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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: A. Seceleanu Email/Phone: asecelanu2@math.unl.edu

Speaker's Name: Idun Reiten

Talk Title: Cluster categories

Date: 09/05/12 Time: 10:30 am / pm (circle one)

List 6-12 key words for the talk: cluster category, quiver, tilting theory, Calabi-Yau

Please summarize the lecture in 5 or fewer sentances: This lecture describes cluster categories associated with finite acyclic quivers, cluster tilting objects and properties of cluster categories.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
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  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
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- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Idun Reiten – Cluster categories II

Recall: Q acyclic, vertices $1,\dots,n$, $\mathcal{F} = \mathbb{Q}(x_1,\dots,x_n)$
First guess consider kQ (finite dim k-alg)
tilting modules $T = T_1 \oplus \dots \oplus T_n$
Two main points: need more indecomposable objects
need more maps
Ex: $1 \to 2 \to 3$ (last lecture)
Have 9 indecomposable objects in $\mathcal{C}_Q$ [6 in mod kQ + 3 extra: $P_1[1]$, $P_2[1]$, $P_3[1]$].

$$\mathrm{Hom}_{\mathcal{C}_Q}(S_1, S_3) \stackrel{\text{def}}{=} \bigoplus_i \mathrm{Hom}_{D^b(kQ)}(S_1, F^i S_3)$$

$\underline{i=1}$ $\mathrm{Hom}_{D^b(kQ)}(S_1, F(S_3)) = \mathrm{Hom}_{D^b(kQ)}(S_1, S_2[1]) = \mathrm{Ext}^1_{kQ}(S_1, S_2) \neq 0$
$F(S_3) = \tau^{-1}(S_3[1])$

$0 \to S_2 \to \begin{smallmatrix} S_1 \\ S_2 \end{smallmatrix} \to S_1 \to 0$

⑦ Analogs of clusters / cluster variables

Def 1) T in $\mathcal{C}_Q$ is rigid if $\mathrm{Ext}^1_{\mathcal{C}_Q}(T,T)=0$ and maximal rigid if T is maximal with this property.
2) T is cluster tilting if in addition $\mathrm{Ext}^1_{\mathcal{C}_Q}(T,X)=0$
$\Rightarrow X \in \mathrm{add}\, T$
T is tilting kQ-module $\Rightarrow$ T is cluster tilting = max. rigid

Thm The desired properties (i), (ii), (iii) hold.
[(iv) $X_2 X_2^{**} = m_1 + m_2$ corresponds to
$T_i \to B' \to T_i^* \to T_i[1]$
$T_i^* \to B \to T_i \to T_i^*[1]$]

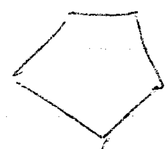
⑧ An application of lifting of quiver mutation

- The graph of cluster tilting objects is connected

$$\begin{array}{ccc} T = kQ & \longmapsto & \mu_i(T) \\ \downarrow & & \searrow \\ Q(=Q_T) & \longmapsto & \mu_i(Q) = Q_{\mu_i(T)} \end{array}$$

<u>Get</u> mutation class of the acyclic quiver Q
$= \{ Q_T ; T$ is a cluster tilting object $\}$
[Buan–Reiten] Mutation class of Q is finite $\iff$
$\iff |Q|$ is Dynkin or extended Dynkin.

(9) <u>A_n case</u> [Caldero–Chapoton–Schiffler]
$n = 2$ ($n+3$-gon)



Cluster variables : diagonals
Clusters sets of non-crossing diagonals
Shared cat. $\mathcal{C} \sim \mathcal{C}_{A_n}$

II <u>2-Calabi Yau categories with cluster tilting objects</u>

① <u>General theory</u>
For $\mathcal{C}_Q$ we have, Hom-finite, triangulated and functorial isomorphism $D\mathrm{Hom}(A,B) \simeq \mathrm{Hom}(B, A[2])$ (where $D = \mathrm{Hom}_k(-,k)$).
[$D\,\mathrm{Ext}^1(A,B) \simeq \mathrm{Ext}^1(B,A)$]
[$F = [2]$ is the Serre functor, so $\mathcal{C}_Q$ is 2 Calabi-Yau].

<u>Thm</u> $\mathcal{C}$ is Hom-finite, triangulated 2 Calabi-Yau
Assume $\exists$ some cluster tilting object
(a) For $T = T_1 \oplus \dots \oplus T_i \oplus \dots \oplus T_n$ cluster tilting, then $\mu_i(T)$ is defined (Iyama–Yoshino)
(b) Have existence of the special triangle (Iyama–Yoshino)
(c) If Q_T (T tilting object) has no loops or 2-cycles then the two mutations commute. [Buan–Iyama–Reiten–Scott]

<u>Note</u> $G \curvearrowright$ may exist
may have max. rigid $\neq$ cluster tilting

③ Preprojective algebra of Dynkin type [GLS]
Q Dynkin $\mapsto \Lambda_Q$ preprojective alg is finite dim,
self injective
mod Λ is 2 Calabi Yau.