Is there one brain for each of us ?

Multistability and symmetry in the dynamics of cortical plasticity

M. Kaschube, M.Schnabel, F. Wolf
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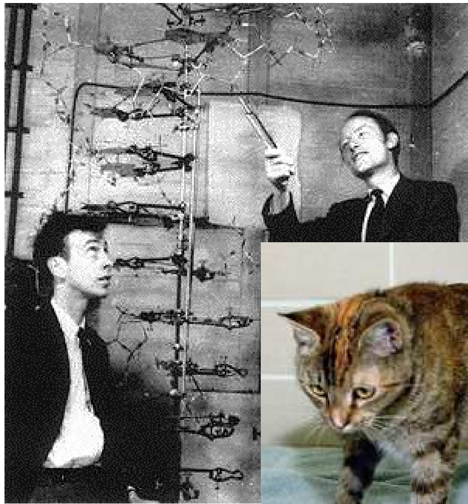
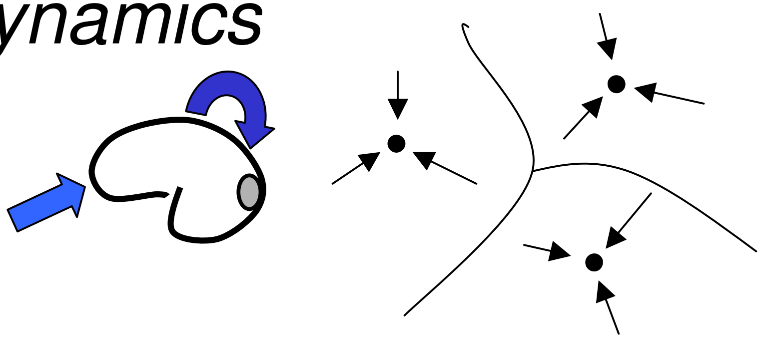
Shaping Neural Circuits

Nurture

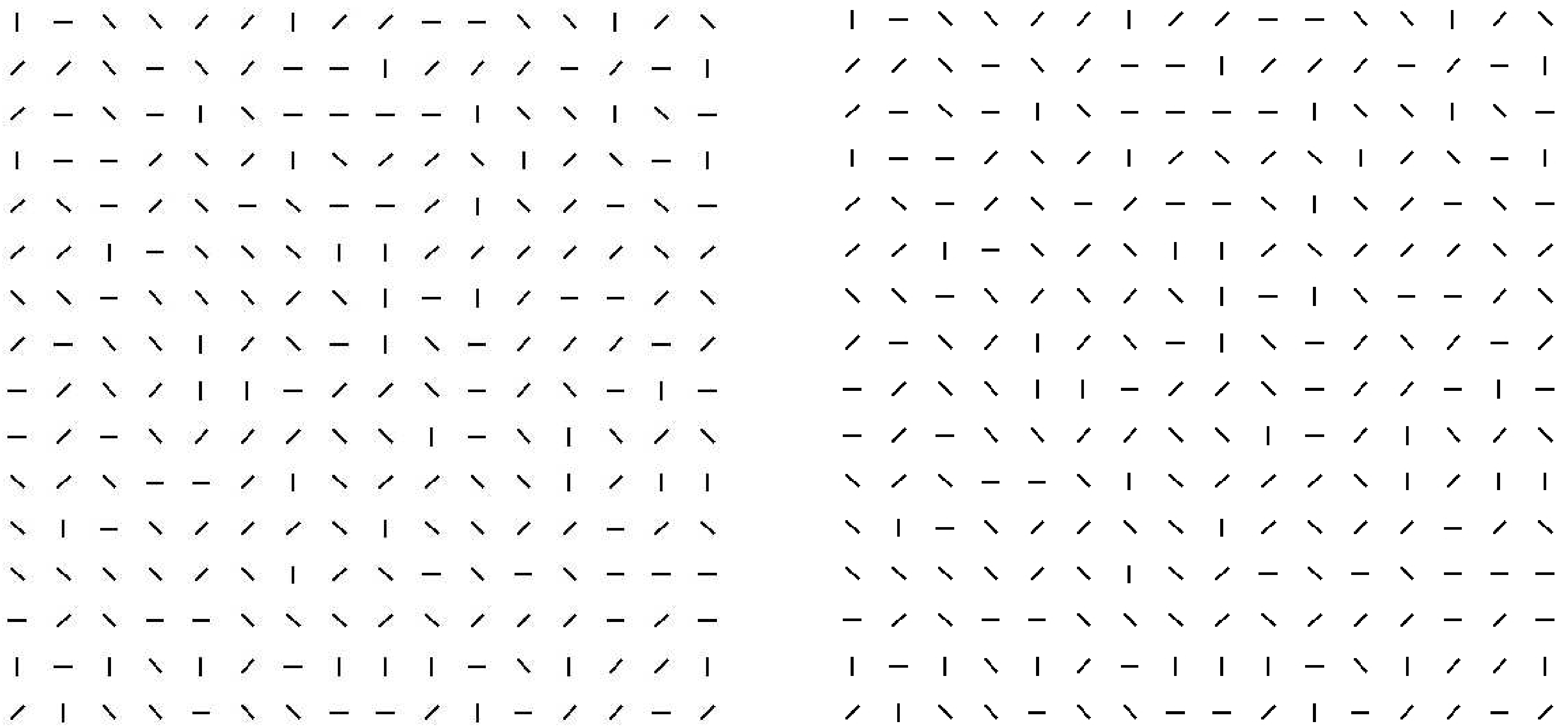
Nature



Dynamics

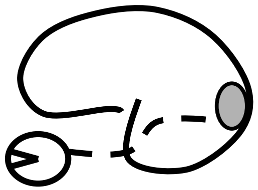


Contour *Processing*

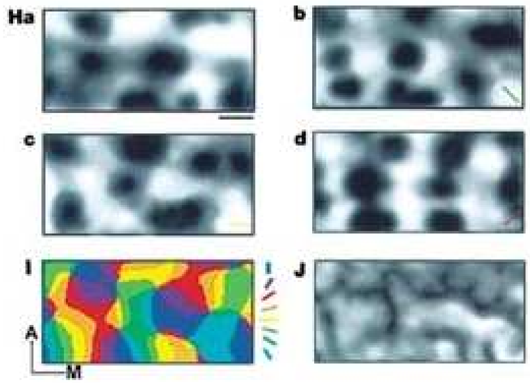


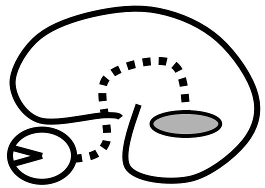
Nonlocal perceptual interactions among contours elements

Misleading Visual Information

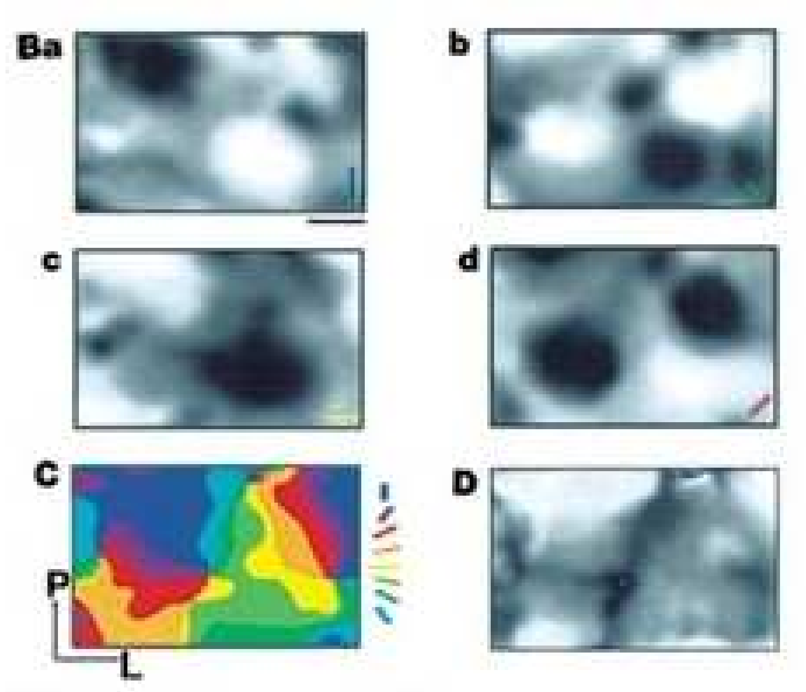


area V1





area A1

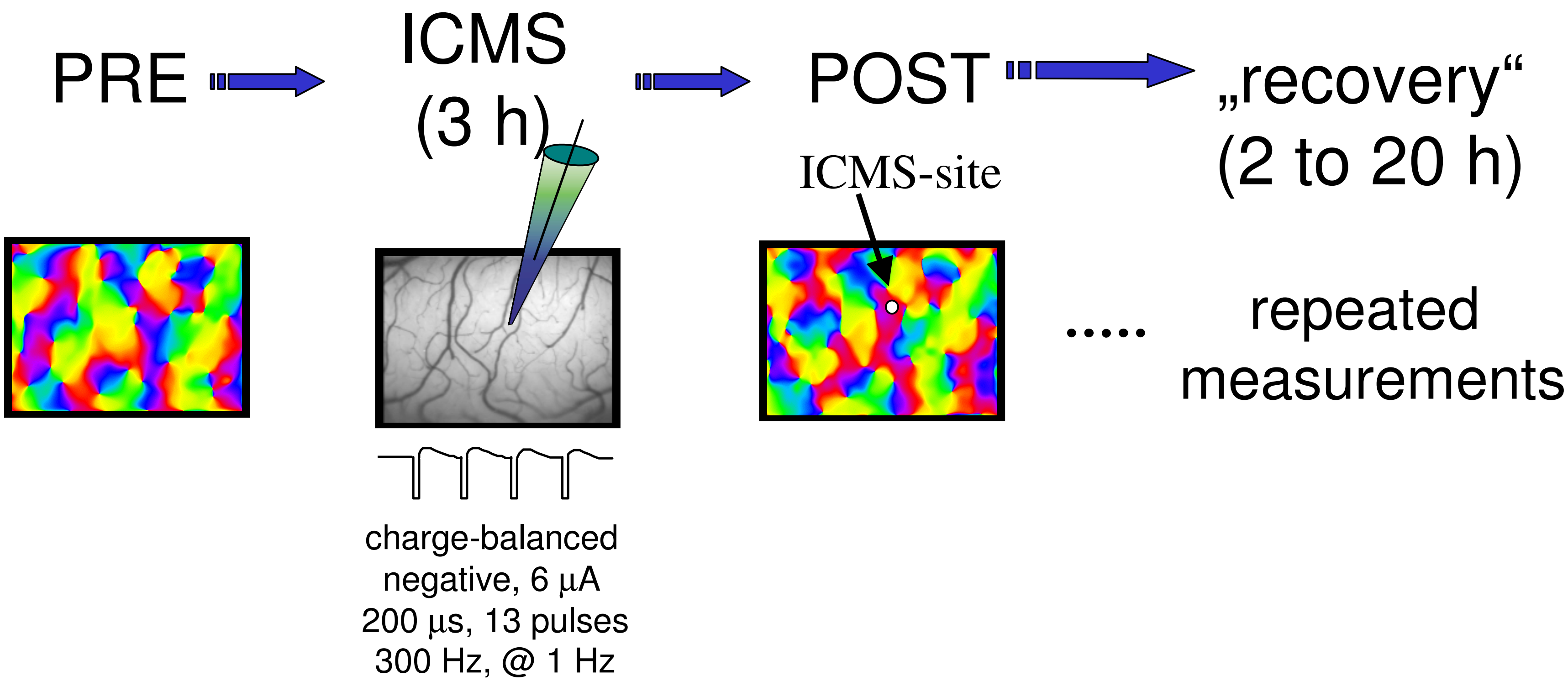


Sharma, Angelucci, Sur (2000)

Formation of orientation columns in ‚auditory' cortex.

Perturbing Maps in Adult VC

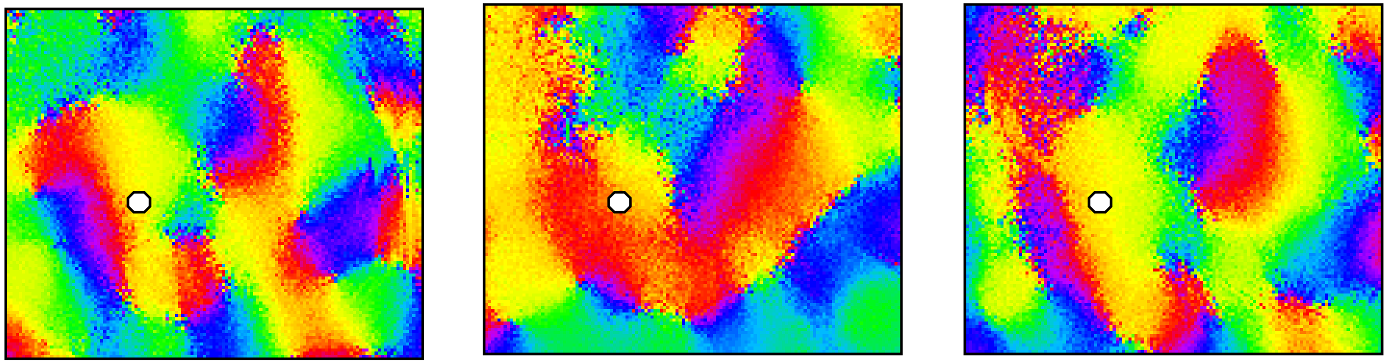
Intra-cortical micro stimulation (ICMS).



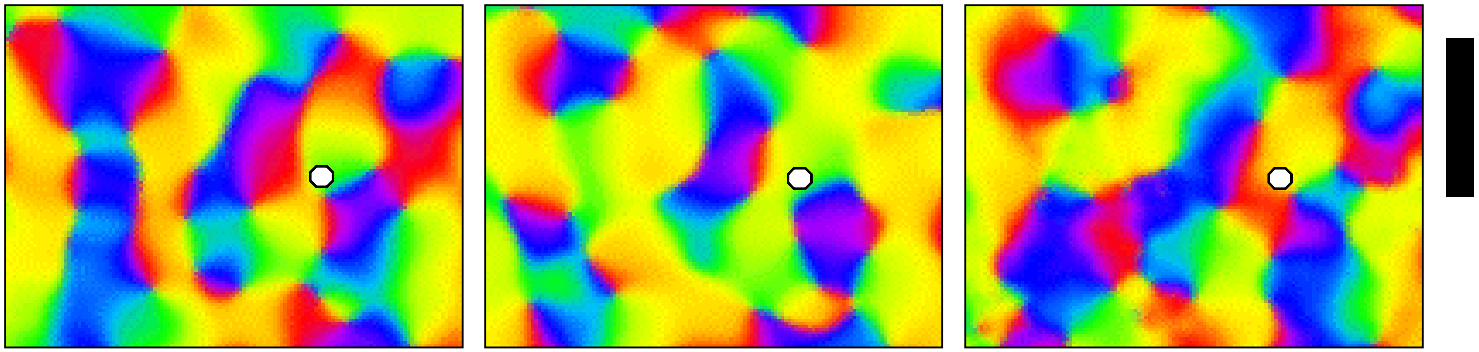
Godde, Dinse et al., PNAS (2002)

Qualitative Variability

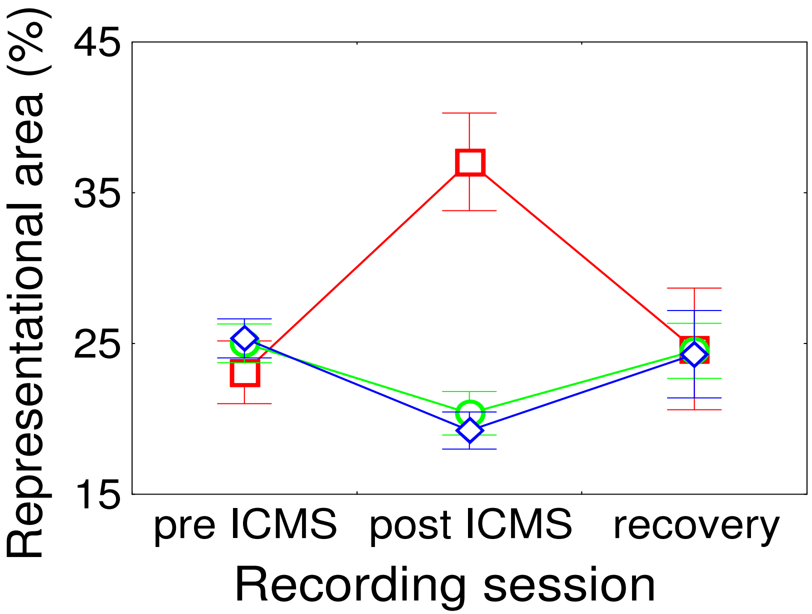
True recovery: pre post recovery

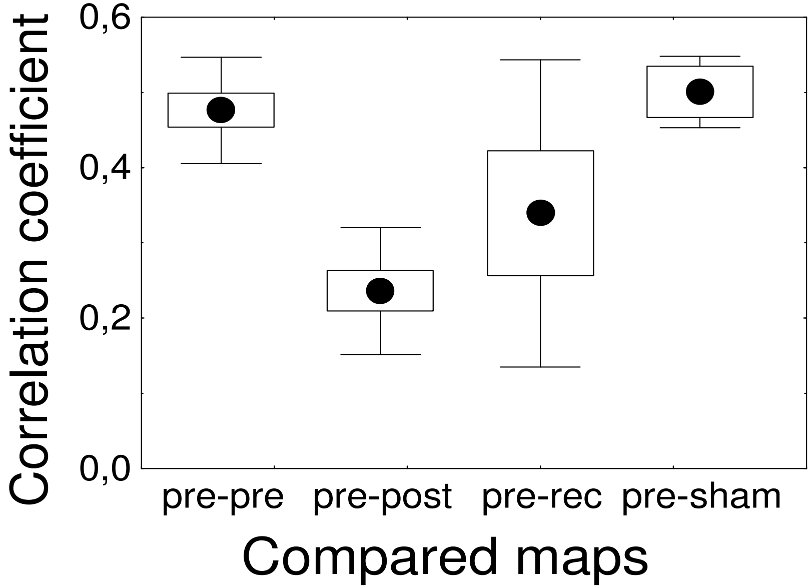


Progressive rearrangement:



Quantifying ‘recovery’





Representational areas always recover, detailed map structure sometimes!

Questions

Orientation maps as attractors ?

Non-periodic spatial order – Why ?

Multistability: discrete vs. continuous, origin ?

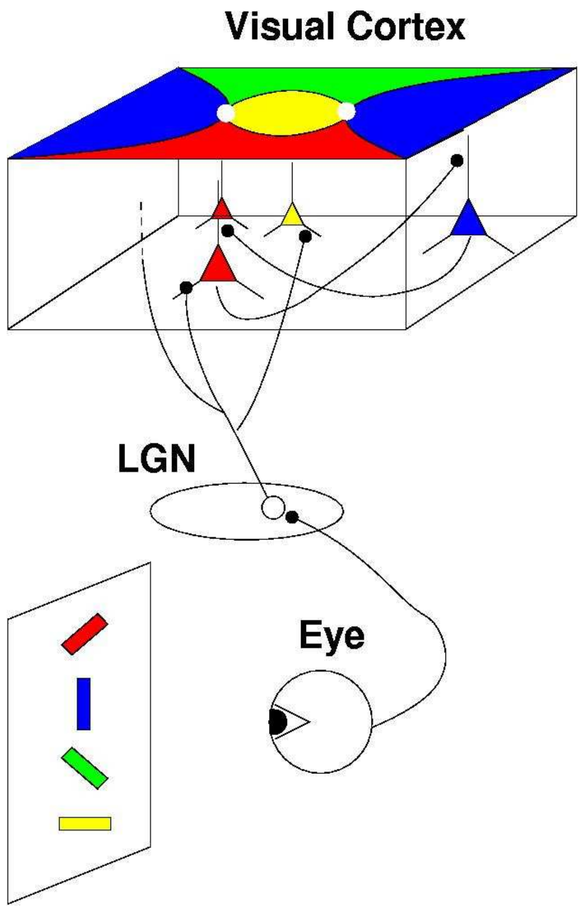
Outline:

(1) Simple model

(2) Numerical experiments

(3) Origin of behaviour: inner symmetry

Modeling Approach



time dependent orientation map

$$z(\mathrm{x,t}) = \left|z(\mathrm{x,t})\right| \exp(2\mathrm{i}\,\vartheta(\mathrm{x,t}))$$

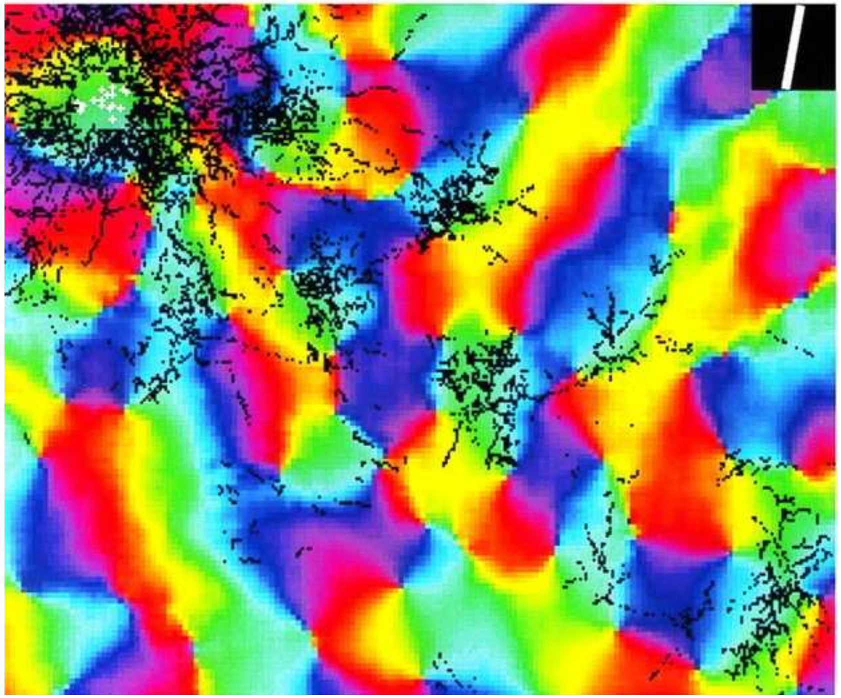
degree of selectivity

preferred angle

Phenomenological dynamics:

$$\frac{\partial}{\partial t} z(\mathrm{x},t) = F[z(\bullet,t)]$$

Long-Range Interactions in V1



Bosking et al., J. Neurosci. (1997)

lin□ □istant iso□orientation □omains

long□range□ $\sigma > \Lambda$

'Like connects to like.'

□□namic connection pattern

e.g. Löwel & Singer, Science (1992)

Tangential *connections* yà x: $W(\mathrm{x},\mathrm{y})$
Dynamics:

$$\frac{\partial}{\partial t} z(\mathrm{x},\mathrm{t}) = F_{local}[z] + g_1 \int d^2 y\, W(\mathrm{x},\mathrm{y})\big(z(\mathrm{y}) - z(\mathrm{x})\big)$$

Leading Order Model

□□□ **neglecting a□ial selecti□it□□**

$$\frac{\partial}{\partial t} z(\mathrm{x},\mathrm{t}) = F_{local}[z] \quad + \quad g_1 \int \frac{d^2 y}{\sigma^2} \exp\left(- \frac{|\,\mathrm{x}-\mathrm{y}\,|^2}{2\sigma^2} \right) \exp\left(- \frac{|\,z(\mathrm{y}) - z(\mathrm{x})\,|^2}{2\sigma_z^2} \right) \left(z(\mathrm{y}) - z(\mathrm{x}) \right)$$

□□□ **supercritical □i□urcation□e□pan□ing in** z

$$\frac{\partial}{\partial t} z(\mathrm{x},\mathrm{t}) = F_{local}[z] \quad + \quad \frac{g_1}{2\sigma_z^2} \int_{ctx} \frac{d^2 y}{\sigma^2} \exp\left(- \frac{|\,\mathrm{x}-\mathrm{y}\,|^2}{2\sigma^2} \right) \left\{ \begin{array}{l} 2\left|z(\mathrm{y})\right|^2 z(\mathrm{x}) + z(\mathrm{y})^2 \overline{z}(\mathrm{x}) - z(\mathrm{y})\left|z(\mathrm{x})\right|^2 \\ \quad - z(\mathrm{y})\left|z(\mathrm{y})\right|^2 - \overline{z}(\mathrm{y}) z(\mathrm{x})^2 + \quad O\left(z^5\right) \end{array} \right\}$$

□□□ □**inite** □ **a**□**elengt**□□ ***essential*** **terms onl**□

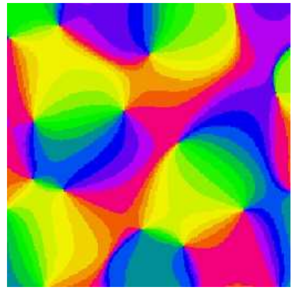
$$\frac{\partial z}{\partial t} = \hat{L}\, z + (1-g)\left|z(\mathrm{x})\right|^2 z(\mathrm{x}) - \frac{(2-g)}{4\pi\sigma^2} \int_{ctx} d^2 y \left\{ 2\left|z(\mathrm{y})\right|^2 z(\mathrm{x}) + z(\mathrm{y})^2 \overline{z}(\mathrm{x}) \right\} \exp\left(- \frac{\left|y-x\right|^2}{2\sigma^2} \right)$$

Symmetries, *more Symmetries!*

□minimalism□

s□i□t □ permutation s□mmetr□

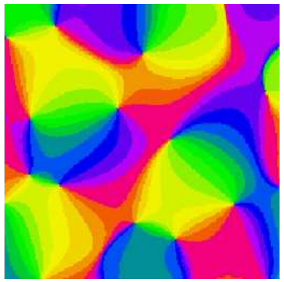
s□i□t s□mmetr□



$$\hat{S}_{\phi} z(x) = \exp(2i\phi)\, z(x)$$

□realism□

s□i□t t□ ist s□mmetr□



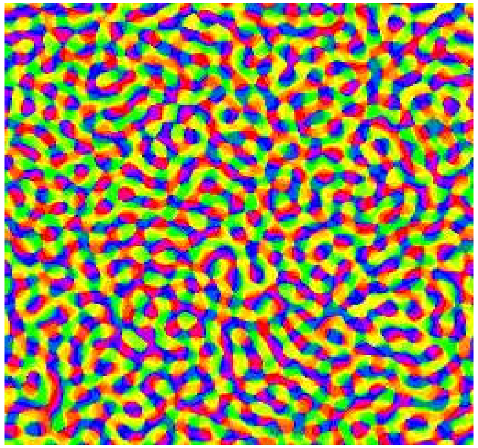
$$\hat{T}_{\phi} z(x) = \exp(i2\phi)\, z(\mathbf{\Omega}_{\phi} x)$$

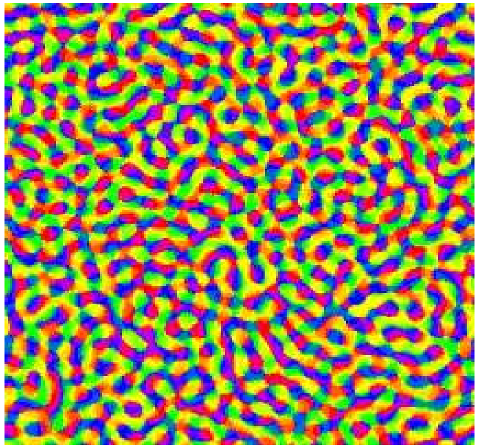
Irregular Patterns by Long Range Interactions

$$\frac{\partial z}{\partial t} = \hat{L}\, z + (1-g)\left|z(\mathrm{x})\right|^2 z(\mathrm{x}) - \frac{(2-g)}{4\pi\sigma^2}\int_{ctx} d^2y \left\{2\left|z(\mathrm{y})\right|^2 z(\mathrm{x}) + z(\mathrm{y})^2\,\overline{z}(\mathrm{x})\right\}\exp\left(-\frac{\left|y-x\right|^2}{2\sigma^2}\right)$$

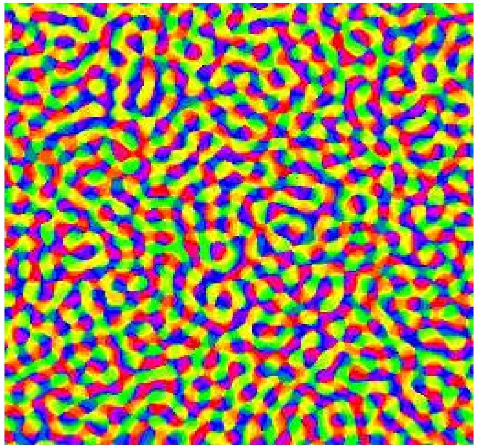
initial state | **dynamics**

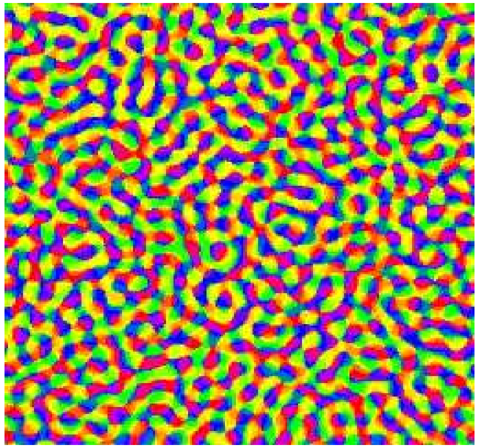
$g = 2$





$g = 0.98, \sigma = 3$

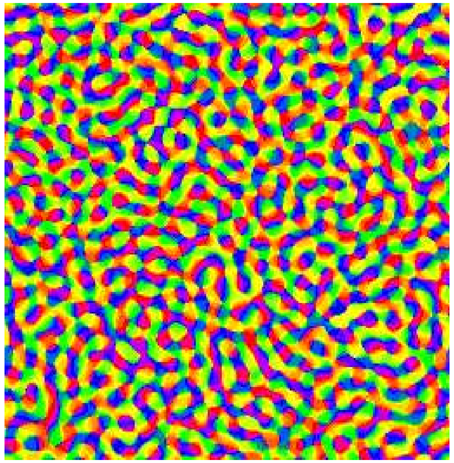


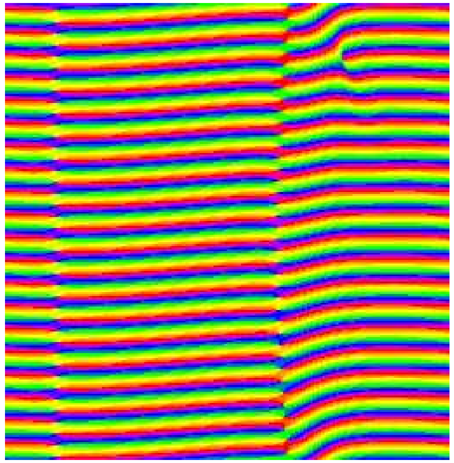


Metastability of Irregular Patters ?

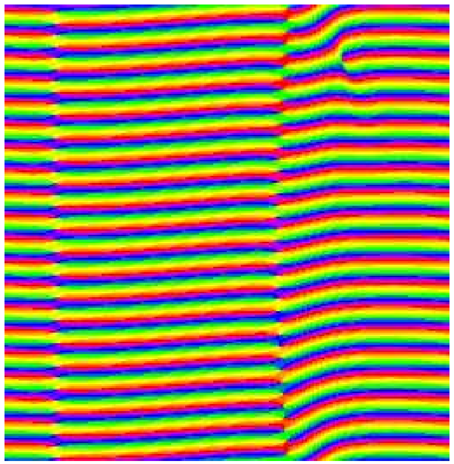
initial state | **dynamics**

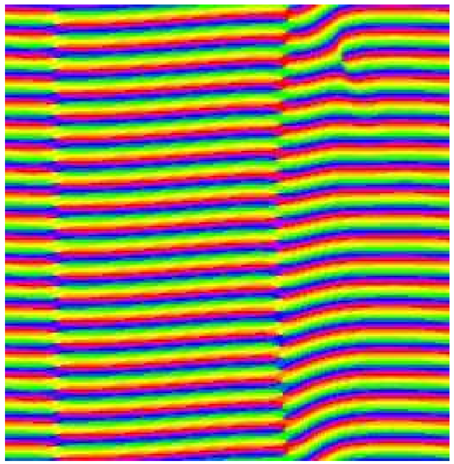
$g = 2$





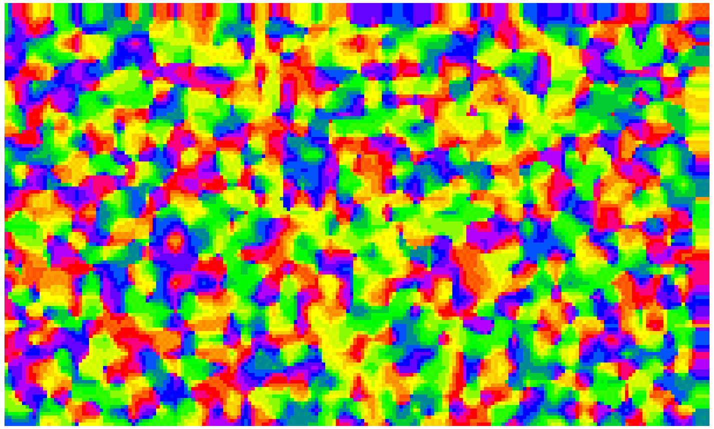
$g = 0.98, \sigma = 3$





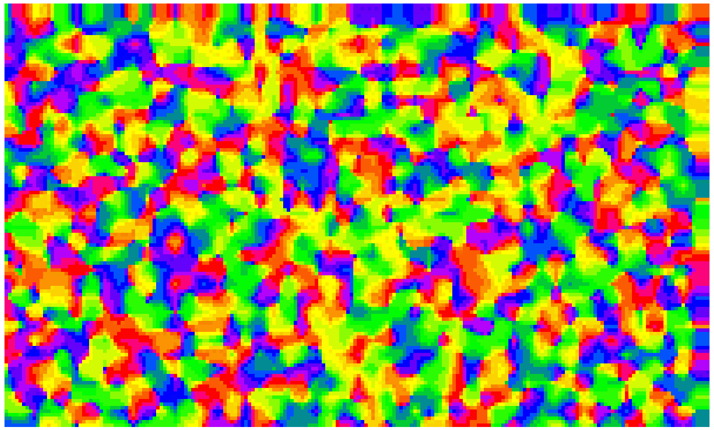
Dynamical Brain Damage

$g = 0.98, \sigma = 1.5$



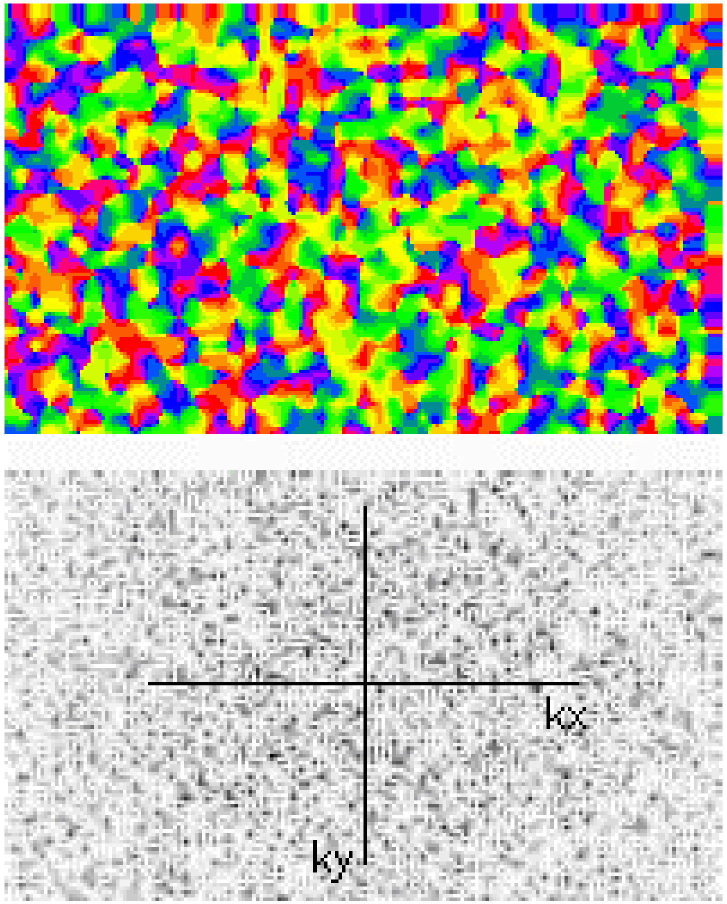
$$\frac{\partial z}{\partial t} = \hat{L}\, z + (1-g)\left|z(\mathrm{x})\right|^2 z(\mathrm{x}) - \frac{(2-g)}{4\pi\sigma^2}\int_{ctx} d^2y \left\{2\left|z(\mathrm{y})\right|^2 z(\mathrm{x}) + \xcancel{z(\mathrm{y})^2 \bar{z}(\mathrm{x})}\right\}\exp\left(-\frac{\left|y-x\right|^2}{2\sigma^2}\right)$$

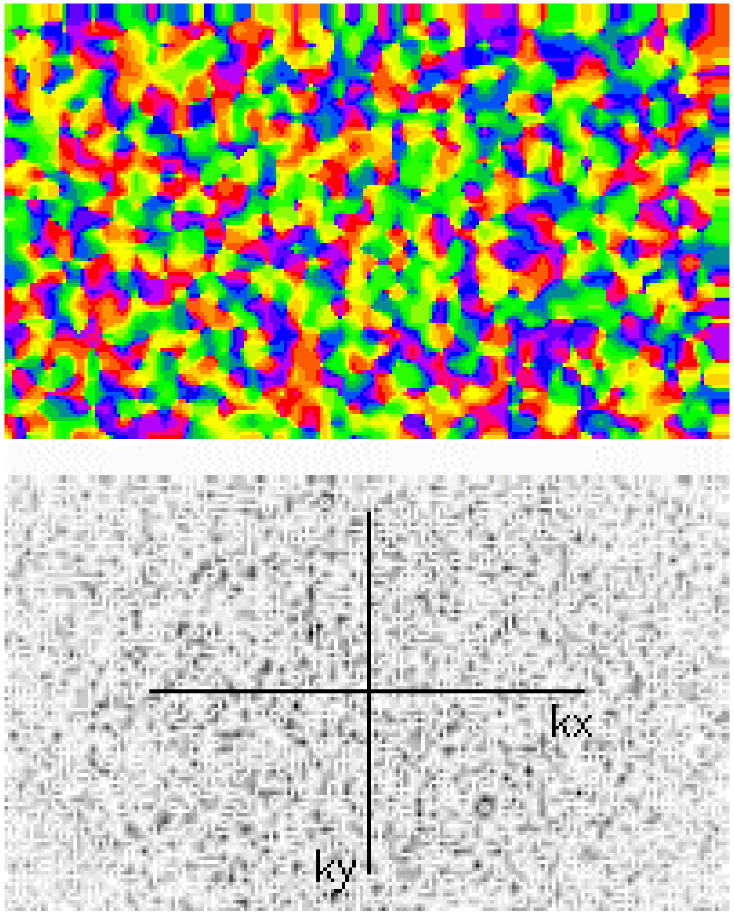
□□namicall□ a□uire□ orientation □lin□ness □*cas□s* □□*a*□*is*□



Capricious Pattern Selection

$g = 0.98, \sigma = 1.5$





Amplitude Equations

Regular planforms:

$$z(\mathrm{x}) = \sum_{j=1}^{n} A_j e^{i\mathrm{k_j x}};\ |A_j| = |A_1|$$

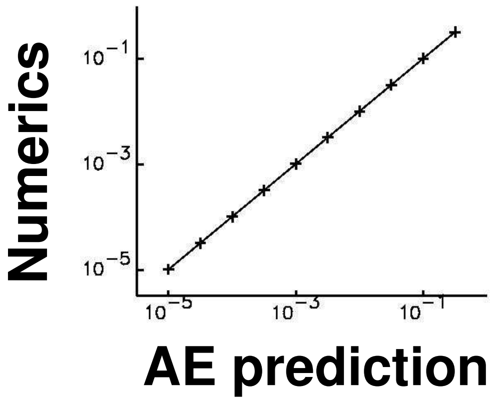
$$\mathrm{k_j} = k_c \left(\cos(2\pi j/n), sin(2\pi j/n)\right)^{\mathrm{T}}$$

Amplitude equations:

$$\dot{A}_i = A_i - \sum_j g_{ij} |A_j|^2 A_i - \sum_j f_{ij} A_j A_{j+n/2} \bar{A}_{i+n/2}$$

Amplitudes well described at onset:

$$g = 0.98, \sigma = 1.5$$



Interaction Functions

$$\dot{A}_i = A_i - \sum_j g_{ij} \left|A_j\right|^2 A_i - \sum_j f_{ij} A_j A_{j+n/2} \bar{A}_{i+n/2}$$

Assume: $g_{ij}, f_{ij} > 0$ **potential dynamics**

Derived from angle dependent interaction functions:

$$g_{ij} = \left(1 - \delta_{ij}/2\right) g\left(\alpha_{ij}\right) \quad f_{ij} = \left(1 - \delta_{ij}\right) f\left(\alpha_{ij}\right)$$

$$\alpha_{ij} = \left|\arg(\mathrm{k}_i) - \arg(\mathrm{k}_j)\right|$$

From Fields to Amplitudes

$$g_{ij} = \left(1 - \delta_{ij}/2\right) g\left(\alpha_{ij}\right) \quad f_{ij} = \left(1 - \delta_{ij}\right) f\left(\alpha_{ij}\right)$$

Trilinear **Notation:**

$$N_3\left(z(\mathrm{x})\right) = N_3[z(\mathrm{x}), z(\mathrm{x}), \overline{z}(\mathrm{x})]$$

Multiscale expansion =>

$$g(\alpha) = -e^{-i\mathrm{k}_0\mathrm{x}}\left(N_3[e^{i\mathrm{k}_0\mathrm{x}}, e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{-i\mathrm{h}(\alpha)\mathrm{x}}] + N_3[e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{i\mathrm{k}_0\mathrm{x}}, e^{-i\mathrm{h}(\alpha)\mathrm{x}}]\right)$$

$$f(\alpha) = -\frac{e^{-i\mathrm{k}_0\mathrm{x}}}{2}\left(N_3[e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{-i\mathrm{h}(\alpha)\mathrm{x}}, e^{i\mathrm{k}_0\mathrm{x}}] + N_3[e^{-i\mathrm{h}(\alpha)\mathrm{x}}, e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{i\mathrm{k}_0\mathrm{x}}]\right)$$

where

$$\mathrm{k}_0 = k_c\left(1,0\right)^{\mathrm{T}} \quad \mathrm{h}(\alpha) = k_c\left(\cos(\alpha), sin(\alpha)\right)^{\mathrm{T}}$$

Curing Orientation Blindness

Requirement: All real solutions unstable !

$$z(\mathrm{x}) \in \mathbb{R} \qquad A_i = \bar{A}_{i+n/2}$$

Amplitude Perturbation:

$$a_i = |A_i| - |A_0|$$

Linearized Amplitude Equations:

$$\dot{a}_i = -|A_0|^2 \sum_j \hat{g}_{ij} a_j, \quad \hat{g}_{ij} = 2g_{ij} + f_{ij} + f_{i,j+n/2} + \delta_{j,i+n/2} \sum_k f_{ik}$$

Sufficient Instability Criterion for $f_{ij} > 0$ **:**

$$\exists (i, j) \quad \text{with} \quad g_{ii} < g_{ij}$$

Anti-DAOB Symmetry

Instability by Permutation Symmetry:

$$N_3[u,v,w] = N_3[w,u,v]$$

Interaction function: $\Rightarrow \quad g(\alpha) = g(\alpha+\pi)$

$$g(\alpha) = -e^{-i\mathrm{k}_0\mathrm{x}}\left(N_3[e^{i\mathrm{k}_0\mathrm{x}}, e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{-i\mathrm{h}(\alpha)\mathrm{x}}] + N_3[e^{i\mathrm{h}(\alpha)\mathrm{x}}, e^{i\mathrm{k}_0\mathrm{x}}, e^{-i\mathrm{h}(\alpha)\mathrm{x}}]\right)$$

Instability criterion for antiparallel modes:

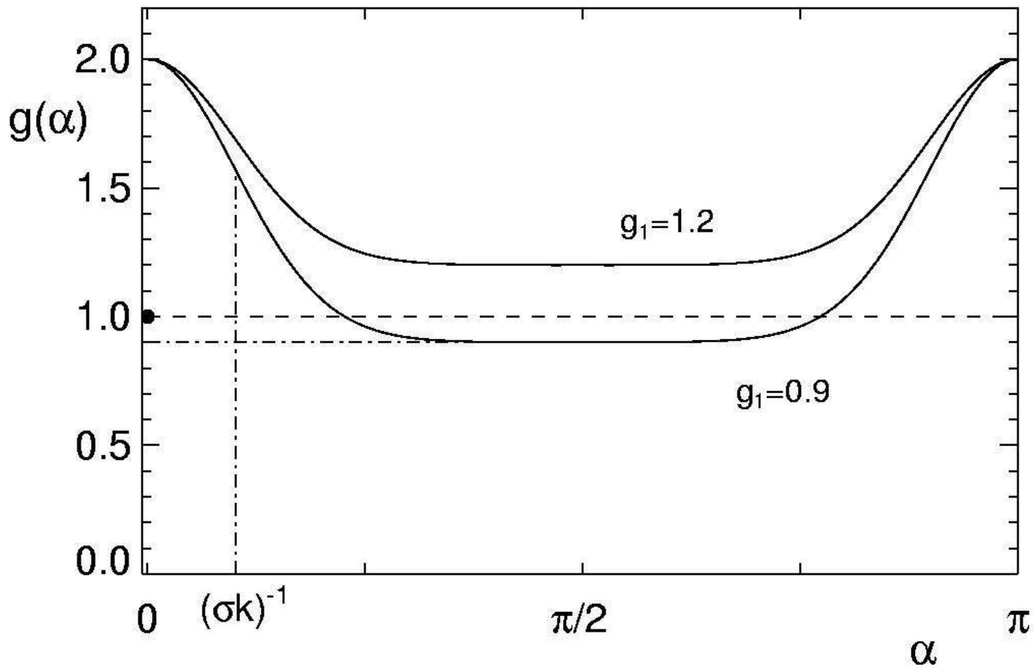
$$g_{i,i+n/2} = g(\pi) > g(0)/2 = g_{i,i}$$

Long Range Model

$$N_3[u,v,w] = (1-g)u(x)v(x)w(x) -$$

$$\frac{(2-g)}{4\pi\sigma^2}\int_{ctx} d^2y\,\{u(\mathrm{y})v(\mathrm{y})w(\mathrm{x}) + u(\mathrm{y})v(\mathrm{x})w(\mathrm{y}) + u(\mathrm{x})v(\mathrm{y})w(\mathrm{y})\}\exp\left(-\frac{|y-x|^2}{2\sigma^2}\right)$$

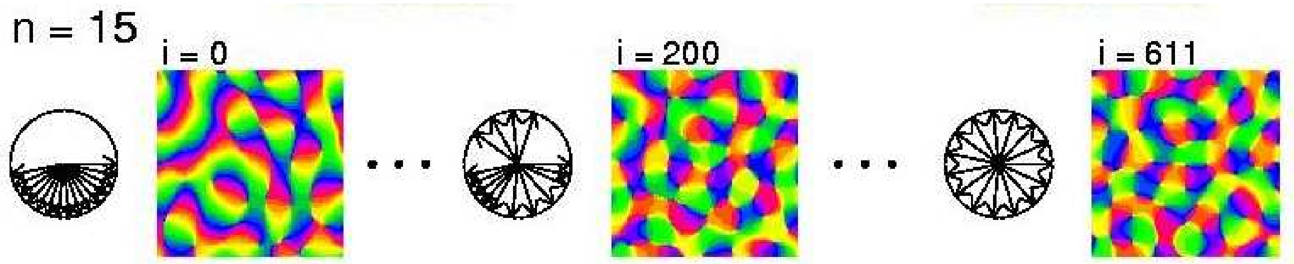
Interaction function:



Permutation Symmetry and Multistability

Essentially Complex Planforms

$$z(\mathrm{x}) = \sum_{j=1}^{n} A_j e^{i l_j \mathrm{k}_j \mathrm{x}}, \quad \mathrm{k}_j = k_c \left(\cos(\pi j/n), sin(\pi j/n)\right)^{\mathrm{T}}, \quad l_j = \pm 1$$



Amplitude Equations l_j **-independent:**

$$\dot{A}_i = A_i - \sum_j g_{ij} \left|A_j\right|^2 A_i$$

Stability and Energy degenerate !

External Stability

$$z(\mathrm{x}) = Be^{i\mathrm{hx}} + \sum_{j=1}^{n} A_j e^{i\mathrm{k}_j\mathrm{x}} \qquad \dot{B} = \left(1 - \frac{\sum_j g(\alpha - \alpha_j)}{\sum_j g_{ij}} \right) B \quad \text{for} \quad B \approx 0$$

test mode

planform

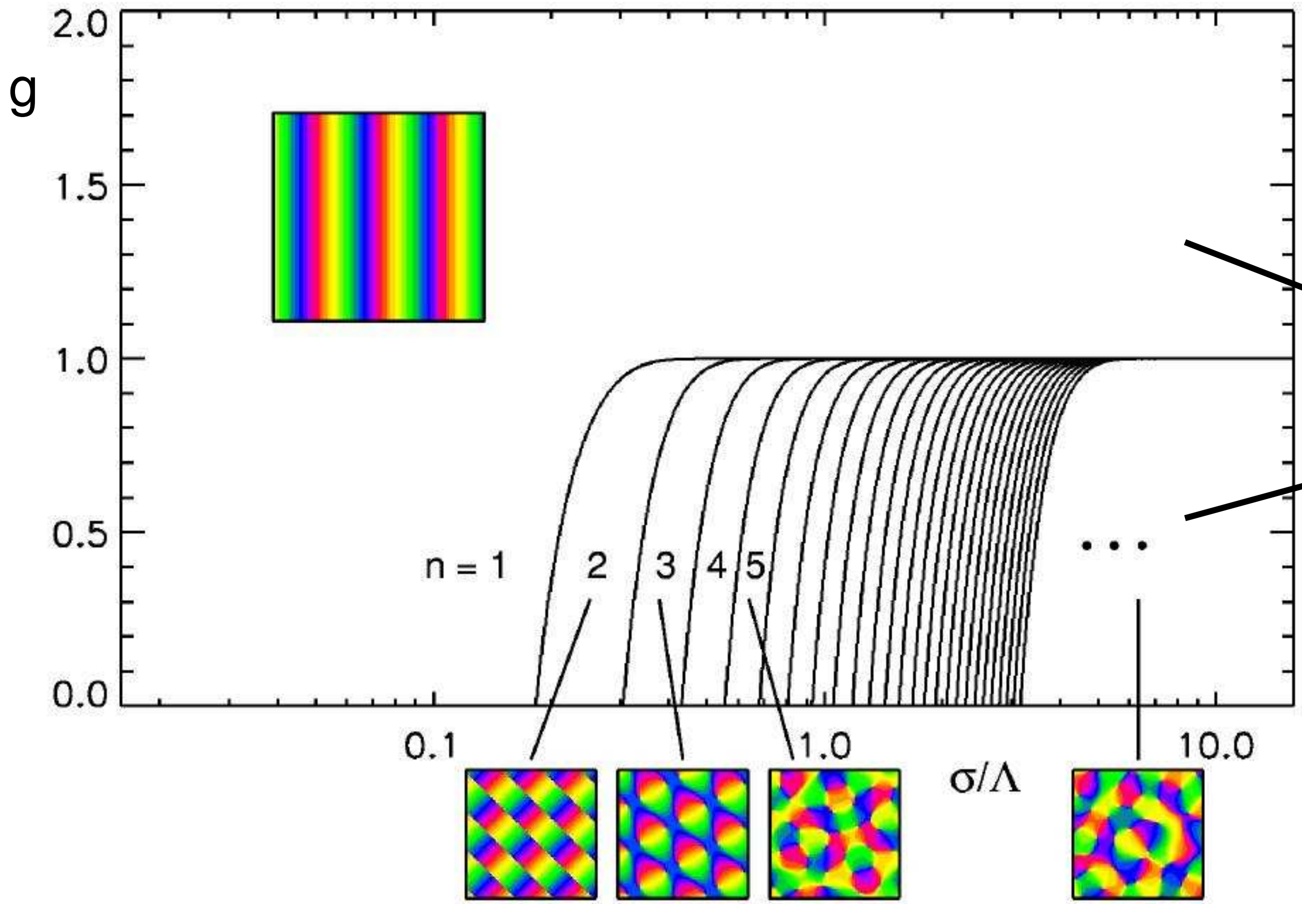
Evaluate for long range model

$$\left(1 - \frac{\sum_j g(\alpha - \alpha_j)}{\sum_j g_{ij}} \right) \xrightarrow[\sigma \to \infty]{} 1 - \frac{ng}{1 + (n-1)g} > 0 \quad \text{for g} < 1$$

If $\sigma > \sigma^*, \mathrm{g} < 1 \quad \Rightarrow$

No stable periodic planforms at threshold !

Phase Diagram



multistability

$$N_{sol} = O(1)$$

$$N_{sol} \propto \exp(\sigma / \Lambda)$$

Modeling ICMS

Add the simplest term that locally increases the representational area of the stimulated orientation.

$\varepsilon > 0$ during INMS

$$\frac{\partial z(x,t)}{\partial t} = F[z(\bullet,t)] + \varepsilon G_{ext}\left[z(\bullet,t) \mid \mathrm{INMS}\right]$$

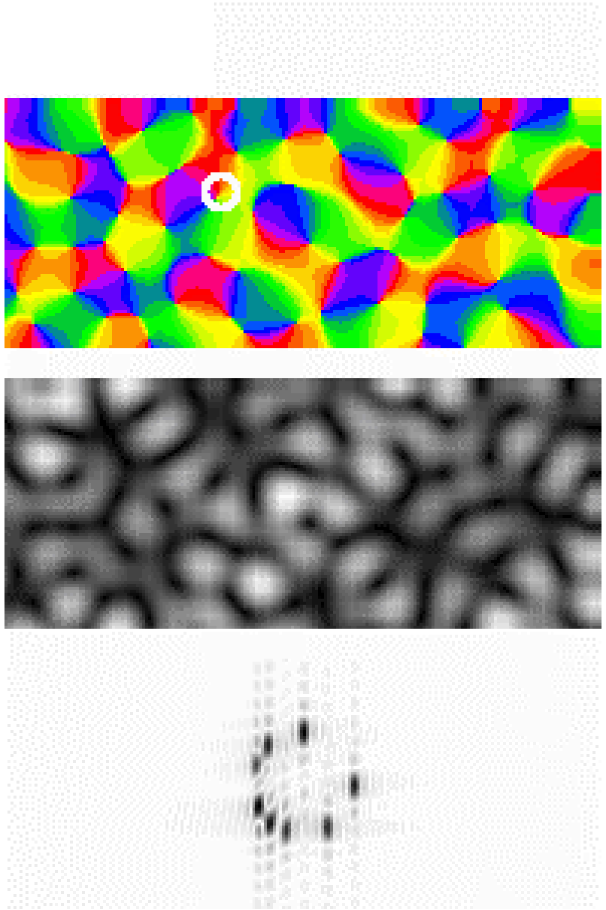
$$G_{ext}\left[z(\bullet,t) \mid \mathrm{INMS}\right] = z(\mathrm{x}_0)\, h(\mathrm{x} - \mathrm{x}_0)$$

INMS location

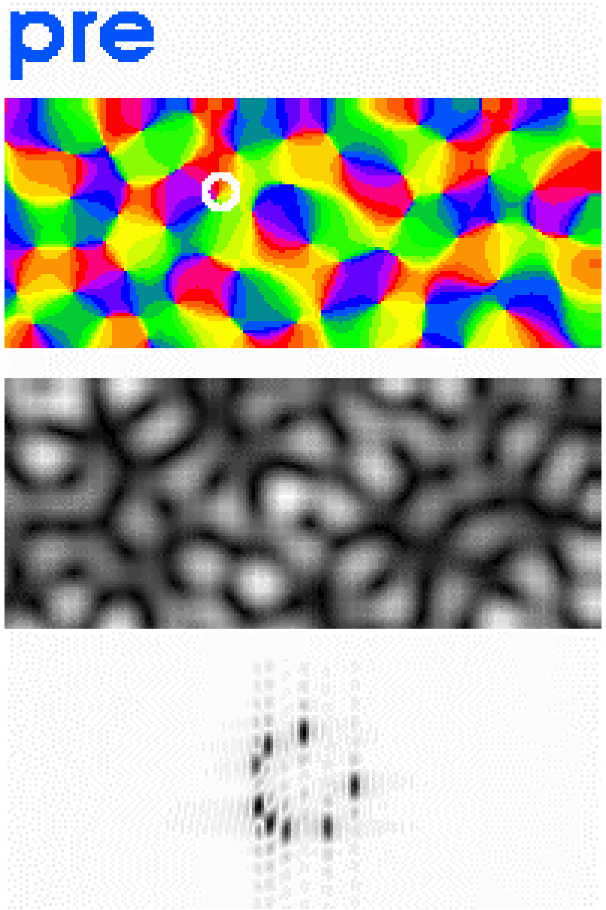
localized window function

Intra Numerical Micro Stimulation

initial state



dynamics



Conclusions

(1) After proper stimulation you might go home with an severely modified brain.

(2) Experimental evidence for multistability of orientation maps from ICMS.

(3) Tractable model: highly multistable

(4) Key ingredients: long range interactions & permutation symmetry

(5) Reproduces ICMS phenomenology

(6) Note: No stable periodic solutions at onset

Acknowledgements

ICMS Experiments:

Siegrid Löwel (Leibniz Inst. F. Neurobiology)
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Hubert Dinse (U Bochum)
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Matthias Kaschube (MPI-SF)
Michael Schnabel (MPI-SF)

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