INVERSE SCATTERING FROM CUSP

We consider an inverse problem for some 2-dimensional non-compact manifolds, or strictly speaking, orbifolds. Our motivating example is a Riemann surface $\mathcal{M} = \Gamma\backslash\mathbf{H}^2$ associated with the Fuchsian group of 1st kind Γ containing parabolic elements. Then, although $\mathcal{M}$ has a finite volume, it is non-compact, and its ends consist of a finite number of cusps. Moreover, it contains elliptic singular points. Thus it is regarded as a hyperbolic orbifold. Our aim is to introduce spectral characteristics by which we can identify this orbifold.

We shall introduce a class of orbifolds whose metric is asymptotically equal to the hyperbolic metric at the cusp. By observing asymptotic behavior of solutions to the Helmholtz equation at the cusp, we define a generalization of Heisenberg's S-matrix for the orbifold. We then show that this generalized S-matrix determines the Riemannian metric and the orbifold structure of $\mathcal{M}$. This is a joint work with Y. Kurylev and M. Lassas.