λ Expansions of Fredholm Determinants and the Borodin-Okounkov Identity

Nicholas Witte
University of Melbourne

Recent work on the diagonal spin correlations of the square lattice Ising model has revealed that the form-factor expansions found in studies from the late 1970's are natural expansions in a parameter λ which appears only in the boundary or initial conditions applied to the integrable Painlevé equations satisfied by the correlations. In this example the relevant solutions are the Picard solutions of the sixth Painlevé equation. This parameter also appears as the natural parameter in the Neumann expansions of Fredholm determinants, a fact implicit in the Jimbo and Miwa result of 1980. Two different Fredholm expansions for the correlations can be made, one with a continuous kernel and another with a discrete kernel, the latter result being an example of the Borodin-Okounkov identity. The observations made in the example of the Ising correlations can in fact be generalised beyond it to those in the class of semi-classical Toeplitz symbols, subject to convergence conditions similar to that applying for Fisher-Hartwig asymptotics. We do this by extending the analysis of the 1979 Geronimo and Case study to non-hermitian Toeplitz matrices, i.e. bi-orthogonal polynomials on the unit circle.

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