Abstracts MSRI Workshop, November 8-12, 2010

Invited Talks

Simon Arridge (UCL) A Model Error Approximation Method for NonLinear Tomography Problems

In several medical imaging problems a PDE is used to model the propagation of a probing radiation whose measurement at specified detectors provides data for which the parameters of the PDE constitute the image sought for within an inverse problem framework. The model for propagation is typically based on either a Green's function, which is limited to certain well defined geometries and usually homogeneous parameter distributions, together with a series approximation for the inhomogeneities, or a numerical solution such as finite elements. Both these models are unreasonable in the sense that they are either too simplistic or too computationally intensive or both. By recognizing that any model is always a limited approximation to real data we may associate the modelling error as a random variable and attempt to compensate for its influence using Bayesian techniques. In this talk I will show some recent results of this approach for diffuse optical tomography and related problems which are both non-linear and ill-posed. The model error approximation method leads to computationally fast reconstruction times that are comparable in accuracy to more detailed models.

Kari Astala (University of Helsinki) Exploring the Limits of Visibility in Calderón's Inverse Problem

In two dimensions the Calderón problem, of determining an arbitrary bounded conductivity from the Dirichlet-Neumann operator, admits a complete solution. This includes computer implementation of the algorithms given by the uniqueness proof method. The method works even for infinite conductivities or infinite resistivities, with explicit control of the singularities. Beyond these bounds the uniqueness fails, thus allowing cloaking and determination of the limits between visibility and invisibility.

In this talk we give an overview of the recent results in the two-dimensional Calderón problem. The talk is based on joint works with M. Lassas, L. Pïvr̈inta, S. Siltanen. J. Müller and A. Perämäki.

Guillaume Bal (Columbia) Hybrid Inverse Problems in Optics

Photo-acoustic tomography is a novel imaging technique that aims to combine the good contrast of optical waves with the good resolution properties of acoustic waves. Mathematically, it consists of solving two successive inverse problems. The first one, and inverse wave problem, reconstructs a spatial map $H(x)$ corresponding to an amount of absorbed radiation. This will be addressed in talks by Plamen Stefanov and Linh Nguyen. The second inverse problem involves the reconstruction of optical parameters from knowledge of $H(x)$. This is the main object of interest in this talk. Similar spatial maps may be reconstructed in other hybrid imaging modalities such as thermoacoustic tomography and acousto-optics. This gives us a wide class of inverse problems with internal data. This talk will present recent results of uniqueness, non-uniqueness, and stability in this class of inverse problems. Numerical simulations confirm the predicted good resolution capabilities that these modalities share.

Fioralba Cakoni (University of Delaware) Transmission Eigenvalues in Inverse Scattering Theory

The transmission eigenvalue problem is a new class of eigenvalue problems that has recently appeared in inverse scattering theory for inhomogeneous media. Such eigenvalues provide information about material properties of the scattering object and can be determined from scattering data, hence can play an important role in a variety of problems in target identification. The transmission eigenvalue problem is non-selfadjoint and nonlinear which make its mathematical investigation very interesting.

In this lecture we will describe how the transmission eigenvalue problem arises in scattering theory, how transmission eigenvalues can be computed from scattering data and what is known mathematically about about these eigenvalues. For the sake of simplicity, most of our mathematical exposition will be done for the case of scattering for isotropic media and at the end of the lecture we will discuss how these results can be extended to the anisotropic media.

Emmanuel Candes (Stanford) Robust Principal Component Analysis and Other Advances in Low-Rank Matrix Modeling: Some Theory and Some Applications

This talk is about a curious phenomenon. Suppose we have a data matrix, which is the superposition of a low-rank component and a sparse component. Can we recover each component individually? We prove that under some suitable assumptions, it is possible to recover both the low-rank and the sparse components exactly by solving a very convenient convex program. This suggests the possibility of a principled approach to robust principal component analysis since our methodology and results assert that one can recover the principal components of a data matrix even though a positive fraction of its entries are arbitrarily corrupted. This extends to the situation where a fraction of the entries are missing as well. In the second part of the talk, we present applications in the area of video surveillance, where our methodology allows for the detection of objects in a cluttered background, and in the area of face recognition, where it offers a principled way of removing shadows and specularities in images of faces. We also show how the methodology can be adapted to simultaneously align a batch of images and correct serious defects/corruptions in each image. We also present promising applications for texture alignment, which opens new perspectives in computer vision.

Joint work with X. Li, Y. Ma and J. Wright.

Margaret Cheney, RPI Waveform-Diverse Moving-Target Spotlight Synthetic-Aperture Radar

This lecture begins with an outline of the standard theory for Synthetic-Aperture Radar (SAR). It then develops the theory for the more general case in which the scene contains multiple moving targets and the radar antenna transmits an arbitrary sequence of waveforms. The targets are assumed to be moving linearly, but the antenna flight path is arbitrary. We include the case of waveforms whose duration is sufficiently long that the targets and/or platform move appreciably while the data is being collected.

A formula for phase space (position and velocity) imaging is developed, and we provide a formula for the point-spread function of the corresponding imaging system. This point-spread function is expressed in terms of the ordinary radar ambiguity function.

As an example, we show how the theory can be applied to the problem of estimating the errors that arise when target and antenna motion is neglected during the transit time of each pulse.

This is joint work with Brett Borden.

Kiril Datchev, MIT Propagation Through Trapped Sets and Semiclassical Resolvent Estimates

The high energy behavior of the resolvent of the Laplacian on a noncompact manifold is closely related to the geometry of the set of trapped geodesics. Typically, estimates when there is trapping exhibit a loss compared to estimates in the nontrapping setting. However, if the resolvent is suitably truncated in space or in phase space, this loss can be removed. We prove a semiclassical propagation theorem which shows that in certain settings this truncation can overlap part of the trapped set. This project is joint work with Andras Vasy.

Fernando Guevara Vazquez (University of Utah) Uncertainty quantification in resistor network inversion

We present a method for finding the electrical conductivity in a domain from electrical measurements at the boundary. Our method consists of two steps. In the first step we find a resistor network that fits the data and then we estimate the conductivity from the resistors by interpreting the network as a finite volumes discretization of the problem. We show through a Monte Carlo study that our discretization of the conductivity reduces the uncertainty in the reconstructions, as compared to a conventional discretization.

Pilar Herreros (University of Münster) Scattering Rigidity for Analytic Manifolds with a Magnetic Field

Consider a compact Riemannian manifold M with boundary endowed with a Riemannian metric g and a magnetic field Ω. Given a point and direction of entry at the boundary, the scattering relation Σ determines the point and direction of exit of a particle of unit charge, mass, and energy. We will show that a magnetic system $(M, \partial M, g, \Omega)$ that is known to be real-analytic and that satisfies some mild restrictions on conjugate points is uniquely determined up to a natural equivalence by Σ. In the case that the magnetic field Ω is taken to be zero, this gives a new rigidity result in Riemannian geometry that is more general than related results in the literature.

Hamid Hezari (MIT) Spectral and resonant uniqueness of radial potentials

We prove that the spectrum of an n-dimensional semiclassical radial Schrodinger operator determines the potential within a large class of potentials for which we assume no symmetry or analyticity. We also show that in odd dimensions the resonances of a compactly supported radial potential determine the potential. Our proof is based on the first two semiclassical trace invariants and on the isoperimetric inequality. This is a joint work with Kiril Datchev and Ivan Ventura.

David Isaacson, RPI Mathematical Problems in the Diagnosis and Treatment of Disease

Mathematical problems arising in the diagnosis and treatment of heart disease and breast cancer will be discussed.

Hyeonbae Kang, Inha University Generalized Polarization Tensors: Mathematics and Applications

The generalized polarization tensors (GPT) are geometric and physical quantities associated with inclusions. Theyappear naturally in multipolar expansions of electric potentials near infinity in the presence of the inclusion. In this talkI will discuss about optimal bounds for GPTs, Kang-Milton's solution to the Polya-Szego conjecture, and connection to the Dirichlet-to-Neumann map. I then discuss their usage for reconstruction of fine details of the shape of the inclusion. I will also mention briefly on their connection to the cloaking (invisibility) which attractsmuch attention these years.

Matti Lassas (University of Helsinki) Stability of Inverse problems for Heat and Wave equations and the Collapse of the Dimension

In classical inverse problems one wants to show that physical measurements can be used to determine coefficients of various partial differential equations modeling macroscopic and microscopic phenomena. Examples of these are the paradigm problems, the inverse problem for conductivity equation and inverse scattering problem for Schrödinger operator. In these problems the structure of the underlying (Euclidean) space is a priori known before measurements Recently many inverse problems have been generalized to invariant settings, for cases where the underlying space is not a priori known but is assumed to be a Riemannian manifold. The measurements one considers to be given can be either measurements on a boundary of a domain or in a subdomain.

In this talk we consider the inverse problems associated to interior measurements: Assume that on a closed manifold M we can do measurement on some open subset U of M. Can these measurements be used to determine the whole manifold M and metric g on it? We analyze the stability of the reconstruction in a class of n dimensional manifolds we which contain manifolds which have almost collapsed to lower dimensional objects.

In the Euclidian space, the stability results for inverse problems for partial differential equations need considerations of non-smooth coefficients. Indeed, smooth coefficients functions can approximate non-smooth coefficients. For geometric inverse problems, we encounter similar phenomena : To understand stability of the solution of inverse problems for smooth manifolds, we need to consider collapse of smooth manifolds. As the limit of such manifolds can be orbifolds, we need to study inverse problems on orbifolds, too. The almost collapsed manifolds and orbifolds is encountered in models of the mathematical physics, in particular in the string theory. To explain this connection, we will consider as an example a predecessor of string theories, the Kaluza-Klein theory, and consider its relations to inverse problems.

The results presented in the talk have been done in collaboration with Yaroslav Kurylev (University College London, UK) and Takao Yamaguchi (Tsukuba University, Japan).

Alexader Mamonov (University of Texas, Austin) Resistor Networks and Optimal Grids for Electrical Impedance Tomography with Partial Boundary Measurements

We present methods to solve the partial data Electrical Impedance Tomography (EIT) problem numerically. Our methods regularize the problem by using sparse representations of the unknown conductivity on adaptive finite volume grids known as the optimal grids. The discretized problem is reduced to solving the discrete inverse problems for resistor networks. Two distinct approaches implementing this strategy are presented. The first approach uses the results for the full data EIT with circular resistor networks. The optimal grids for such networks are essentially one dimensional objects, which can be computed explicitly. We solve the partial data problem by reducing it to the full data case using the theory of extremal quasiconformal mappings. The second approach is based on pyramidal resistor networks. The optimal grids in this case are computed using the sensitivity analysis of both the continuum and the discrete EIT problems. Numerical results show two main advantages of our approaches compared to the traditional optimization-based methods. First, the inversion based on resistor networks is much faster than any iterative algorithm. Second, we are able to reconstruct the conductivities of ultra high contrast, which usually presents a challenge to inversion methods.

Adrian Nachman (University of Toronto) Reconstruction in the Calderón Problem with Partial Data

We consider the problem of recovering an isotropic conductivity in a body from measurements of the Cauchy data on possibly very small subsets of its surface. The lecture will explain a constructive proof of the uniqueness results of Bukhgeim-Uhlmann and Kenig-Sjöstrand-Uhlmann. We construct a uniquely specified family of solutions such that their traces on the boundary can be calculated by solving an integral equation which involves only the given partial Cauchy data. The construction introduces a new family of Green's functions for the Laplacian, and corresponding single layer potentials, which may be of independent interest. This is joint work with Brian Street.

Frank Natterer (University of Münster) Consecutive time reversal in wave equation imaging

Wave equation imaging plays an important role in ultrasound tomography and in seismic exploration. Most of the work in this field is done by linearization. In this talk we deal with iterative methods for the fully nonlinear problem. We start with a survey on the results obtained by linearization. For the fully nonlinear problem we describe in detail the Kaczmarz method in the time domain. It turns out that Kaczmarzs method, whose linear version is widely used in X-ray tomography, can be viewed in a very intuitive way as consecutive time reversal. We show by numerical examples that Kaczmarz easily solves the standard problems, such as transmission tomography and reflection imaging with broadband data. We also study the behavior of the method in non-standard situations, such as caustics, trapped rays, and, in particular, missing low frequencies in the source pulse. We discuss the role of reflectors, be they known or unknown. A heuristic condition for convergence is presented.

George Papanicolaou (Stanford) Imaging with Intensities Only

At high frequencies echographic imaging becomes impractical since only scattered intensities can be recorded effectively. This means that we do not have time resolved echoes recorded at an array so as to image reflectors. The usual migration methods and their variants do not work well in this case. However, analogs of L^1 based optimization techniques can be used and, under certain conditions, produce good images. I will introduce the intensity-only imaging problem and the associated optimization methodology, compare it with full echographic imaging, and then show the results of some numerical simulations. This is joint work with A. Chai and M. Moscoso.

Linh Nguyen (University of Idaho) Some Problems of Thermoacoustic Tomography (TAT)

TAT is a recently developed hybrid modality of medical imaging. A brief pulse of electromagnetic (EM) radiation slightly heats up the biological tissue, which leads to an ultrasound (pressure) wave propagation. The pressure is measured by transducers placed around the body. From this data, one tries to reconstruct the initial pressure distribution, and thus the EM absorption inside the body.

This talk addresses the following two issues arising in TAT:

1. We prove that the reconstruction is not Hölder stable if a natural visibility condition is violated. This complements a recent result by V. Palamodov (for constant speed) and P. Stefanov and G. Uhlmann (for variable speed), which states that under the visibility condition, the reconstruction is Lipschitz stable.

2. Most of the work done in TAT assumes that the ultrasound speed is known. However, it is

usually not known. One might wonder whether the TAT data could determine both the ultrasound speed and the initial pressure. We will discuss some partial results obtained in this direction (joint work with M. Agranovsky and P. Kuchment).

Barbara Romanowicz (UC Berkeley) Recent Advances in Full Waveform Global Seismic Tomography of the Earth's Mantle

Over the last 15 years, we have been developing global and continental scale models of shear velocity, anisotropy and attenuation of the earth's mantle based on full waveform inversion of long period ($T > 30s$) three component teleseismic records. For many years, our several generations of models were based on the computation of synthetics and kernels using a normal mode based approach, NACT (non-linear asymptotic coupling theory, Li and Romanowicz, 1995), which is theoretically limited to wave propagation in weak and smooth heterogeneity. Recently, it has become possible to compute accurate teleseismic synthetics in spherical geometry for arbitrary 3D structures using numerical approaches. In particular, the Spectral Element Method is well suited for our applications. The challenge has been shifted from theoretical limitations to the length of computations involved. Computing both the forward and inverse problem numerically, in the presence of a realistic crustal model, with the many iterations and the large number of sources required, is still prohibitive in practice, especially if we want to utilize information at higher frequencies. Our philosophy has been to approach the problem progressively, so that we can learn something about the Earth at each step. In particular, we have chosen to compute the forward model accurately using SEM, but keeping approximate NACT kernels recomputed at each iteration. I will discuss the construction of our first global upper mantle shear velocity model obtained using SEM for forward computations, periods down to 60s, and an approximate scheme for crust homogeneization. I will also discuss our current steps towards the development the next generation, whole mantle model, using waveforms down to 30s and the corresponding construction of a more realistic crustal model.

Hart Smith (University of Washington) Decoupling of Modes for the Elastic Wave Equation in Media of Limited Smoothness

We establish a decoupling result for the P and S waves of linear, isotropic elasticity, in the setting of twice-differentiable Lamé parameters. Precisely, we show that the P to S components of the wave propagation operator are regularizing of order one on L^2 data, by establishing the diagonalization of the elastic system modulo a L^2 bounded operator. Effecting the diagonalization in the setting of twice differentiable coefficients depends upon the symbol of the conjugation operator having a particular structure.

Mikko Salo (University of Helsinki) Inverse Problems for the Anisotropic Maxwell Equations

We prove that the electromagnetic material parameters are determined by boundary measurements for the time-harmonic Maxwell equations in certain anisotropic settings. This extends an earlier result with Dos Santos Ferreira, Kenig and Uhlmann for the scalar conductivity equation to the case of the Maxwell system. To do this we establish Sylvester-Uhlmann type estimates for transversally anisotropic manifolds, and obtain a simple new proof for the Euclidean case as a byproduct. This is a joint work with Carlos Kenig (University of Chicago) and Gunther Uhlmann (University of Washington).

Plamen Stefanov (Purdue University) Thermoacoustic and Photoacoustic Tomography with a

Variable Continuous or Discontinuous Sound Speed

We study the mathematical model of Thermoacoustic and Photoacoustic Tomography with a variable sound speed. We consider both a smooth speed, and a speed having jump type of discontinuities across a smooth surface, that leads to a transmission problem. The latter model arises in brain imaging, where the speed in the skull jumps by about a factor of 2. In case of data on the whole boundary, for a smooth speed, we present an explicit solution as a convergent Neumann series. In case of partial data, we give sufficient and necessary conditions for uniqueness, and sufficient and necessary conditions for stability. We also characterize the observation operator as a Fourier Integral Operator, and characterize the range microlocally. In case of a discontinuous speed, we show that there are invisible singularities. Under the condition that all singularities in the support of the source are visible, then the Neumann series in the smooth case still converges and provides an explicit solution but the convergence can be expected to be slower due to rays that reflect from the "skull". This talk is based on a joint work with Gunther Uhlmann.

Leo Tzou (University of Arizona) The Inverse Calderón Problem for Schrödinger Operator on Riemann Surfaces

We show that on a smooth compact Riemann surface with boundary (M, g) the Dirichlet-to-Neumann map of the Schrödinger operator $\Delta_g + V$ determines uniquely the potential V . This seemingly analytical problem turns out to have connections with ideas in symplectic geometry and differential topology. We will discuss how these geometrical features arise and the techniques we use to treat them.

This is joint work with Colin Guillarmou of CNRS Nice. The speaker is partially supported by NSF Grant No. DMS-0807502 during this work.

Andras Vasy (Stanford) Wave Propagation on Asymptotically De Sitter and Anti-de Sitter Spaces

In this talk I describe the asymptotics of solutions of the wave equation on asymptotically De Sitter and Anti-de Sitter spaces. This is part of a larger program to analyze hyperbolic equations on non-product, non-compact manifolds, similarly to how various types of 'ends' have been studied for the Laplacian and other elliptic operators on Riemannian manifolds. The AdS setting is particularly interesting from the point of view of propagation phenomena, since for the conformally related incomplete metric, there are null-geodesics which are tangent to the boundary. I will also mention some natural inverse problems in this context.

Steve Zelditch (Northwestern) Spectral rigidity of ellipses among C^{infty} plane domains with the ellipse symmetry

We prove that ellipses are infinitesimally spectrally rigid among smooth plane domains with the same two symmetries. The advance is that we do not need to assume analyticity of the competing domains. Joint work with Hamid Hezari.

Hongkai Zhao (UC Irvine) A Phase Space Method for Traveltime Tomography

A phase space method is developed for reconstructing the index of refraction of a medium from travel time measurements. The method is based on the so-called Stefanov-Uhlmann identity which links two Riemannian metrics to the scattering relations. The phase space formulation can deal with multiple arrival times naturally. The numerical algorithm uses a hybrid formulation that follows rays in phase space and reconstruct the velocity field in physical space. We also developed an adaptive strategy that uses geodesics that produces smaller mismatch first in the spirit of layer stripping but without the need to define the layers physically. The adaptive approach improves stability, efficiency and accuracy. We then extend our method to reflection traveltime tomography by incorporating broken geodesics. In particular we show that our method can distinguish and utilize measurements from non-broken and broken geodesics accordingly in reflection traveltime tomography.

Ting Zhou (University of Washington) Reconstructing Electromagnetic Obstacles by the Enclosure Method

We show that one can determine Perfectly Magnetic Conductor obstacles, Perfectly Electric Conductor obstacles and obstacles satisfying impedance boundary condition, embedded in a known electromagnetic medium, by making electromagnetic measurements at the boundary of the medium. The boundary measurements are encoded in the impedance map that sends the tangential component of the electric field to the tangential component of the magnetic field. We do this by probing the medium with complex geometrical optics solutions to the corresponding Maxwells equations and extend the enclosure method to this case. Moreover, using complex spherical waves, constructed by the inversion transformation with respect to a sphere, the enclosure method can recover some non-convex part of the obstacle.