## Boundary Control Approach to Inverse Problems on Graphs

The Boundary Control (BC) method is based on the deep connection between control theory for partial differential equations and inverse problems of mathematical physics and offers an interesting and powerful alternative to previous identification techniques based on spectral or scattering methods. This approach has several advantages: (i) it is applicable to a wide range of linear lumped and distributed systems and reconstruction situations; (ii) it is, in principle, dimension-independent; (iii) it lends itself to straightforward algorithmic implementations. In the first part of this talk we discuss connections between the BC method and the classical Gelfand–Levitan and Krein theories, and the recently proposed Simon and Remling approaches. In particular, we describe the local version of the Gelfand–Levitan equations.

The second part of the talk devoted to inverse problems on graphs. Differential equations on graphs are used to describe many physical processes such as mechanical vibrations of multi-linked flexible structures usually composed of flexible beams or strings, propagation of electro-magnetic waves in networks of optical fibers, heat flow in a wire mesh, and also electron flow in quantum mechanical curcuits.

As an important example, we consider the in-plane motion of elastic strings on a tree-like network. The two-velocity wave equation for a two component vector displacement is assumed to hold on each edge of a tree. We investigate the inverse problem of recovering not only the physical properties, i.e. the velocities and lengths of each string, but also the topology of the tree and the angles between branching edges. It is shown that under generic assumptions the inverse problem can be solved by applying measurements at all leaves, the root of the tree being fixed.

We use a new version of the Boundary Control method proposed in [1,2] (which combines the spectral and dynamical approaches to inverse problems for PDEs on graphs) and develop a constructive procedure for the recovery tree's parameters. This procedures is recursive — it allows recalculating efficiently the inverse data from the original tree to the smaller trees, 'removing' leaves step by step up to the rooted edge. Because of its recursive nature, this procedure may serve as a base for developing effective numerical algorithms.

1. S. Avdonin and P. Kurasov, *Inverse problems for quantum trees,* Inverse Problems and Imaging, **2** (2008), 1–21.

2. S. Avdonin, G. Leugering and V. Mikhaylov, *On an inverse problem for tree-like networks of elastic strings*, ZAMM, **90** (2010), 136–150.