Finsler Geometry and Dynamics

Patrick Foulon
Université de Strasbourg

We will investigate the geometry of some typical examples of Finsler structures such as Katok-Ziller metrics and Hilbert geometries. Katok-Ziller metrics on spheres are prototypes of smooth non- reversible metrics of constant positive flag curvature (boat sailing metrics). Following Thomas Barthelme's work we introduce a new solid angle associated with the Holmes-Thompson volume. An immediate byproduct is the existence of a linear natural Finsler Laplace operator. I shall present some of Barthelme's results concerning the spectrum of these metrics and emphasize the interplay between the λ_1 and the Finsler volume. I shall also comment briefly on the existence of harmonic measures for negatively curved smooth Finsler metrics. Hilbert geometry and some quotients, the co-compact convex projective structures, are very nice examples of reversible non-smooth Finsler metrics for which the idea of extending the construction of a part of Chern's connection dynamically turns out to be very fruitful. To show this, I shall report on work by Mickael Crampon which gives a sharp estimate of the volume entropy and thus provides a very neat approach for the counting problem of closed geodesics.