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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: A. Seceleanu **Email/Phone:** asecelenu2@math.unl.edu

Speaker's Name: Michel Van den Bergh

Talk Title: Non-commutative resolutions

Date: 09/07/12 **Time:** 2:00 am/pm (circle one)

List 6-12 key words for the talk: non-commutative resolution, Gorenstein, crepant, endomorphism ring

Please summarize the lecture in 5 or fewer sentances: Non-commutative resolutions generalize the algebraic geometric concept of crepant resolutions of singularities. Existence and non-existence results are presented.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Non-Commutative Resolutions #3 9/7/12

Michel Van den Bergh

Correction: $k[[x,y,z,t]]^{\mathbb{Z}/2\mathbb{Z}}$ is $\mathbb{Q}$-factorial.

Explicit Models of some MCM(R)

(Amiot, Iyama, Reiten)

MCM(S^G) ?

Reminder: Let Q be an acyclic quiver. Then $\mathcal{C}_Q = D^b(\text{mod } kQ)/(\tau[-1])^{\mathbb{Z}}$ is a 2-CY, cluster-tilting object (Reiten's talk)

More generally (Claire Amiot, Keller)
quiver with potential ~~$\widehat{kQ}/[kQ,kQ]$~~
Q W $W \in kQ^\wedge/[kQ,kQ]^\wedge$
$\rightsquigarrow$ Ginzberg Algebra.

Define $\tilde{Q}$ with arrows $\begin{cases} a & a \in Q_1 \\ a^* & a \in Q_1, \text{ opposite} \\ c_i & i \in Q_0, \text{ a loop} \end{cases}$

grading: $\begin{cases} |a| = 0 \\ |a^*| = -1 \\ |c_i| = -2 \end{cases}$

$\Gamma(Q,W) = (kQ, d)$

$da^* = \frac{\partial w}{\partial a}$

$dc = \sum_{a \in Q_1} [a^*, a]$, $c = \sum_i c_i$

$H_0(\Gamma(Q,W)) = J(Q,W) = kQ/\left(\left(\frac{\partial w}{\partial a}\right)_a\right)$

Theorem (Amiot)
Assume $H^0(\Gamma(Q,W))$ is f.d. $\forall i$. ($=A$)
then $\mathcal{C}_{Q,W} \stackrel{\text{def}}{=} \text{per} A/\langle (S_i)_{i \in Q_0} \rangle$ is 2-CY
and A is a cluster tilting object
simpler

Facts:

- $\mathcal{C}_Q = \mathcal{C}_{Q,w=0}$
- $\Lambda = kQ/(r_1, \dots, r_n)$, $\dim \Lambda$, gldim $\Lambda = 2$

$\mathcal{C}_\Lambda = \mathcal{C}_{\tilde{Q},w}$ $\tilde{Q} = Q \cup \{x_1, \dots, x_n\}$, $W = \sum_{i=1}^n x_i r_i$

$[D^b(\text{mod } \Lambda)/[\tau[-1]]]$ "triangulated hull"

Relation with MCM modules

(R,m) complete local Gorenstein, isolated singularity, k-dim 3.

Assume $\exists$ NCCR $A = \text{End}_R(M)$
$\rightsquigarrow \mathcal{C} \in \underline{\text{MCM}}(R)$ 2-CY, cluster tilting object.
Moreover, ?a name? ~~Write~~
Write $\mathcal{C} = \mathcal{C}_{Q,W}$.

(+ de Thanhoffer de Völcsey)

$\longrightarrow$

The relative singularity category (TVdB, Bewbai-Kalck, Kalck-Yang)

$M = \underbrace{M_0 \oplus \cdots \oplus M_u}_{\text{no repetition}}$ projective A-modules.
$R^{\oplus}$

Let $P_i = \mathrm{Hom}_R(M_i, M)$
$S_i = P_i / \mathrm{rad}\, P_i$

Recall $\underline{\mathrm{MCM}}(R) \simeq D_{sg}(R) \stackrel{\text{def}}{=} D^b(\mathrm{mod}\, R)/\langle R \rangle$ (Buchweitz, Orlov)

$D_{sg}(R)$ ← singularity category; $\langle R \rangle$ ← per R

Relative singularity category:

$$\Delta_R A = D^b(\mathrm{mod}\, A) / \langle P_0 \rangle$$

$\boxed{\text{Theorem}}$ (TVdB, Kalck-Yang)

- $\Delta_R A$ is Hom finite.
- $\Delta_R A / \langle S_1, \ldots, S_u \rangle = D_{sg}(R)$ $(= \underline{\mathrm{MCM}}(R))$

Recall A is (topological) Ginzberg 3-CY

$\exists\, Q, W$ s.t. $H^*(\Gamma(Q,W)) \simeq A$

$$\Delta_R A = \mathrm{per}\, A / \langle P_0 \rangle \quad (P_0 \to Ae_0,\ e_0 \to \text{idempotent corr. to } 0)$$
$$\simeq \mathrm{per}\, \Gamma(Q,W) / \langle \Gamma(Q,W) e_0 \rangle$$
$$\underset{\text{Keller}}{\simeq} \mathrm{per}\left(\Gamma(Q,W) / \Gamma(Q,W) e_0 \Gamma(Q,W)\right)$$
$$\simeq \mathrm{per}\, \Gamma(Q', W')$$

Q' is obtained from Q by deleting 0 and all adjacent arrows.

$W' = W$-terms going through 0

Conclusion: $\underline{\mathrm{MCM}}(R) = \Delta_R A / \langle S_1, \ldots, S_u \rangle$

$= \mathrm{per}\, \Gamma(Q', W') / (S_1, \ldots, S_u) = \mathcal{C}_{(Q', W')}$.

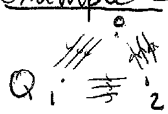
Example 1 $R = k[[x,y,z,t]]/(xy - zt)$

Q  $\quad Q' = \cdot 1$

W = something $\quad W' = 0$

$\underline{\mathrm{MCM}}(R) = \mathcal{C}_{\cdot 1, W=0} = D^b(\mathrm{vect}(k))/[-2]$
$= \mathbb{Z}/2\mathbb{Z}$ graded vector spaces

Example 2 $R = k[[x,y,z]]^G \quad G = \begin{pmatrix} \zeta & & \\ & \zeta & \\ & & \zeta \end{pmatrix} \quad \zeta^3 = 1$

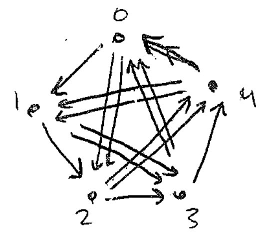
Q  $\quad Q' \cdot \equiv$

$W = ? \quad W = 0$

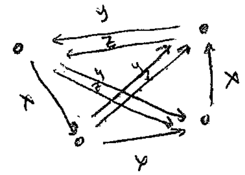
$\underline{\mathrm{MCM}}(R) = \mathcal{C}_{\cdot \equiv}$. (Keller - Reiten)

<u>Example 3</u> $R = k[[x,y,z]]^G \quad G = \left(\begin{smallmatrix} \zeta & & \\ & \zeta^2 & \\ & & \zeta^3 \end{smallmatrix} \right) \quad \zeta^5 = 1$

Q

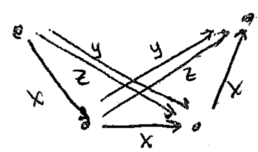


Q'



W = sum of 3-cycles $\quad W' = z(xy - yx) + y(xz - zx)$

Λ:



$$xy - yx = 0$$
$$xz - zx = 0$$

$\mathcal{C}_{Q', W'} = \mathcal{C}_{\Lambda}$

<u>Final Result:</u> $\underline{\mathrm{MCM}}(R) \cong \mathcal{C}_{\Lambda}$