UNIQUENESS AND STABILITY FOR THE INVERSE CONDUCTIVITY PROBLEM WITH INTERNAL DATA

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Abstract

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ be a bounded, open and connected domain with boundary of class C^3, and $\gamma(x)$ be a real function that lies in $C^2(\overline{\Omega})$. For boundary data $(g_i)_{1\leq i\leq d}$ in $C^{2,\alpha}(\partial\Omega)$, where α is a fixed real in $(0,1)$, we consider the following conduction equations

$$\begin{array}{llll} \operatorname{div}(\gamma\nabla u_i) & = 0 & \text{in} & \Omega, \\ u_i & = g_i & \text{on} & \partial\Omega, \end{array} \tag{0.1}$$

$i = 1, \cdots, d$. We assume that there exists a real constant c_0 such that

$$0 < c_0 \leq \gamma(x) \qquad x \in \Omega. \tag{0.2}$$

Under these assumptions it is well known that there exists a unique $u_i \in C^{2,\alpha}(\overline{\Omega})$, solution to the problem (0.1).

Let ω be a subdomain of Ω, that is $\omega \subset\subset \Omega$. The internal data are in the form

$$E_{ij}(\gamma) \;=\; \gamma\nabla u_i(x).\nabla u_j(x) \qquad x \in \omega, \quad i, j = 1, \cdots, d. \tag{0.3}$$

The inverse conductivity problem with internal data consists on the reconstruction of $\gamma(x)$ on ω from the knowledge of $(E_{ij}(\gamma))_{1\leq i,j\leq d}$.

In a recent work [1], Ammari *et al.* have showed that taking boundary measurements while perturbing the medium with ultrasound waves is asymptotically equivalent to measuring $(E_{ij}(\gamma))_{1\leq i\leq d}$. The first inversion algorithm [1] proposed to recover γ on the whole two dimensional domain Ω from internal data $(E_{ij}(\gamma))_{1\leq i,j\leq 2}$ proves remarkably accurate. This algorithm is based on a perturbation approach and uses at least two chosen voltage potentials on the boundary. In [2], the authors have reformulated the inversion as a minimization problem, and proposed a second algorithm that successfully determine the conductivity γ on subdomains ω of Ω.

In this work we study the inverse conductivity problem with internal data. We show that knowledge of a finite number of energy densities uniquely determines the conductivity. A local Lipschitz stability of the reconstruction is also derived. We aim by the derived stability result to understand how and why the previously cited algorithms showed very efficient. This work has been done in collaboration with Eric Bonnetier.

References

[1] H. Ammari, E. Bonnetier, Y. Capdeboscq, M. Tanter, and M. Fink, *Electrical impedance tomography by elastic deformation,* SIAM J. APPL. MATH. Vol. 68, No. 6, pp. 1557-1573 (2008)

[2] Y. Capdeboscq, J. Fehrenbach, F. de Gournay, and O. Kavian, *An optimal control approach to imaging by modification*, SIAM Journal on Imaging Sciences, 2, pp. 1003-1030 (2009).

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