FREE BOUNDARY PROBLEMS, THEORY & APPLICATIONS

MSRI, March 7–11, 2011

ABSTRACTS

Avner Friedman, *MBI, Ohio State University*

A Free Boundary Problem Modeling Wound Healing

Abstract. We consider a 3-D model of dermal wound. The tissue surrounding the wound is slowly healing, and is viewed as an upper convected Maxwell fluid. The velocity satisfies a free boundary problem for a strongly elliptic system with inhomogeneous terms which depend on the Fluid pressure P. But P depends on the density of the tissue, which satisfies, together with a number of other variables (densities of cells and signaling molecules) a system of parabolic/hyperbolic equations outside the wound. We shall discuss questions of existence of a solution and properties of the free boundary.

(*2011-03-07, 9:15-10:15 am, MSRI, Simons Auditorium*)

Luis Silvestre, *University of Chicago*

On Unique Continuation for Nonlinear Elliptic Equations

Abstract. We will discuss the following issue: if a solution to a nonlinear elliptic equation vanishes in a small ball, is it necessarily identically zero? The problem is fairly well understood in the linear setting, but it is open for most interesting nonlinear elliptic equations. We will analyze the difficulties of the problem and prove a result in arguably the simplest case in which one cannot linearize the equation a priori. We will also relate it to a partial regularity result for fully nonlinear elliptic equations.

(*2011-03-07, 11:00-12:00 noon, MSRI, Simons Auditorium*)

Yuval Peres, *Microsoft Research*

Laplacian Growth and the Mystery of the Abelian Sandpile

Abstract. Abstract: We compare several growth models on the two dimensional lattice. In some models, like internal DLA and rotor-router aggregation, the scaling limits are universal; in particular, starting from a point source yields a disk. Other sources yield a free-boundary problem for the Laplacian. In the abelian sandpile, particles are added at the origin and whenever a site has four particles or more, the top four particles topple, with one going to each neighbor. Despite similarities to other models, for the sandpile, the intriguing pattern that arises is not circular and depends on the particular lattice. It is an open problem to prove a scaling limit exists for the sandpile, though some bounds are known. I will describe

some open problems, including a connection to conformal mapping which has not been established yet. Talk based on joint works with Lionel Levine.

(*2011-03-07, 2:00-3:00 pm, MSRI, Simons Auditorium*)

Juan Luis Vázquez, *Universidad Autónoma de Madrid*

Nonlinear diffusion and free boundaries. From porous media to fractional diffusion (*Evans Lecture*)

Abstract. In the talk we will make a presentation of the theory of Nonlinear Diffusion centered on one of the popular models, the porous medium equation and its close relative, the fast diffusion equation. The existence of free boundaries is one of the most peculiar properties of the former equation.

In the final section, we will present recent work that combines degenerate nonlinear diffusion with nonlocal operators of fractional Laplacian type. Apart from the unexpected existence of free boundaries, the model admits mass preserving self-similar solutions that are found by solving an elliptic fractional-Laplacian obstacle problem. We use entropy methods to show that the asymptotic behaviour is described after renormalization by these selfsimilar solutions.

(*2011-03-07, 4:10-5:10 pm, UC Berkeley, Evans Hall 60*)

Luis Caffarelli, *University of Texas at Austin*

Regularity Properties of the Non Local Parabolic Obstacle Problem

Abstract. We intend to give an idea of the methods used to show regularity for parabolic obstacle involving fractional diffusion in the case where the obstacle coincides with the initial data. This is a case concerning an American option. We will review the stationary case, where some of the ideas generate, to then pass to the evolution case.

(*2011-03-08, 9:00-10:00 am, MSRI, Simons Auditorium*)

Gieri Simonett, *Vanderbilt University*

On the Rayleigh-Taylor Instability for the Two-Phase Navier-Stokes Equations

Abstract. We consider the free boundary problem of two superposed, immiscible, viscous, incompressible fluids. Allowing for gravity to act on the fluids, we prove local well-posedness of the problem. In particular, we obtain well-posedness for the case where the heavy fluid lies on top of the light one, that is, for the case where the Rayleigh-Taylor instability is present. Additionally we show that solutions become real analytic instantaneously, and we study the Rayleigh-Taylor instability. The approach relies on the theory of maximal regularity. (Joint work with Jan Prüss.)

(*2011-03-08, 10:30-11:30 am, MSRI, Simons Auditorium*)

Juhi Jang, *University of California, Riverside*

Vacuum in Gas and Fluid Dynamics

Abstract. An interesting problem in gas and fluid dynamics is to understand the behavior of vacuum states, namely the behavior of the system in the presence of vacuum. A particular interest is so called physical vacuum which naturally arises in physical problems. I'll report on a recent progress in a rigorous study of a physical vacuum. If time permits, I'll also discuss stability theory of Lane-Emden equilibrium stars under Euler-Poisson or Navier-Stokes-Poisson system.

(*2011-03-08, 11:30-12:30 pm, MSRI, Simons Auditorium*)

Antoine Mellet, *University of Maryland*

A Free Boundary Problem for Thin Films

Abstract. The lubrication approximation leads to a fourth order degenerate equation modeling the evolution of small viscous droplets on a solid support (the thin film equation). Along the free boundary (contact line), the solution must satisfy a gradient condition (contact angle condition). While many existence and regularity results are known for solutions with zero contact angle, the only existence result with non-zero contact angle is due to Otto and only holds in some particular framework (Hele-Shaw cell). Following Bertsch, Giacomelli and Karali, we take a different approach (regularization) to this free boundary problem to attempt to generalize Otto's result.

(*2011-03-08, 2:00-3:00 pm, MSRI, Simons Auditorium*)

Nicola Garofalo, *Purdue University*

Optimal Regularity and the Study of the Free Boundary in the Parabolic Signorini Problem

Abstract. I will survey some recent joint work with D. Danielli, A. Petrosyan and T. To. We give a comprehensive treatment of the parabolic Signorini problem based on a generalization of Almgren's monotonicity of the frequency to parabolic equations. This includes the proof of the optimal regularity of solutions, classification of free boundary points, the regularity of the regular set and the structure of the singular set.

(*2011-03-08, 3:30-4:30 pm, MSRI, Simons Auditorium*)

Charles Elliott, *University of Warwick, UK*

Computational Surface Partial Differential Equations

Abstract. I will discuss computational approaches to the problem of solving PDES on evolving surfaces. The motivation is applications such as phase separation on biomembranes, surface dissolution, pattern formation on biological surfaces, transport of surfactants on fluid interfaces and surface etching of binary alloys.

(*2011-03-09, 9:00-10:00 am, MSRI, Simons Auditorium*)

John Ockendon, *University of Oxford, UK*

Free Boundary Problems in Elastoplasticity

Abstract. After a brief review of the history of free boundary problems in elastoplasticity, this talk will describe recent work on one-dimensional elastoplastic wave propagation when the applied stress greatly exceeds the yield stress. In these waves, free boundaries can occur both within regions of plastic flow and separating elastic and plastic regions.

(*2011-03-09, 10:30-11:30 am, MSRI, Simons Auditorium*)

Kaj Nyström, *Umeå University, Sweden*

Regularity and Free Boundary Regularity for the p-Laplace Operator in Reifenberg flat and Ahlfors Regular NTA-Domains

Abstract. In this talk I will discuss joint work with John Lewis concerning regularity and free boundary regularity below the continuous threshold for positive p-harmonic functions, $1 < p < \infty$, vanishing on a portion of a Reifenberg flat or Ahlfors regular NTA-domain. Our results generalize previous works of Kenig and Toro, valid in the case $p = 2$ and for harmonic functions, to the non-linear setting of the p-Laplace operator.

(*2011-03-09, 11:30-12:30 pm, MSRI, Simons Auditorium*)

Ricardo Nochetto, *University of Maryland*

Electrowetting on Dielectric: Modeling, Analysis and Computation

Abstract. Electrowetting on dielectric (EWOD) refers to a parallel-plate micro-device that moves fluid droplets through electrically actuated surface tension effects. These devices have potential applications in biomedical 'lab-on-a-chip' devices (automated DNA testing, cell separation) and controlled micro-fluidic transport (e.g. mixing and concentration control).

We present two models that account for several physical effects. We first model the fluid dynamics using Hele-Shaw type equations (in 2d) with a focus on including the relevant boundary phenomena, such as viscous damping and contact line pinning (sticking of the interface). The latter leads to a variational inequality on the liquid-gas interface. We analyze this approach, present simulations, and compare them to experimental videos of EWOD driven droplets exhibiting pinching and merging events.

We also discuss a diffuse interface 3d model for incompressible two-phase flows with moving contact lines. The latter are governed by a generalized Navier boundary condition. We show that the fully discrete scheme, based on fractional time stepping, satisfies an energy law, and we document its performance with several simulations.

This is joint work with S. Walker, A. Bonito, B. Shapiro, and A. Salgado.

(*2011-03-10, 9:30-10:30am, MSRI, Simons Auditorium*)

Ovidiu Savin, *Columbia University*

Boundary Regularity for a Class of Solutions to the Monge-Ampere Equation

Abstract. We discuss boundary regularity in 2D for minimizers of a linear integral

$$L(u) = \int_{\partial\Omega} u d\sigma - \int_{\Omega} u dA$$

among the solutions to the Monge-Ampere equation

$$\det D^2 u = 1.$$

This problem can be viewed as a particular case of minimizing more general convex functionals of the type

$$\int_{\Omega} \Phi(\det D^2 u) dx + \int_{\partial\Omega} u d\sigma - \int_{\Omega} u dA$$

which appear in other contexts. This is a joint work with N. Le.

(*2011-03-10, 11:00-12:00 noon, MSRI, Simons Auditorium*)

Régis Monneau, *École Nationale des Ponts-et-Chaussées - ENPC, France*

Free Boundary Pointwise Estimates for the Obstacle Problem

Abstract. We will present some results about pointwise regularity of the free boundary for the obstacle problem. This can be in particular applied when the coefficients of the problem are Dini. The results cover both regular and singular points of the free boundary. We will also present generalizations in the case of the parabolic obstacle problem.

(*2011-03-10, 2:00-3:00 pm, MSRI, Simons Auditorium*)

Stephen Gardiner, *University College Dublin, Ireland*

Two-Phase Quadrature Domains

Abstract. Recent work on two-phase free boundary problems has led to the investigation of a new type of quadrature domain for harmonic functions. This talk will discuss such two-phase quadrature domains and, in particular, how they may be constructed. The construction is based on the technique of partial balayage, which has proved to be a useful tool in the study of one-phase quadrature domains and Hele-Shaw flows. (Joint work with Tomas Sjödin.)

(*2011-03-10, 3:30-4:30 pm, MSRI, Simons Auditorium*)

Gui-Qiang Chen, *University of Oxford, UK*

Free Boundary Problems in Conservation Laws

Abstract. n this talk we will present several free boundary problems for the stability of discontinuity waves and the existence of fundamental wave patterns in hyperbolic conservation laws. The discontinuity waves include shock waves, vortex sheets and entropy waves. Some recent developments will be reviewed and discussed. Further trends, perspectives, and open

problems in this direction will also be addressed.

(*2011-03-11, 9:30-10:30 am, MSRI, Simons Auditorium*)

Lionel Levine, *Massachusetts Institute of Technology*

Logarithmic Fluctuations from Circularity

Abstract. Start with n particles at the origin in $\mathbb{Z}^d$, and let each particle in turn perform simple random walk until reaching an unoccupied site. This process, known as internal DLA, is a discrete analogue of the stable direction of Hele-Shaw flow. Lawler, Bramson and Griffeath proved that with high probability the random set of n occupied sites formed by internal DLA is close to a ball. We show that its fluctuations from circularity are, with high probability, at most logarithmic in the radius of the ball, answering a question posed by Lawler in 1995. These logarithmic fluctuations were predicted numerically by chemical physicists in the 1980's. Joint work with David Jerison and Scott Sheffield.

(*2011-03-11, 11:00-12:00 noon, MSRI, Simons Auditorium*)

Ki-Ahm Lee, *Seoul National University, Korea*

Homogenization of the Oscillating Data on a Lower Dimensional Surface

Abstract. In this talk, I would like to discuss the homogenization of the partial differential equations with a oscillating data on a lower dimensional surfaces. Such problems arise on standard Dirichlet or Neumann problem with oscillating boundary data and Free boundary problems. The lower dimensional character creates similar issues: oscillating of boundary in mod 1, multiple limits on rational direction and unique behavior on the irrational direction, which will be discussed through examples. And we find out the effective data by some estimates on the so-called correctors which will be used to correct super- or sub-solutions of the homogenized equation to be a super- or sub-solutions of ϵ-problems. And then we will discuss the general lower dimensional surface.

(*2011-03-11, 2:00-3:00 pm, MSRI, Simons Auditorium*)

Norayr Matevosyan, *University of Cambridge, UK*

A Boundary Obstacle Problem Near Fixed Boundary

Abstract. We study a Signorini type thin obstacle problem near a fixed boundary, that is on a part of the boundary we "clamp down" the solution. We show that in general the solutions are at least $C^{0,1/2}$ regular and that regularity is sharp. However, we are able to prove that near the actual points of contact of the free boundary with the fixed one the solution had to be at least $C^{1,1/2}$ regular, while away from it the global solutions are homogeneous with discrete homogeneity degrees $1/2, 3/2, 5/2, \ldots, (2n+1)/2, \ldots$.

Joint work with A. Petrosyan.

(*2011-03-11, 3:30-4:30 pm, MSRI, Simons Auditorium*)
