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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: A. Seceleanu Email/Phone: aseceleanu2@math.unl.edu

Speaker's Name: Idun Reiten

Talk Title: Cluster categories

Date: 09/06/12 Time: 10:30 am/ pm (circle one)

List 6-12 key words for the talk: cluster category, quiver, tilting theory, Calabi-Yau

Please summarize the lecture in 5 or fewer sentances: This lecture describes cluster categories associated with finite acyclic quivers, cluster tilting objects and properties of cluster categories.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
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- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Idun Reiten - Cluster category III

II 2 - CY categories with cluster tilting objects

Recall

Let $\mathcal{C}$ be a Hom-finite triangulated category.
Then an equivalence $F: \mathcal{C} \to \mathcal{C}$ is a Serre functor if we have functorial isomorphisms
$D(\mathrm{Hom}(A,B)) \simeq \mathrm{Hom}(B, F(A))$ $\quad (D = \mathrm{Hom}_k(-,k))$
$\mathcal{C}$ is 2-CY if $F = [2]$ (equiv. $\tau = [1]$)

Thm (Reiten, Van den Bergh)
$\mathcal{C}$ has a Serre functor $\iff$ $\mathcal{C}$ has almost split triangles

Examples ① Cluster category $\mathcal{C}_Q$
② Preprojective alg. of Dynkin type $Q: 1 \xleftarrow{\alpha} 2 \xleftarrow{\beta} 3$

$\overline{Q} = 1 \underset{\alpha^*}{\overset{\alpha}{\rightleftarrows}} 2 \underset{\beta^*}{\overset{\beta}{\rightleftarrows}} 3$ (~~[illegible]~~)

$k\overline{Q}/\langle -\alpha^*\alpha, \alpha\alpha^* - \beta^*\beta, \beta\beta^* \rangle = \Lambda$ is the preprojective alg. of Dynkin type A_3

Λ is finite dim. since Q is Dynkin.
All these are selfinjective

[GLS] mod Λ is Hom-finite, triangulated, 2-CY
$\left(\iff \tau = [1] \xrightarrow[\text{(Gorenstein)}]{\text{here}} \Omega^{-1}\right)$

$\left(\mathrm{Hom}_{\underline{\mathrm{mod}\Lambda}}(X,Y) \stackrel{\text{def}}{=} \mathrm{Hom}_\Lambda(X,Y) / \mathcal{P}(X,Y) \right)$

③ 2-CY category associated with elements in Coxeter groups
[GLS], [BIRS]

Q = finite acyclic quiver $\rightsquigarrow W = W_Q$ Coxeter group

$w \in W \longmapsto \Lambda_w$ = fin. dim. alg, id $\Lambda_w \leq 1$

$\underline{\mathrm{Sub}\,\Lambda_w}$ = the submodules of direct sums of Λ_w-proj

$\underline{CM}^{1}(\Lambda_w)$ modules is a Hom-finite triangulated 2-CY.

④ Generalised cluster cat [Amiot]

(a) Let Λ be a f.d. alg, gldim $\Lambda \leq 2$

Let $C_\Lambda = D^b(\Lambda)/\tau^{-1}[1]$ not nec. triangulated

If f.dim Hom-spaces, then $\mathcal{C}_\Lambda$ is OK

(Q, W) quiver w/ potential $\rightsquigarrow \mathcal{C}_{(Q,W)}$ 2-CY [Amiot] [Keller-Yang].

⑤ Formula for f.dim. algebra

$\mathrm{Ext}^1_\Lambda(A, \tau B) \simeq D\,\underline{\mathrm{Hom}}(B, A)$

$\tau\ \underline{\mathrm{mod}}\,\Lambda \longrightarrow \overline{\mathrm{mod}}\,\Lambda$ is equiv.

Let R be a local commutative complete isolated Gorenstein singularity.

[Assume $T = k[[x_1, .., x_d]] \subset R$ s.t. R is fin. gen. free T-mod.

$CM(R) = \{X$ in mod R; X is f.g. free T-module $\}$]

Candidate $\underline{CM}(R)$ is Hom-finite, triangulated.

[Auslander] $\tau = \Omega^{2-\dim R}$

Want $\tau = [1] = \Omega^{-1}$, OK if $2 - \dim R = -1 \Rightarrow \dim R = 3$

⑥ Ex from CM-mod (k field, char $= 0$, $k = \bar{k}$)

(i) $S = k[[X, Y, Z]]$, G (finite) $\subset SL(3, k)$, $R = S^G$

R = Gorenstein isolated sing, $\dim R = 3$

$\underline{CM}(R)$ is Hom-finite, triangulated 2-CY

(Iyama) S in $\underline{CM}(R)$ is cluster tilting.

(ii) Hypersurface isolated singularities in dim $= 1$ $(\Omega^2 \simeq \mathrm{id})$

(including the simple ones $\Leftrightarrow$ Dynkin diagrams)

Thm (Amiot–Iyama–Reiten) (cf. Thanhoffer–Wollsley, Van den Bergh)

Assume $\zeta^n = 1$ (primitive n^{th} root)

$$g = \begin{pmatrix} \zeta^{a_1} & & \\ & \zeta^{a_2} & \\ & & \zeta^{a_3} \end{pmatrix} \in SL_3(k)$$

with $0 < a_j < n$, $(n, a_j) = 1$ for $1 \leq j \leq 3$

$S = k[x, y, z]$

$R = S^{\langle g \rangle}$

Then $\underline{CM}(R) \overset{\text{triangle}}{\sim} \mathcal{C}_{\bar{A}}$ ($\mathrm{gl.dim}\, \bar{A} \leq 2$)