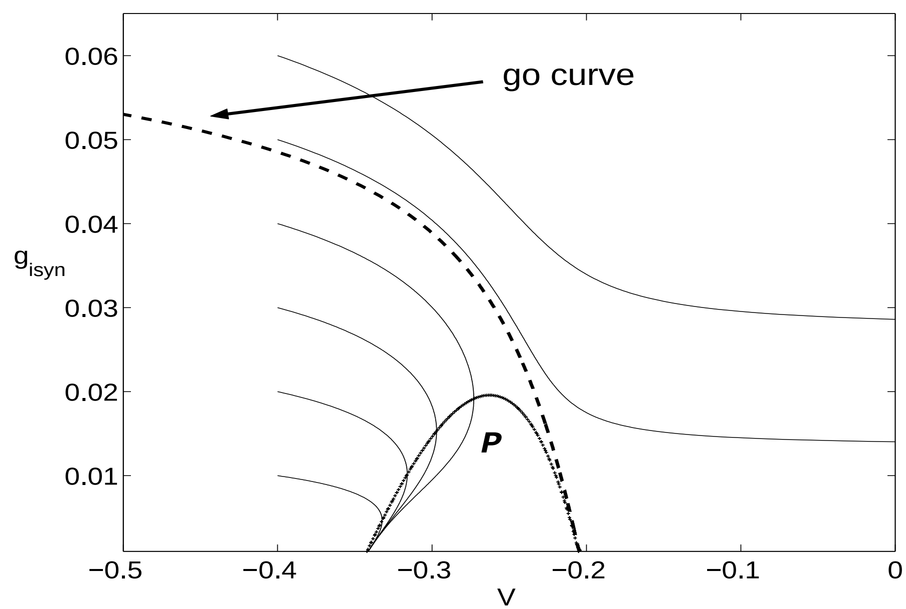
- threshold of synaptic decay dynamics determines result of stimulation
- delay to firing depends on proximity to threshold
- variable delays desynchronize and stop spread

Geometry - Morris-Lecar with $w' = 0$:

$$\begin{cases} v' = f(v,w) - g_{i_{syn}}(v - E_{syn}) \\ w' = 0 \\ s_i' = \alpha[1 - s_i]H(v_i - v_\theta) - \beta s_i \end{cases}$$

decay dynamics: replace s_i-equations with $g_{i_{syn}}' = -\beta g_{i_{syn}}$

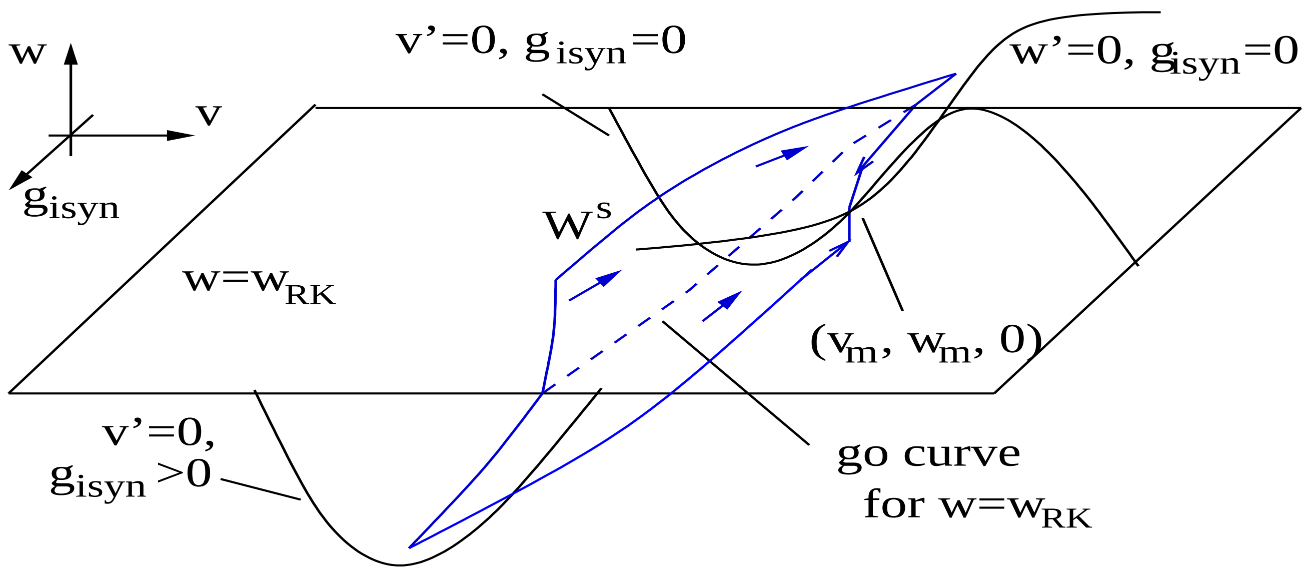




Geometry - Morris-Lecar full system:

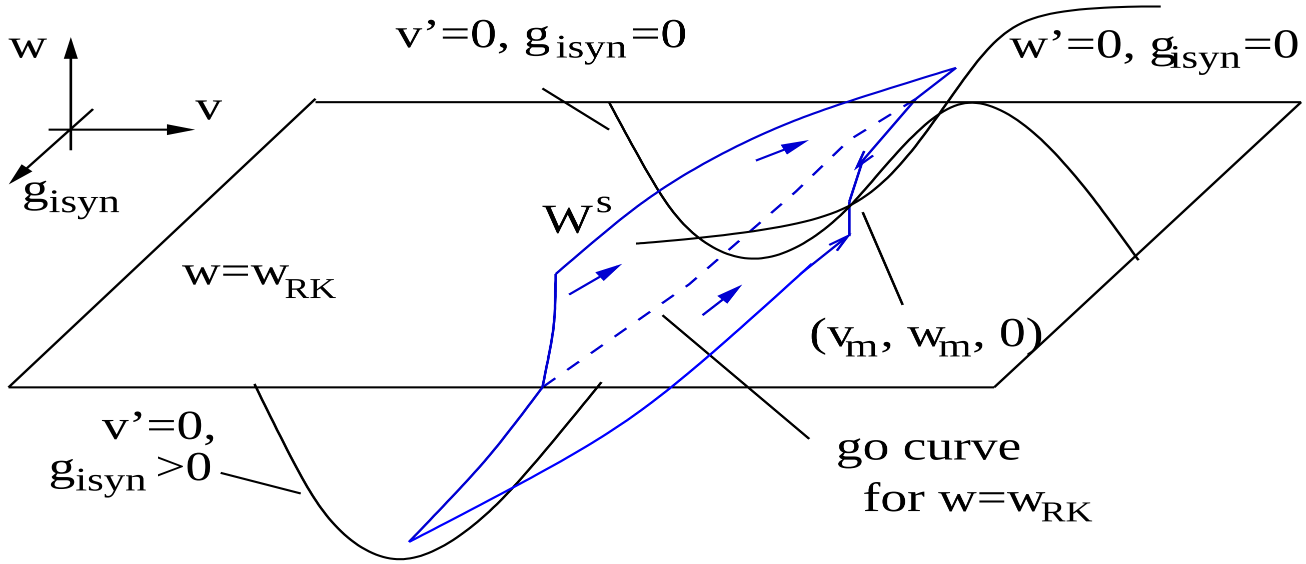
$$\begin{cases} v' = f(v,w) - g_{i_{syn}}(v - E_{syn}) \\ w' = g(v,w) \\ s_i' = \alpha[1 - s_i]H(v_i - v_\theta) - \beta s_i \end{cases}$$

decay again: replace s_i-equations with $g_{i_{syn}}' = -\beta g_{i_{syn}}$



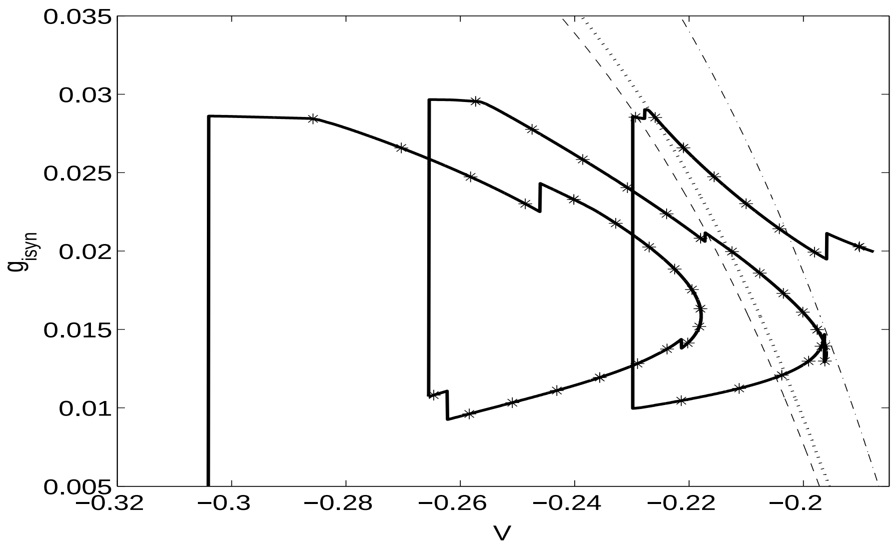
note: can still project into $(v, g_{i_{syn}})$ phase plane!

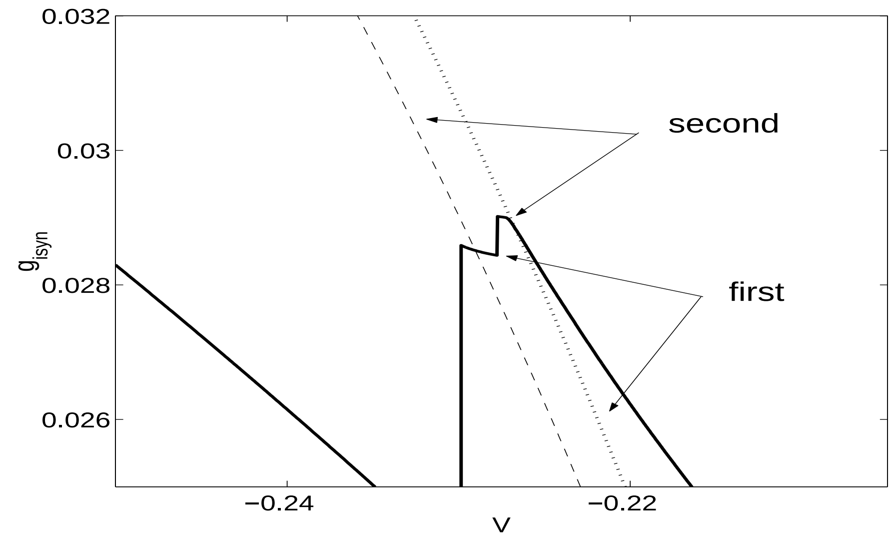
Analysis: who jumps?

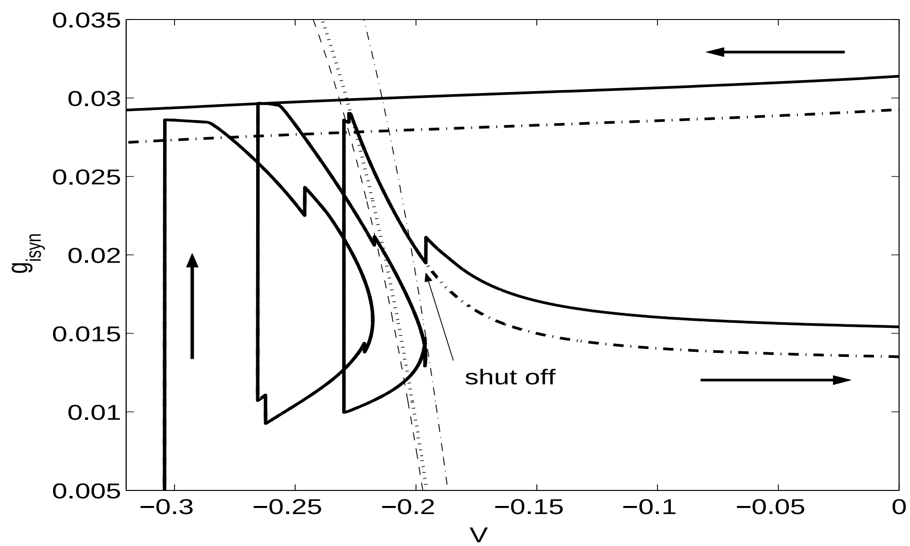


- suppose that cell i gets a synaptic input from cell j
- check the projected $(v, g_{i_{syn}})$ phase plane for cell i at the moment cell j falls down through v_{thresh}
- position of cell i relative to go curve determines recruitment

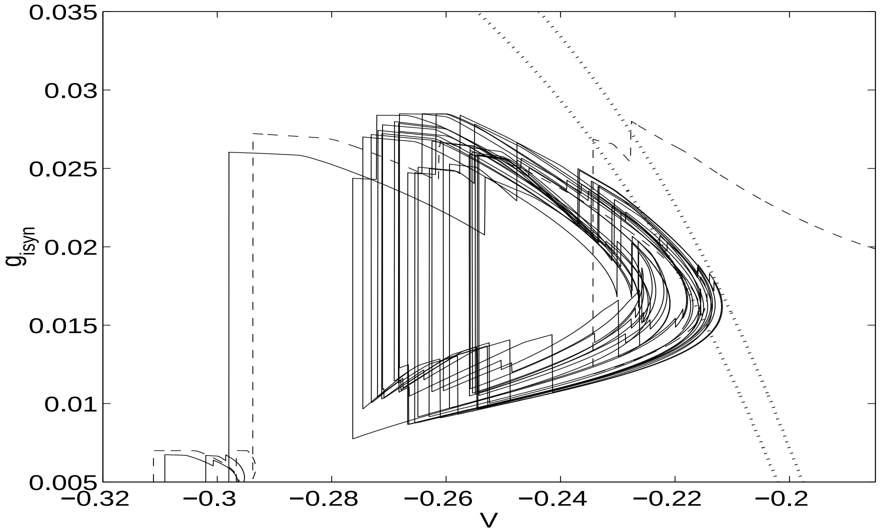
Implications - identify recruitment

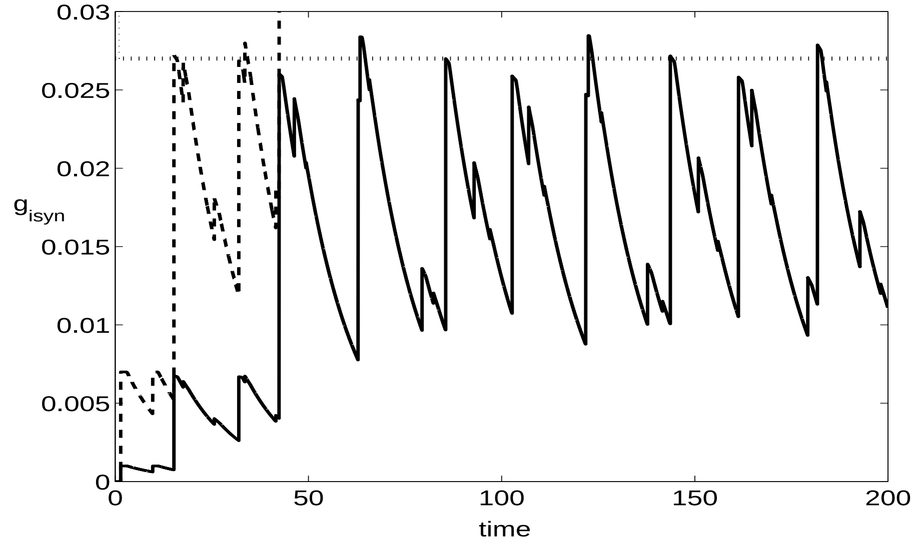






Implications - recruitment depends on more than $|g_{i_{syn}}|$





- **also, faster synaptic decay hurts in two ways...**
- **...and bump size is non-unique and nonrobust**

Bump formation:

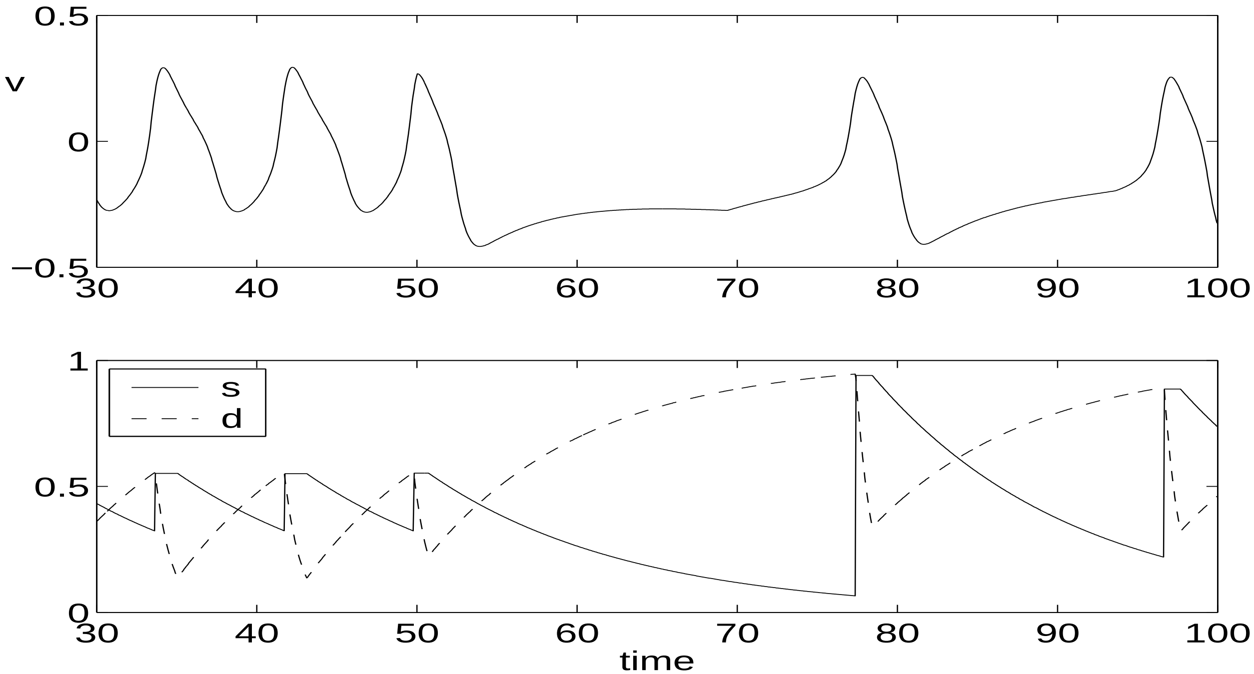
- **recruitment as above; initial synchrony helps**
  - **—— start near same rest state**
  - **—— common input overrules synaptic coupling**
- **localization via desynchronization after shock (delayed escape from go surface)**
- **sustainment via self-coupling (not sufficient!) and asynchrony**

Synaptic depression:

Bose et al., SIAP, 2001:

$$s' = \alpha[d - s]H(v - v_{thresh}) - \beta s$$

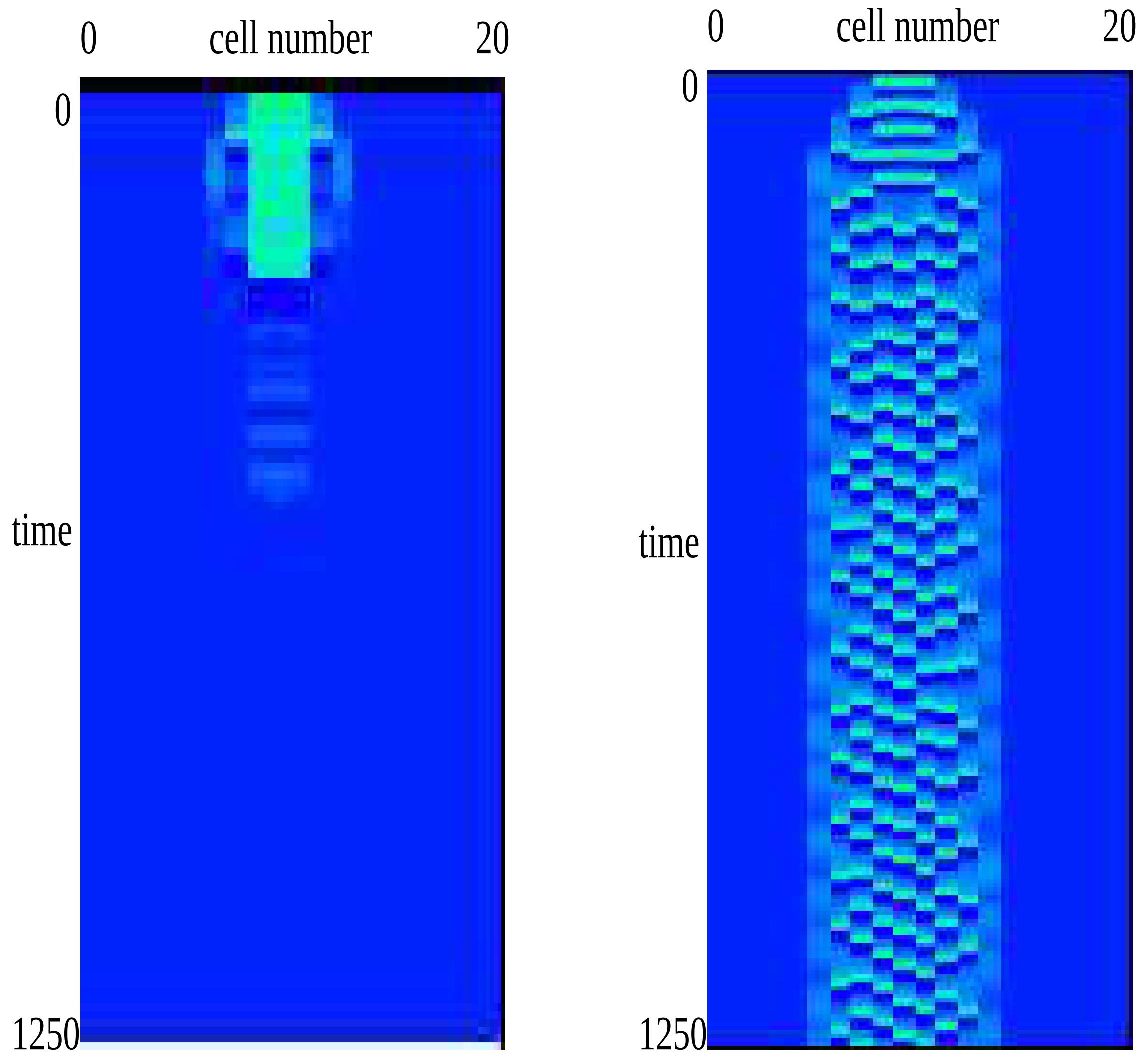
$$d' = ([1 - d]/\tau_\gamma)H(v_{thresh} - v) - (d/\tau_\eta)H(v - v_{thresh})$$



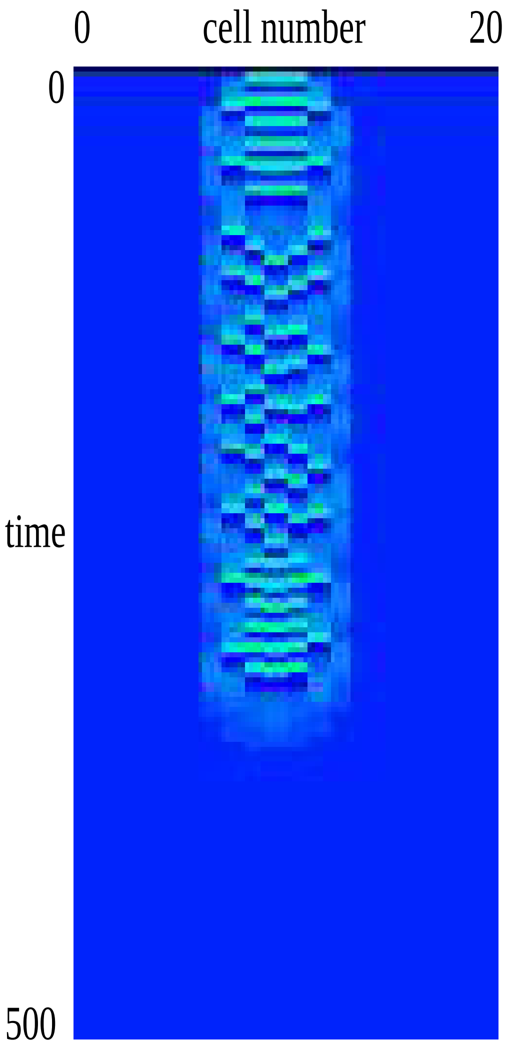
Results: ***filtering*** **and** ***toggling***

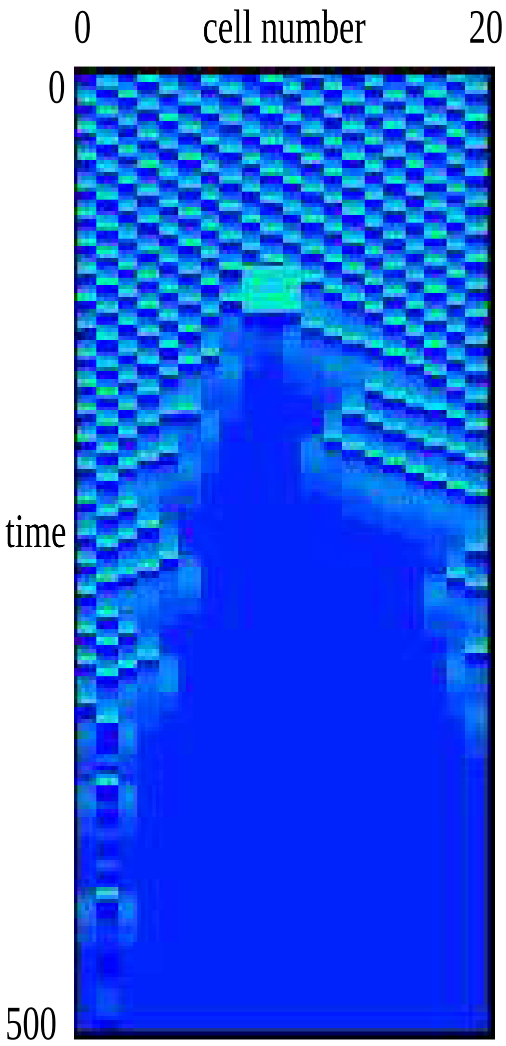
Filtering:

strong input **moderate input**



Toggling:





Discussion

- **bumps can exist in purely excitatory networks - inhibition is not necessary**
  - **initiated by transient input**
  - **go curve forms recruitment threshold**
  - **localized by desychronization (Type I dynamics)**
  - **sustained by self-coupling (or e.g. I_{CaL}) and desynchronization**
- **bump details are sensitive to parameters**
- **synaptic depression $\Rightarrow$ band-pass filter, toggling**
- **OPEN: rigorous analysis, role of short-term spike frequency changes, bump mechanism for Type II (D/E)**
- **IDEA: go curve/surface is useful for predicting the impact of transient inputs**
- **IDEA: intrinsic dynamics can be important, and excitatory networks can do interesting things**

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