The Mathematical Education of Inservice Teachers: Lessons Learned Opportunities & Challenges

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Goals

- Highlight selected challenges for professional development
- Review lessons learned from research on professional development
- Indentify some productive starting points for addressing the professional development needs posed by CCSS-M.

What is NCSM?

International organization of and for mathematics education leaders:

Coaches and mentors
Curriculum leaders
Department chairs
District supervisors/leaders
Mathematics consultants
Mathematics supervisors
Principals
Professional developers
Publishers and authors
Specialists and coordinators
State and provincial directors
Superintendents
Teachers
Teacher educators
Teacher leaders

Cents and Non-Sense

http://www.youtube.com/watch?v=ANDk0SW_

Challenges

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like?

CCSS-M, p. 4

Challenges

One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from . . . Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

CCSS-M, p. 4

Tasks Matter!

"Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking***."

Stein, Smith, Henningsen, & Silver, 2000

"***The level and kind of thinking in which students engage determines what they will learn***."

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Tasks Matter

Solve: 1 ¾ ÷ ½ = ______

Write a story problem that can be solved using 1 ¾ ÷ ½ = ______

Explain why 8 ÷ 1/3 = 24 and 8 ÷ 2/3 = 12.

Why is 8 ÷ 2/3 exactly half of 8 ÷ 1/3?

Tasks Matter

To solve $\frac{3}{4} \div \frac{2}{5}$, Joey writes

"$\frac{3}{4} \div \frac{2}{5}$ is the same as $\frac{15}{20} \div \frac{8}{20}$. So the answer to $\frac{3}{4} \div \frac{2}{5}$ is the same as $15 \div 8$."

Is Joey correct? Will his method always work? Explain.

Tasks Matter

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

1. **If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?**
2. **How long would each of the sides of the pen be if they had only 16 feet of fencing?**
3. **How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.**

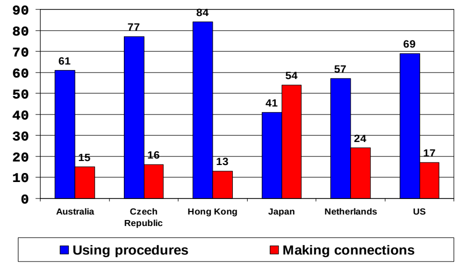
Stein, Smith, Henningsen, & Silver, 2000, p. 2

Standards for Mathematical Practice

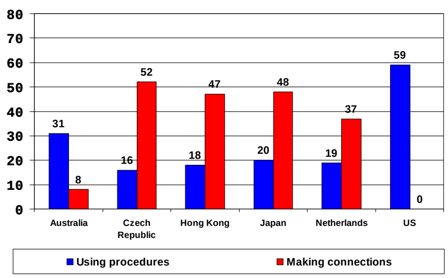
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Types of Math Problems Presented

1999 TIMSS Video Study



How Teachers Implemented *Making Connections* Math Problems



Challenges

- Our “mathematics teaching culture” emphasizes learning procedures over developing understanding.
- Engaging students with challenging tasks that involve active meaning-making promotes conceptual understanding.
- Too little attention is paid to analyzing tasks and sequences of tasks for their potential to support mathematics learning.
- Teachers tend to reduce the cognitive demand of tasks in implementation. In fact, we traditionally have encouraged them to “adapt tasks to meet the needs of their students.”
- How to orchestrate summary discussions so that they result in mathematics learning for all students.

Five Practices for Orchestrating Effective Discussions

- ***Anticipating*** likely student responses
- ***Monitoring*** students' actual responses
- ***Selecting*** particular students to present their mathematical work during the whole class discussion
- ***Sequencing*** the student responses
- ***Connecting*** different students' responses—to each other and to key mathematical ideas.

Smith & Stein, 2011

Tools for Analyzing Instructional Materials

- CCSSO-NCSM collaboration; Project Director: Bill Bush, University of Louisville.
- Tools for analyzing:
  - Mathematics Content
  - Standards for Mathematical Practice
  - Overarching Elements (Equity, Assessment, Technology, Teacher Support)
- Related professional development

An Emerging Consensus Effective PD:

- Focuses on content knowledge and how students learn content
- Involves a substantial number of hours
- Sustains focus over time
- Models effective practice, including active learning experiences
- Engages teachers in communities of learning
- Involves active participation of school leaders

Weiss, 2010

PD Approaches

One line of reasoning:

- Teachers can't teach what they don't know. Therefore, it is important to start with mathematics content, and only after teachers themselves have a sufficiently deep understanding of the content, move to considering classroom application.

Weiss, 2010

PD Approaches

Another line of reasoning:

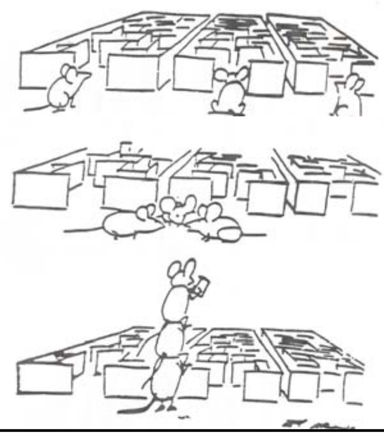
- Teachers are by their very nature practitioners. Starting with classroom applications, e.g., trying to analyze student work, provides a context for engaging the teachers in learning mathematics content.
- It is important to note that the available research doesn't help in making this kind of decision.

Weiss, 2010

Design PD to facilitate transfer to the classroom

- Point out connections between what teachers are learning and what they are expected to teach.
- Help teachers apply what they are learning to their classrooms, with opportunities for practice and feedback.

Weiss, 2010



Collaborate!

Engage teachers in working in collaborative teams

- Grade level/course/department meetings
  - Common assessments
  - Common unit planning
  - Differentiating instruction

Potential Productive Starting Points

- Standards for Mathematical Practice
  - In learning content
- Selected “Key Advances” in content

Write a word problem that could be modeled by a + b = c.

- Result or total unknown; e.g. 5 + 3 = ?
  - Mike has 8 pennies. Sam gives him 3 more. How many does Mike have now?
- Change or part unknown; e.g., 5 + ? = 8
  - Mike has 5 pennies. Sam gives him some more. Now he has 8. How many did he get from Sam?
- Start unknown; e.g., ? + 3 = 8
  - Mike has some pennies. He gets 3 more. Now he has 11. How many did he have at the beginning?

Common Addition and Subtraction Situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

Common Addition and Subtraction Situations

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

	Operations and Algebraic Thinking	Numbers and Operations in Base Ten	Fractions
1	Understand and apply properties of operations and the relationship between addition and subtraction.	Use place value understanding and properties of operations to add and subtract.	
2		Use place value understanding and properties of operations to add and subtract.	
3	Understand properties of multiplication and the relationship between multiplication and division.	Use place value understanding and properties of operations to perform multi-digit arithmetic. *A range of algorithms may be used.*	
4		Use place value understanding and properties of operations to perform multi-digit arithmetic. *Fluently add and subtract multi-digit whole numbers using the standard algorithm.*	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5		Perform operations with multi-digit whole numbers and with decimals to hundredths. *Fluently multiply multi-digit whole numbers using the standard algorithm.*	Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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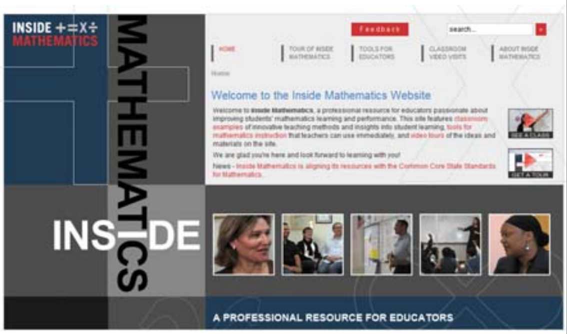
Potential Productive Starting Points

- Standards for Mathematical Practice
  - In learning content
- Selected “Key Advances” in content
- Assessments
  - Balanced Assessment in Mathematics released tasks (insidemathematics.org)
  - Released performance assessments from selected states
  - Mathematics Assessment Project (MAP)

Insidemathematics.org



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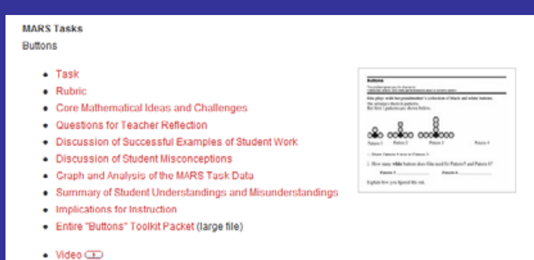

Thank You!