Introduction to Frobenius splitting #1

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We begin by recalling the famous Hochster-Roberts theorem, which states that when a linearly reductive group acts linearly on a polynomial ring, the resulting ring of invariants is a Cohen-Macaulay ring. The key idea in proof is the idea of splitting. We will define what it means for a homomorphism of rings to split and give many examples of the power of this idea. In characteristic p, the splitting of the Frobenius map has especially nice consequences. Hochster and Roberts essentially called this "F-purity" but nowadays Frobenius splitting is more common. We introduce the related notion of F-regularity, explaining how F-regularity can be used to prove Cohen-Macaulayness, and ultimately the Hochter-Roberts theorem in characteristic p. The theorem holds in any characteristic by reduction to characteristic p.