MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: A. Seceleanu **Email/Phone:** aseceleanu2@math.unl.edu

Speaker's Name: Dylan Thurston

Talk Title: Cluster Algebra and Triangulated Surfaces

Date: 09/07/12 **Time:** 9 : 00 (am) / pm (circle one)

List 6-12 key words for the talk: cluster algebra, triangulation, surface

Please summarize the lecture in 5 or fewer sentances: This lecture discusses connections between cluster algebras and triangulations of surfaces.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

History:

Baer, Dehn : tropical relation, curves

W. Thurston, Penner : λ lengths, Teich. space

Fock-Goncharov
Gekhtman-Shapiro-Vainshtein } connection to cluster algebras

Fomin Shapiro Thurston : Properties

Cluster Algebras and Triangulated Surfaces #2

Dylan Thurston

9/7/12

Surface cluster algebras II

Structure & Properties.

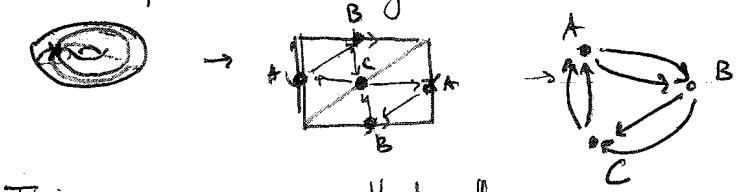
Recall : a marked surface (S, M) — S surface, M marked points

Triangulation → Quiver

→ Cluster algebra $\mathcal{A}(S, M)$

change the triangulation by a quadrilateral flip ⇝ cluster mutation.

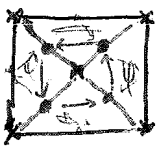
What if not all edges are distinct?

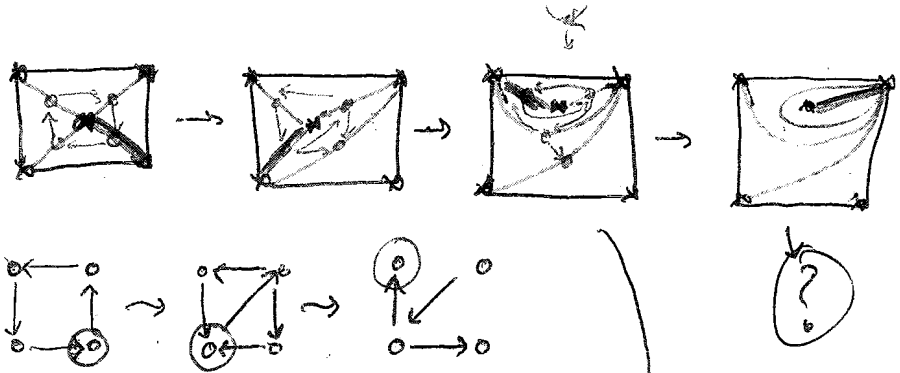


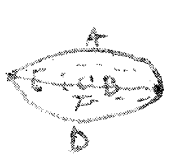
This quiver is called the "Markov Triple" and it's A-O-K!

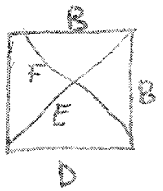
No problem!

S = disk
4 pts on bdry
1 on interior







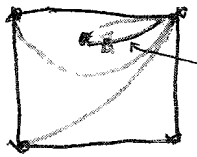


$\lambda(E)\lambda(F) = \lambda(A)\lambda(B) + \lambda(D)\lambda(B)$

no good for cluster mutation!

New cluster variable should be $\frac{\lambda(F)}{\lambda(B)} = \lambda(F')$

triangulation? well...



not parallel b/c arc is tagged.

Tagged arcs

Def - A tagged arc on (S, M) is an arc possibly marked with a notch at one or both ends.

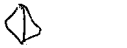
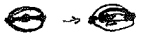
- Notches can occur only at internal punctures.
- Arcs surrounding only one puncture are forbidden

No! Bad!

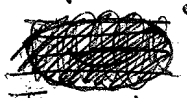
Definition Two tagged arcs are compatible if
a) they are disjoint (after isotopy) and
b) Either
(i) they are tagged the same way at all common endpoints
(ii) they look like  or 

Definition A tagged triangulation is a maximal collection of compatible tagged arcs.

Not compatible

Surface
tagged arcs
tagged triangulations
boundary arcs
quad flips 
or diag 

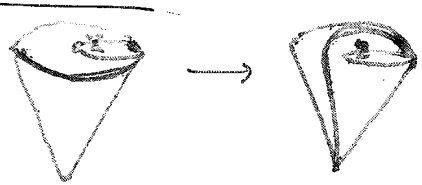
Cluster algebra
cluster variables
clusters
Frozen variables
mutations



The set of tagged triangulations is maybe not connected under mutation.

Lemma
Every tagged triangulation is obtained from an ordinary triangulation (possibly with ) by
a) replace  by  , and then
b) flip markings on some subset of interior points

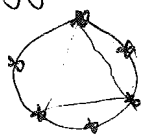
Lemma
The set of tagged triangulations is connected by mutations unless S is closed, M = one point.



Properties of the resulting cluster algebra.

When do you get cluster algebras of finite type?
- Finite # of tagged arcs.

Example: disk with k ~~tagged~~ marked points on the boundary
"k-gon"



A_{k-3}

disk with k marked points on the boundary and 1 interior point

D_k

Any time you have a simple nontrivial closed loop can twist arcs around it.

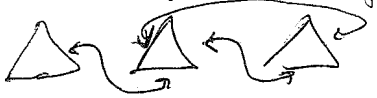


etc.

How many different quivers appear?
combinatorial type of triangulation $\rightsquigarrow$ quiver

Eg. torus 

Triangulation, combinatorially, is given by gluing triangles. Only a finite # of possibilities exist with a fixed # of triangles, determined by surface



Theorem (Fomin-Shapiro-Thurston)
Cluster algebras from surfaces are mutationally finite.

Theorem (Felikson Shapiro-Tumarkin)
All but 11 skew-symmetric, mutationally finite cluster algebras come from surfaces.

Exceptions: $E_6, E_7, E_8, \tilde{E}_6, \tilde{E}_7, \tilde{E}_8$ ← affine
or are rank 2.