A free boundary problem arising in the context of Bose-Einstein condensation

Benedetta Noris, Università degli Studi di Milano Bicocca

We consider the positive solutions of the Gross–Pitaevskii system

$$\begin{cases} -\Delta u_\beta + \lambda u_\beta = \omega_1 u_\beta^3 - \beta u_\beta v_\beta^2 & \text{in } \Omega \\ -\Delta v_\beta + \mu v_\beta = \omega_2 v_\beta^3 - \beta u_\beta^2 v_\beta & \text{in } \Omega \\ u_\beta, v_\beta \in H_0^1(\Omega) \end{cases}$$

where β is a positive parameter, $\lambda, \mu, \omega_1, \omega_2 \in \mathbb{R}$ and $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain ($N = 2, 3$). This system arises in the Hartree-Fock approximation theory for binary mixtures of Bose–Einstein condensates in different hyperfine states. We prove that L^∞–boundedness of the solutions implies $C^{0,\alpha}$–boundedness, uniformly as $\beta \to +\infty$, for every $\alpha \in (0, 1)$. Moreover we prove that the limiting profile, as $\beta \to +\infty$, is Lipschitz continuous. The proof relies upon the blow–up technique and the monotonicity formulae by Almgren and Alt–Caffarelli–Friedman. Work in collaboration with Hugo Tavares, Susanna Terracini and Gianmaria Verzini.