A REFRACTION PROBLEM AND MONGE-AMPÈRE TYPE EQUATIONS

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The physical phenomena of refraction and reflection occur simultaneously: if a light ray strikes a boundary separating to media with different refractive indices, then the ray splits into an internally reflected ray and a refracted (or transmitted) ray, each one having certain intensity. A precise description these intensities, depending on the angle of incidence, is given by the Fresnel formulas, a consequence of Maxwell's equations. We present a new model to construct refractive surfaces that takes into account this splitting of energy and solve the following problem. Let Ω, Ω' be domains in the sphere S^{n-1}, and let n_1 and n_2 be the refractive indices of two homogeneous and isotropic media I and II, respectively, for example, glass and air. Suppose that from a point O surrounded by medium I, radiation emanates with intensity $f(x)$ for $x \in \Omega$. We prove the existence of a surface $\mathcal{R}$ parameterized by $\mathcal{R} = \{\rho(x)x : x \in \overline{\Omega}\}$, interface between media I and II, such that all rays refracted by $\mathcal{R}$ into medium II have directions in Ω' and the prescribed illumination intensity received at each direction $m \in \Omega'$ is $g(m)$. In the model, we introduce energy conditions relating the input intensity f and the output intensity g that take into account the energy used in internal reflection. This yields the existence of a lens refracting radiation in a prescribed way. We show also that the function ρ solves a Monge-Ampère type pde.