Optimization Problems for Domain Functionals

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Let *D* be a domain in the Euclidean space or on a manifold and consider a functional F(D). Examples are for instance the eigenvalues of the Laplace operator, the electrostatic capacity and the best constants in Sobolev inequalities. We ask for the minimum of *F* if *D* varies in a certain class *C* of domains characterized by a certain geometrical property, for instance domains with the same volume. In this survey talk we shall present different approaches for solving such problems, starting with the direct methods of symmetrizations. Another way to study the dependence of a functional on the domain is, in the spirit of classical analysis, to investigate its change under an infinitesimal perturbation of the domain. This method is useful to single out extremal domains. It leads in general to over-determined boundary value problems. The main difficulties related to optimal domains are: existence, its regularity and symmetry. We shall describe some of the most important tools to treat these questions.