How A Focus on Children's Mathematical Thinking Supports the Professional Development of Elementary School Teachers

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ABSTRACT: Teachers in the United States do not have built-in means by which to continue to grow professionally. One promising means for supporting teacher professional development is for teachers to learn from their own practice in general, and from their students' mathematical thinking in particular. This presentation will share results of how focusing on children's mathematical thinking supported the professional development of elementary school teachers. In addition to considering mathematical content knowledge and beliefs, we will also look at what teachers notice from instructional settings. Video examples of students' mathematical thinking will be used to ground the conversation about mathematics, teaching, and learning.

Professional Development

The way we are currently teaching mathematics in the U.S. is problematic.

But our biggest long-term problem related to mathematics teaching in the United States is not how we teach now, but that we have no way of getting better.

-Stigler, J. W., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction: An Overview of the TIMMS Video Study. *Phi Delta Kappan* (September), 14-21.

Stance of Inquiry

- learning to question one's own and others' assumptions and beliefs about teaching, learning, and schooling

- "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (Wells, 1999)

- teachers with an inquiry stance look more deeply than at test scores or correct answers for evidence of student learning; they may look for other indicators of students' understandings, including what students say and how they reason about problems or questions. (Cochran-Smith and Lytle; 1999, 2001)

Where is it useful to focus on children's mathematical thinking?

- With practicing teachers
- With prospective teachers

What Knowledge of Mathematics Do Teachers Need?

First, what do we mean by mathematics?

Define Mathematical Proficiency

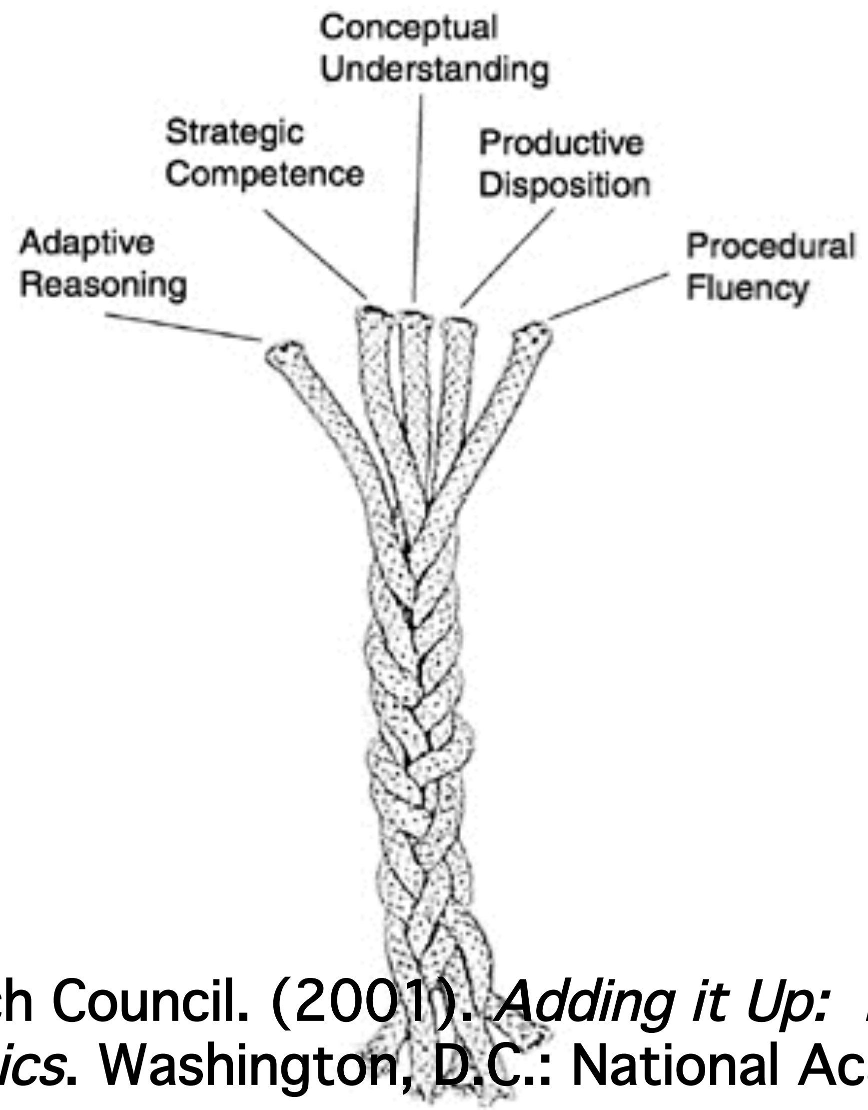
- Concepts
- Procedures
- Problem Solving
- Reasoning and Justifying
- Positive Outlook

Define Mathematical Proficiency

- Concepts (Conceptual Understanding)
- Procedures (Procedural Fluency)
- Problem Solving (Strategic Competence)
- Reasoning and Justifying (Adaptive Reasoning)
- Positive Outlook (Productive Disposition)

National Research Council. (2001). *Adding it Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press.

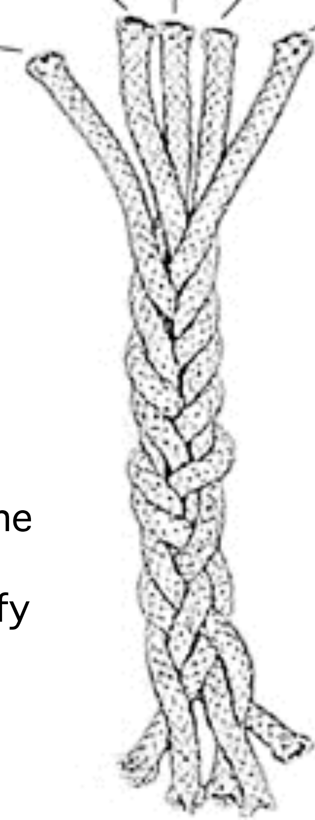
The Strands of Mathematical Proficiency



National Research Council. (2001). *Adding it Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press.

The Strands of Mathematical Proficiency

Integrated and functional grasp of mathematical ideas



The ability to formulate mathematical problems, represent them, and solve them.

Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

The capacity to think logically about the relationships among concepts and situations, including the ability to justify one's reasoning both formally and informally.

The CCSS Process Standards

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision (in language and math)

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

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1 Make sense/persevere

2 Reason abstractly and quantitatively

3 Construct/critique viable arguments

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7 Look for structure.

8 Look for repeated reasoning.

1 Make sense/persevere

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6 Attend to precision (in language and math)

7 Look for structure.

8 ~~Look for repeated reasoning~~ Generalizing

1 Make sense /persevere

2 Reason abstractly & quantitatively

3 Construct/Critique viable arguments

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8 Generalizing

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8 Generalizing

Reasoning & Explaining

1 Make sense /persevere

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Reasoning & Explaining

Modeling & Using Tools

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Make Sense & persevere, & Attend to Precision (in language & math)	Reasoning & Explaining Modeling & Using Tools Look for Structure & Generalizing

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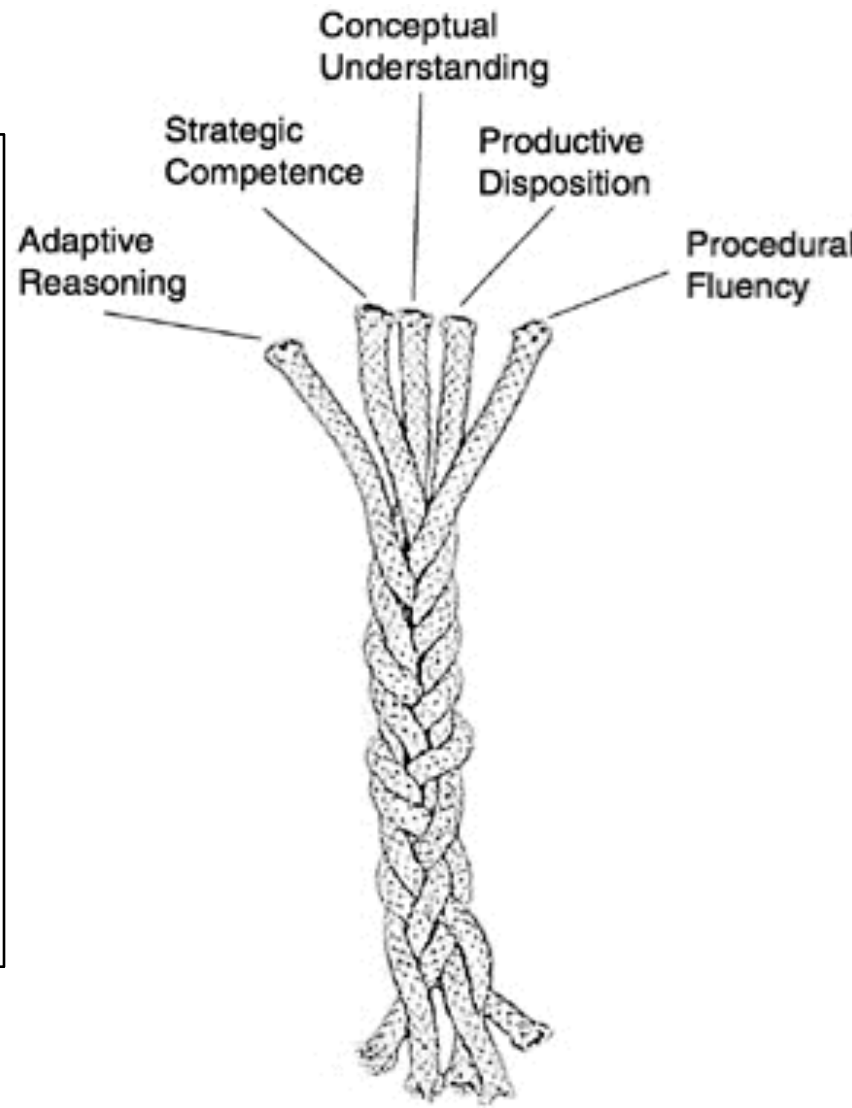


Make Sense & persevere, & Attend to Precision (in language & math)	Reasoning & Explaining Modeling & Using Tools Look for Structure & Generalizing

Conceptual Understanding

Procedural Fluency

Adaptive Reasoning

Strategic Competence

Productive Disposition

Make Sense & persevere, & Attend to Precision (in language & math)	Reasoning & Explaining Modeling & Using Tools Look for Structure & Generalizing

Conceptual Understanding
Make Sense

Procedural Fluency
•Reasoning abstractly and quantitatively
•Attend to precision in math

Adaptive Reasoning
•Attend to Precision (in language)
•Reasoning & Explaining
•Generalizing

Strategic Competence
•Make Sense
•Modeling & Using Tools
•Reasoning abstractly and quantitatively

Productive Disposition
persevere

Conceptual Understanding & Procedural Fluency are ubiquitous throughout the CCSS Content Standards: Example, Grade 3, Page 23

Conceptual Understanding

"Identify arithmetic patterns … and explain them using properties of operations."

Procedural Fluency

"Solve two-step word problems using the four operations."

Conceptual Understanding & Procedural Fluency

"Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division"

Make Sense & persevere, & Attend to Precision (in language & math)	Reasoning & Explaining Modeling & Using Tools Look for Structure & Generalizing

Conceptual Understanding

Make Sense

Procedural Fluency

- Reasoning abstractly and quantitatively
- Attend to precision in math

Adaptive Reasoning

- Attend to Precision (in language)
- Reasoning & Explaining
- Generalizing

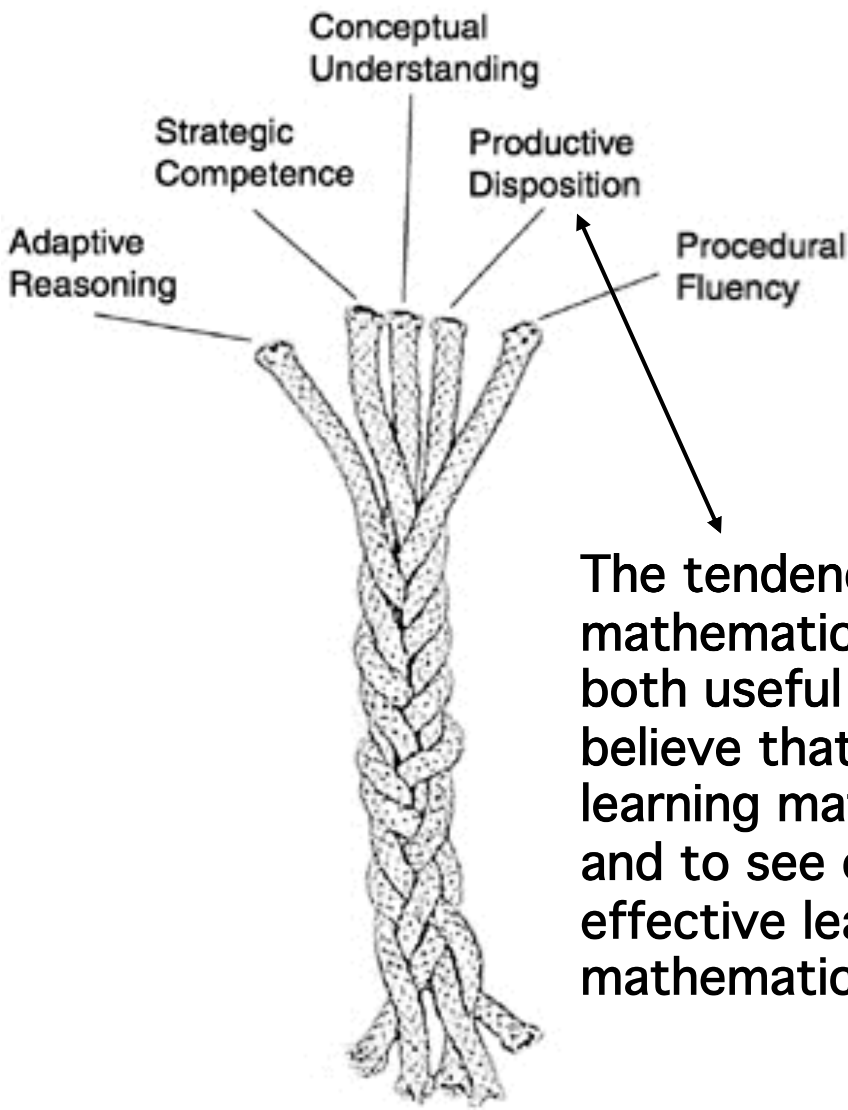
Strategic Competence

- Make Sense
- Modeling & Using Tools
- Reasoning abstractly and quantitatively

Productive Disposition

persevere

The Strands of Mathematical Proficiency



?

The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.

Example of *Mathematical Disposition*

70	76
- 23	- 23
53	53

"Yes, math is like that sometimes."

Mathematical Integrity

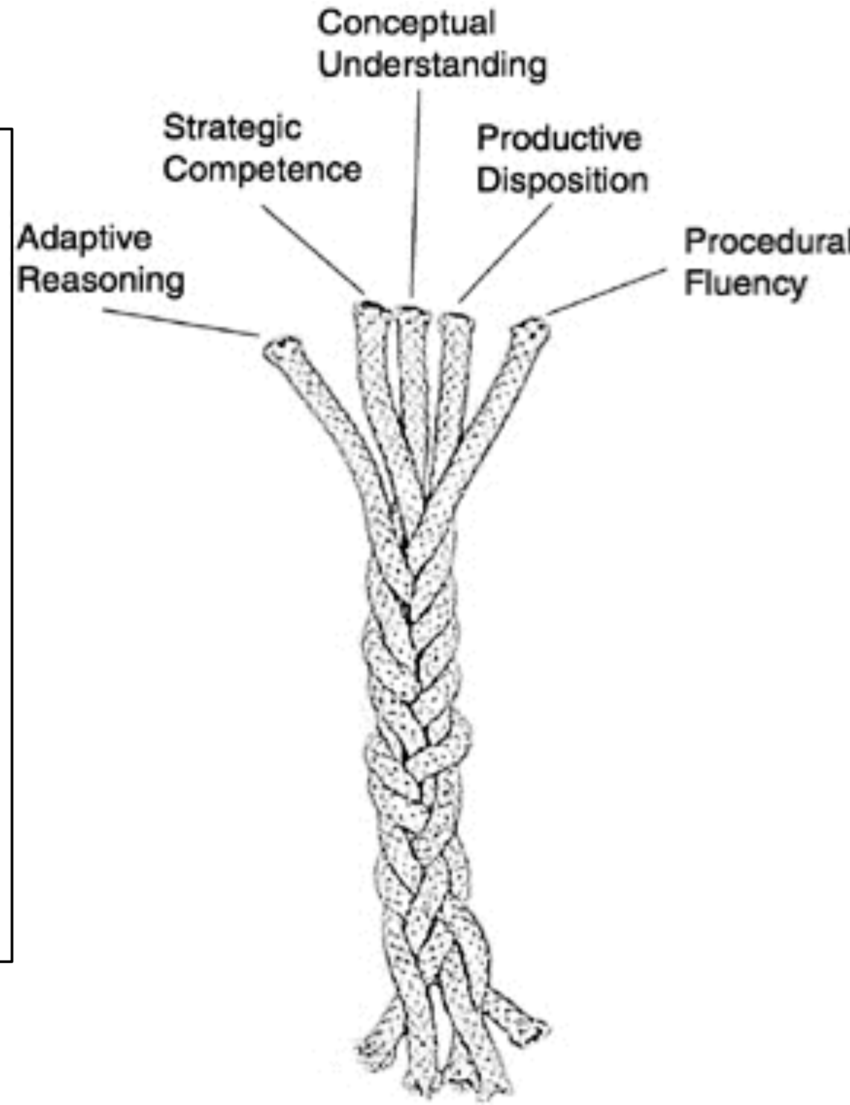
(Another component of Productive Disposition)

"An education isn't how much you have committed to memory, or even how much you know. It's being able to differentiate between what you do know and what you don't."

(Anatole France, Nobel Prize-winning author)

With which of these process standards/strands do *your* students engage regularly?

Make Sense & persevere, & Attend to Precision (in language & math)	Reasoning & Explaining Modeling & Using Tools Look for Structure & Generalizing



What Knowledge of Mathematics Do Teachers Need?

Mathematical Knowledge for Teaching

Ball, Hill, & Bass, 2005; Hill, Sleep, Lewis, & Ball, 2007

- ***Common Content Knowledge***—the mathematical knowledge teachers are responsible for developing in students
- ***Specialized Content Knowledge***—mathematical knowledge that is used in teaching, but not directly taught to students
- ***Pedagogical Content Knowledge***—*"*the ways of representing and formulating the subject that make it comprehensible to others" ***(Shulman, 1986)***

Common Content Knowledge

Evaluate and understand the meaning of $12 \div 3$.

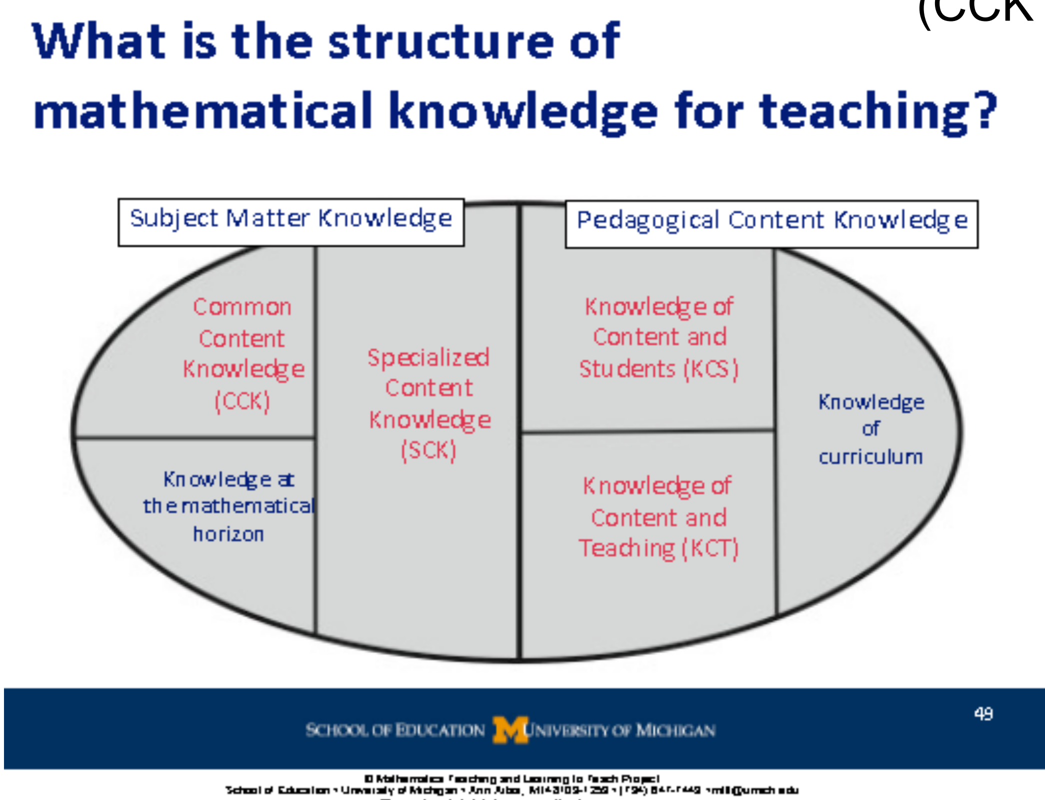
Specialized Content Knowledge

Write a real-life story problem that could be represented by the expression $12 \div 3$.

Pedagogical Content Knowledge

How might children think about the problem you wrote?

Let's Scramble the Egg

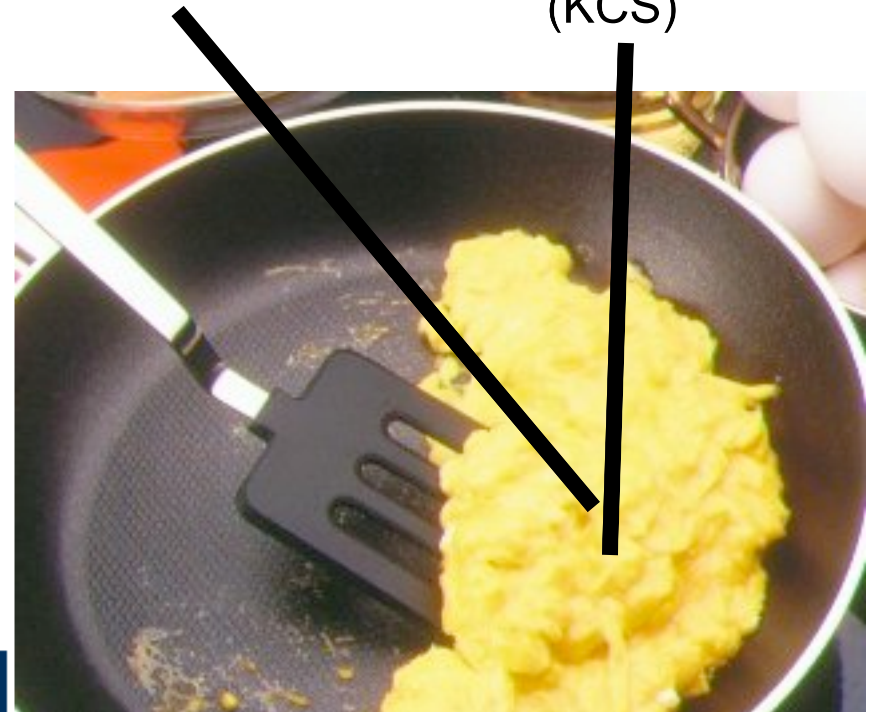


MCK

(CCK and SCK)

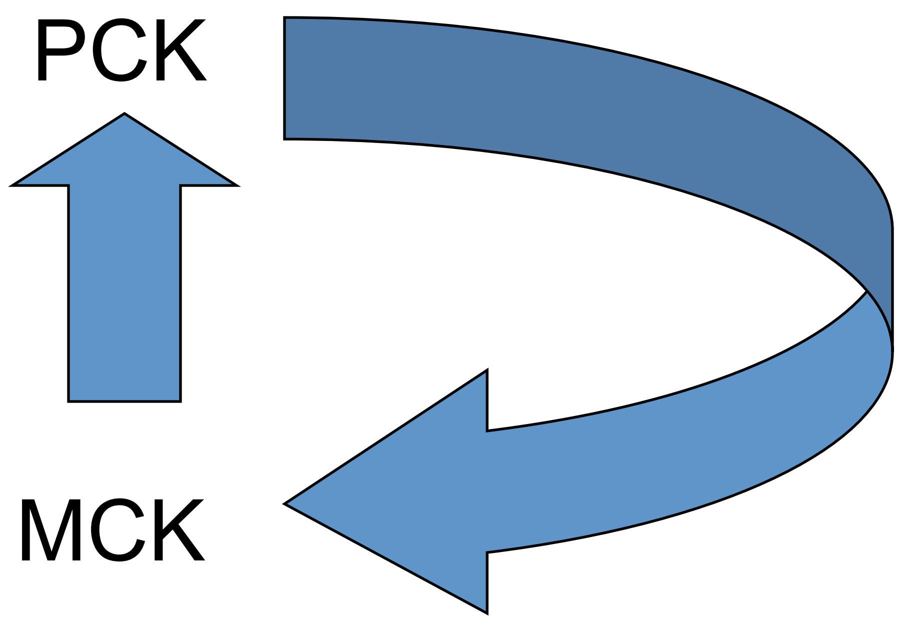
PCK

(KCS)



“Those who can, do. Those who understand, teach.”

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.



The Problem

Even when students attend a thoughtfully planned mathematics course for prospective elementary school teachers (PSTs), too many go through the course in a perfunctory manner.

(Ball, 1990; Ma, 1999; Sowder, Philipp, Armstrong, & Schappelle, 1998)

Many PSTs do not seem to value the experience.

Why?

Many PSTs hold a complex system of beliefs that supports their not valuing the learning of more mathematics. For example, consider the effect of believing the following:

"If I, a college student, do *not* know something, then children would not be expected to learn it. And if I *do* know something, then I certainly don't need to learn it again."

In other words…

Mathematics instructors of PSTs are often answering questions that PSTs have yet to ask.

Should we try to get PSTs to care more about mathematics for mathematics sake?

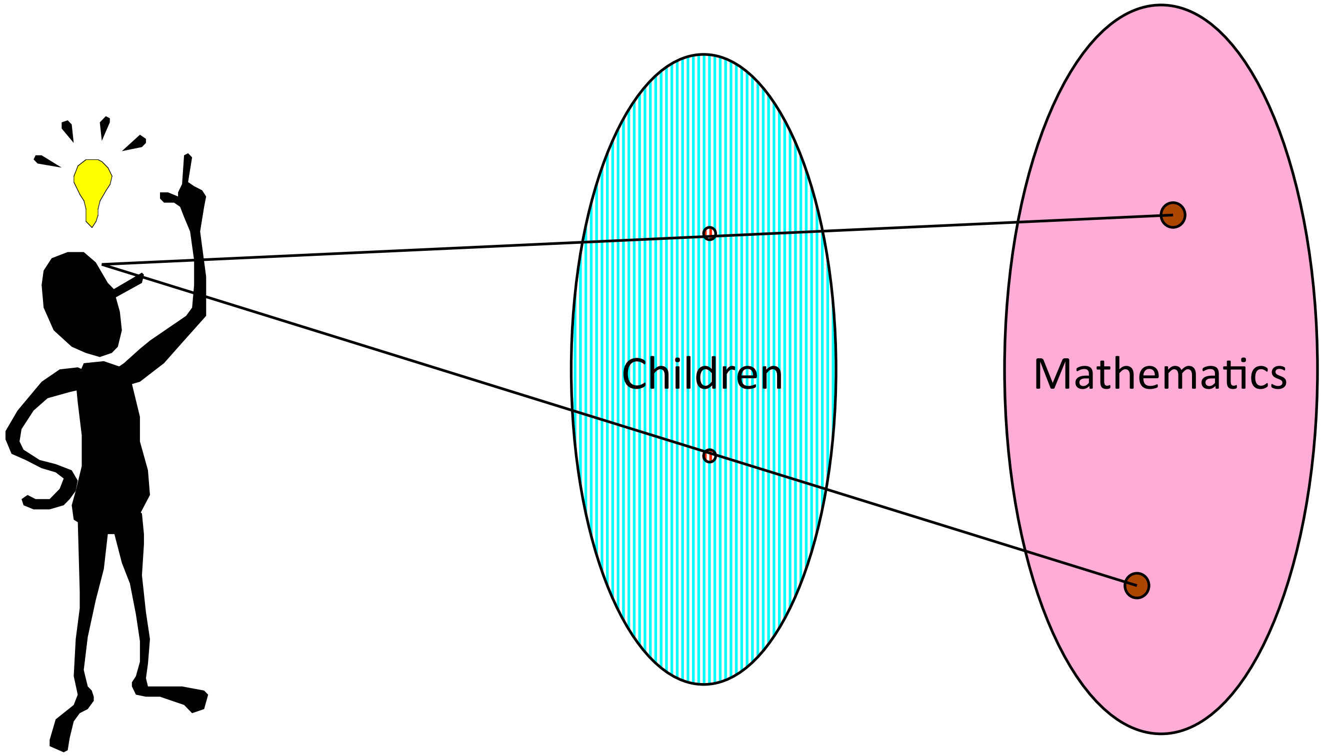
This is one starting point.

We took a different starting point, building on the work of Nel Noddings (1984).

Why not start with what PSTs care about?

Children!

-Darling-Hammond & Sclan, 1996



Looking at mathematics through the lens of children's mathematical thinking helps PSTs come to care about mathematics, not as mathematicians, but as teachers.

We tested this theory.

Major Result*

When compared to PSTs who did *not* learn about children's mathematical thinking, PSTs who *did* learn about children's mathematical thinking

a) improved their mathematical content knowledge *and*

b) developed more sophisticated beliefs.

*Philipp, R. A., Ambrose, R., Lamb, L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., et al. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438 - 476.

How did the focus on children's thinking help the PSTs?

PSTs Speak for Themselves

Reflecting upon the class

CMTE Clip, 0:14 - 0:56

George Poole, personal communication, November 12, 2001

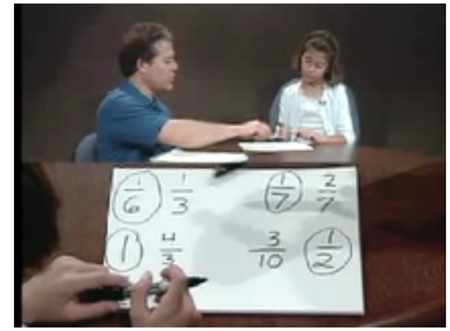
I have used the tape to show my prospective elementary teachers the kind of creative and "different" thinking students use to reason and make calculations. The video clips became motivational clips and saved me having to make the argument for PUFM (Profound Understanding of Fundamental Mathematics, Ma, 1999).

Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children's mathematical thinking.

1. The way most students are learning mathematics in the United States is problematic.

Ally, End of Grade 5

(*This is video clip #11 on IMAP CD.)



An average 5th-grade student at a high-performing local school

Ally, VC#302, 1:30 - 2:30

Questions for PSTs

How did Ally come to hold these conceptions?

What do teachers need to know to reduce the occurrence of these conceptions in their students?

NAEP ITEM: 13-year-olds

Estimate the answer to $\frac{12}{13}+\frac{7}{8}$. You will not have time to solve the problem using paper and pencil.

a) 1	b) 2	c) 19	d) 21	e) I don't know
14%	24%	28%	27%	7%

$\frac{12}{13}+\frac{7}{8}$ Is thought of as

12 7

13 8

Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children's mathematical thinking.

1. The way most students are learning mathematics in the United States is problematic.

2. Learning concepts is more powerful and more generative than learning procedures.

Felisha, End of Grade 4

1) This is a delicious cookie that four children at a party want to share. Show how the four children might share the cookie. How much does each person get?

2) Just before the cookie is handed out, one child is picked up from the party without eating or taking her piece. The three people remaining want to share the cookie. How could they do that fairly? How much does each person get?

Question for PSTs

- Explain Felisha's Thinking.
- What do teachers need to know to understand Felisha?... to teach Felisha?

Consider two problems

- A problem for secondary-school algebra students
- A problem for primary-grade students (K-2)

A Secondary-School Algebra Problem

19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

x = # of seats with 2 children; y = # of seats with 3 children

$$x + y = 7 \quad \text{and} \quad 2x + 3y = 19$$

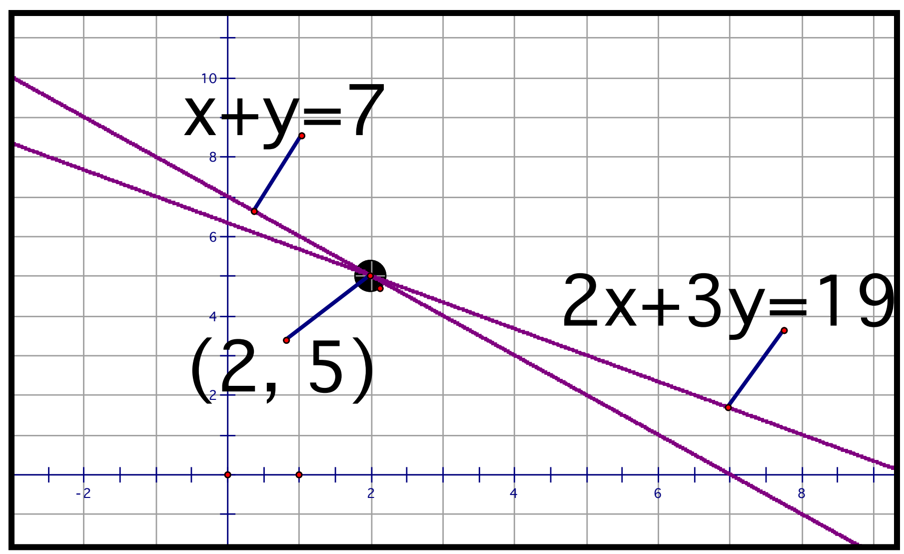
$2x + 3y = 19$ and $y = 7 - x$

$2x + 3(7 - x) = 19$

$2x + 21 - 3x = 19$

$-x = -2$

$x = 2$ and $y = 5$



A Primary-Grade Problem

19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

51% of kindergarten students in six classes (36/70) correctly solved this problem in May.

How do you think they did that?

"*Throughout the year children solved a variety of different problems. The teachers generally presented the problems and provided the children with counters. . . but the teachers typically did not show the children how to solve a particular problem. Children regularly shared their strategies…*

—Carpenter et al., 1993, p. 433. (JRME v.24, #5)

Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children's mathematical thinking.

1. The way most students are learning mathematics in the United States is problematic.

2. Learning concepts is more powerful and more generative than learning procedures.

3. Students' reasoning is varied and complex, and generally it is different from adults' thinking.

Often What We Think We are Teaching is Not What Students are Learning

After explaining to a student through various lessons and examples that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example. This was the result:

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$$\lim_{x \to 5} \frac{1}{x-5} =$$

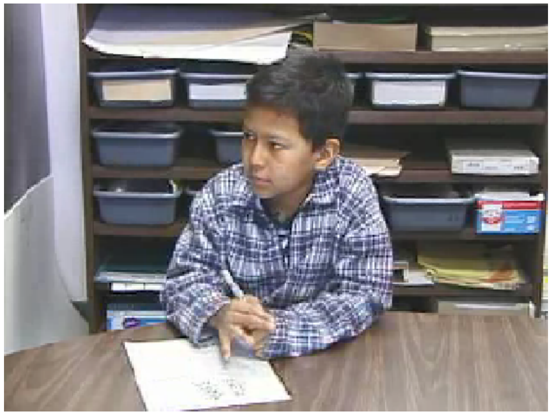
Often What We Think We are Teaching is Not What Students are Learning

After explaining to a student through various lessons and examples that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example. This was the result:

$$\lim_{x \to 5} \frac{1}{x-5} = 5$$



Javier, Grade 5

(*This is video clip #6 on IMAP CD.)

At the time of this interview, Javier had been in the United States about one year, and he did not speak English before coming to this country.

Javier, VC #158, 0:00 - 1:10

One Representation of Javier's Thinking

6×12

$= (5 \times 12) + (1 \times 12)$ (Distributive prop. of x over +)

$= [(\frac{1}{2} \times 10) \times 12] + 12$ (Substitution property)

$= [\frac{1}{2} \times (10 \times 12)] + 12$ (Associative property of x)

$= [\frac{1}{2} \times (120)] + 12$

Place value

$= 60 + 12$

$= 72$

What would a teacher need to know to understand Javier's thinking, and where do teachers learn this?

Students' Reasoning is Often Different From Adults' Reasoning

5^{th}-graders

Circle the larger or write "=“

4.7 4.70

7^{th}-grader

$6 - -2 = \square$

•What thoughts do you have about these students' reasoning?

•In mathematics, what *changes* are we allowed to make, and why?
•When something (e.g., a decimal, an equation) changes, what is invariant?
•What do students think is happening here? Does it matter what they think?

Four principles of mathematics and mathematics teaching and learning addressed by focusing upon children's mathematical thinking.

1. The way most students are learning mathematics in the United States is problematic.

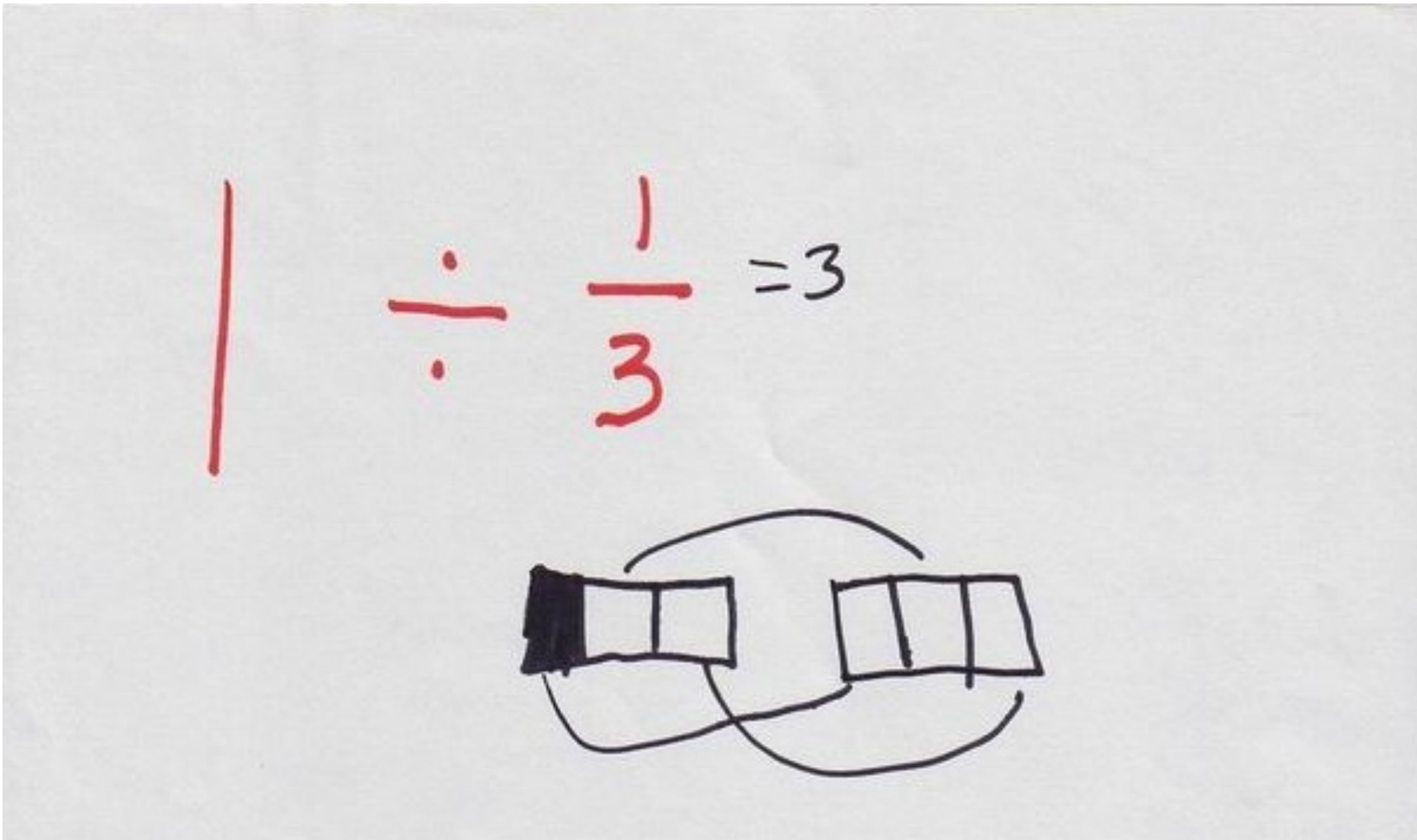
2. Learning concepts is more powerful and more generative than learning procedures.

3. Students' reasoning is varied and complex, and generally it is different from adults' thinking.

4. Elementary mathematics is not elementary.

Elliot, Grade 6: 1 ÷ 1/3?*

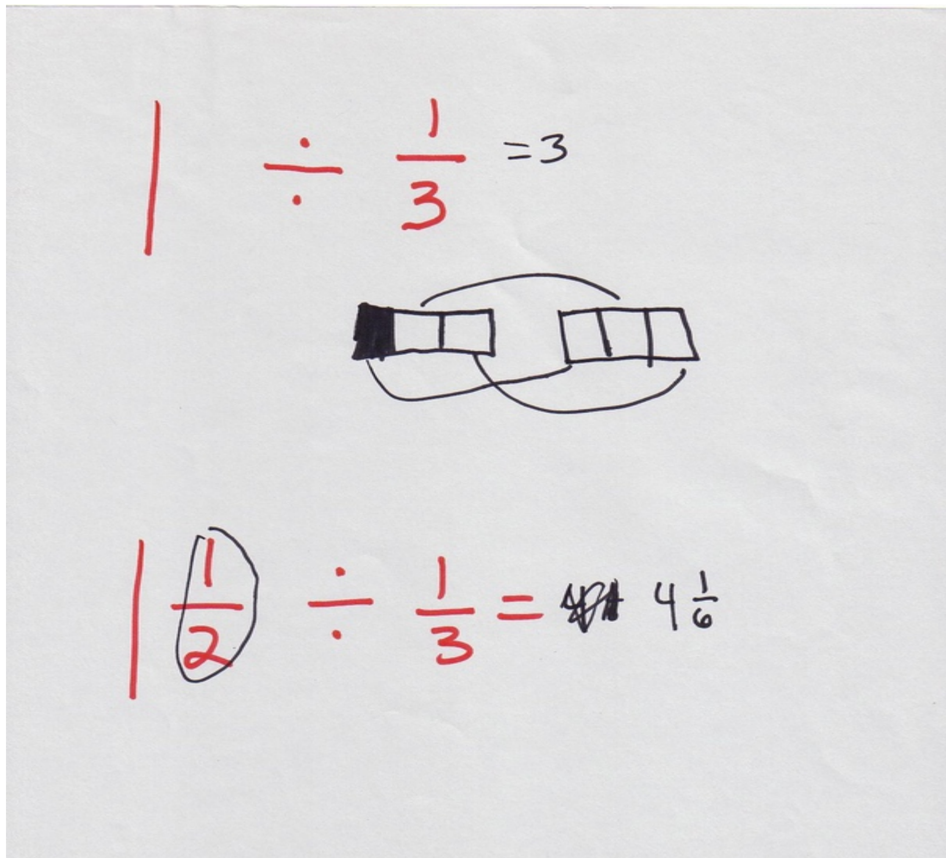
(*This is video clip #16 on IMAP CD.)



“One third goes into 1 three times because there is three pieces in one whole.”

Follow-up: 1 1/2 ÷ 1/3?

"There are three one-thirds in 1, and another 1/3 in 1/2, and there is 1/6 left over."



Elliot does not realize that the remaining 1/6 needs to be seen as 1/2 of 1/3.

Elliot, Grade 6

Question for PSTs

Explain Elliot's reasoning.
What does one need to understand to be in the position to support Elliot?

Issue

Understanding is seldom black and white. Everybody understands something about anything, and no one understands everything about anything.

Turn To Practicing Teachers

STEP
STUDYING TEACHERS' EVOLVING PERSPECTIVES

- 5-year NSF-funded-project (2005–2010)

- Cross-sectional examination of the effects of sustained professional development focused on children's mathematical thinking

We are studying the perspectives of four groups of teachers:

- Beliefs
- Interactions with children
- **Professional Noticing**
- Mathematical Content Knowledge

P.D. Matters!

Why Noticing?

Teachers faced with a "blooming, buzzing, confusion of sensory data" during instruction
- B. Sherin & Star, 2011, p. 69

- Teaching involves deciding where to pay attention and where not to pay attention
- Mathematics teaching in particular calls for adaptive and responsive style of instruction
- Assessing on-going instruction requires "expert noticing"

Noticing Children's Mathematical Thinking

Child says or
does something → Teacher Move

What goes into a teacher's decision making before deciding upon a move?

- Seems simple…but it is not
- Often happens fast

Let's try it….

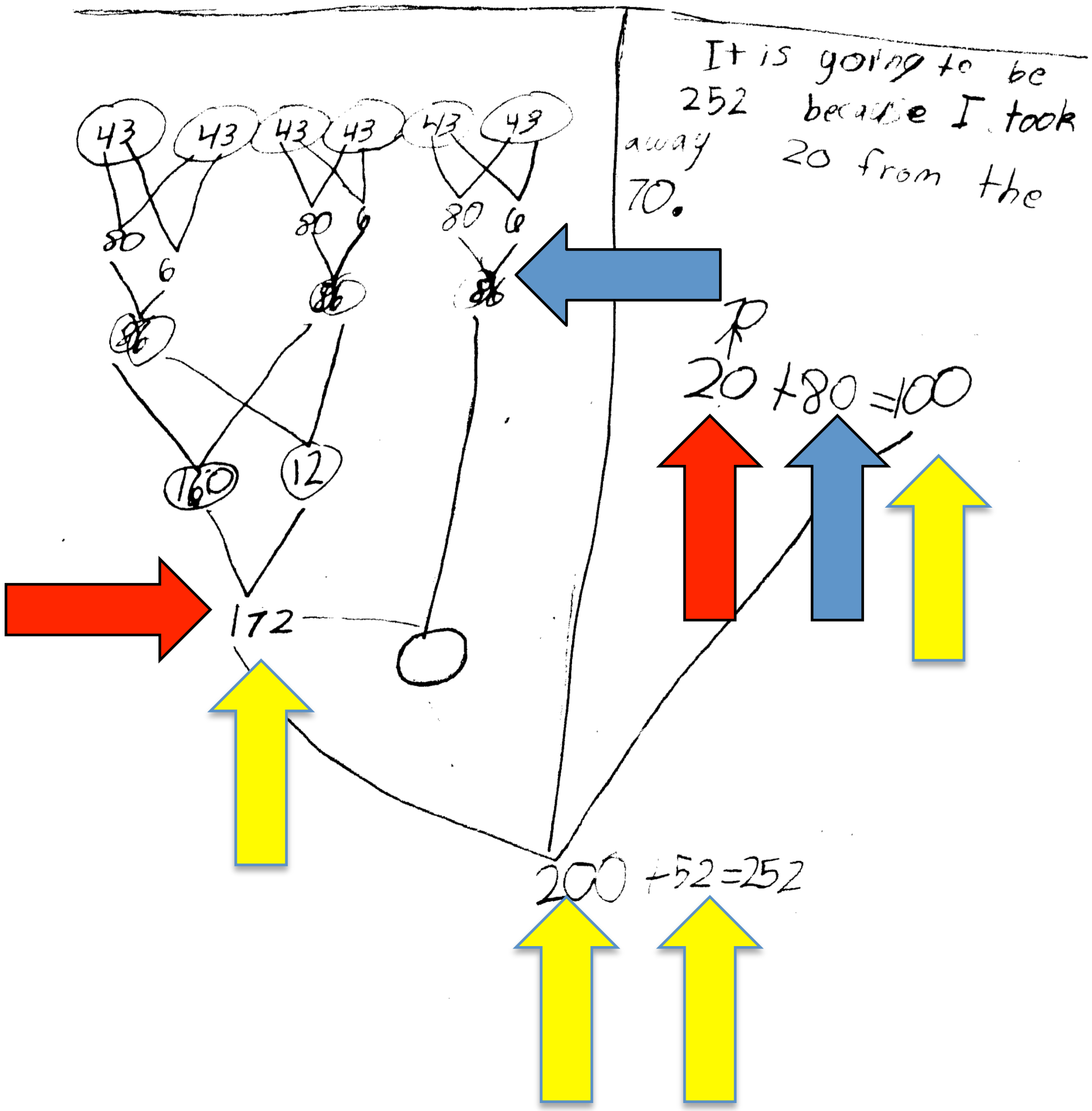
- Grade 2 student work—Josie, Alexis, Cassandra
- Think about how you might respond—What problem or problems might you pose next?
- What do you notice?

Interesting Range of Strategies

- Alexis – direct modeling by 1s with tally marks; unclear how she counted
- Josie – skip counting by 40s and then 3s (or 1s)
- Cassandra – breaking apart numbers and putting them back together; minor error of forgetting 6

Cassandra

Todd has 6 bags of M&Ms. Each bag has 43 M&Ms. How many M&Ms does Todd have?



Three component skills of *noticing children's mathematical thinking*

1. **Attending** to children's strategies

    Please describe in detail what you think each child did in response to this problem.

2. **Interpreting** children's understandings

    Please explain what you learned about these children's understandings.

3. **Deciding how to respond** on the basis of children's understandings

    Pretend that you are the teacher of these children. What problem or problems might you pose next?

What do teachers' responses look like?

- Asked 131 prospective & practicing teachers to respond to these prompts (in writing)
- Give you a sense of responses that focused on children's thinking vs. those that did not

  ➡ Meet two participants: A & B
- Look at extremes so think about what responses in-between might look like
- What role did children's thinking play when you were discussing ideas related to each of these prompts?

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Attending to children's strategies

<u>Prompt:</u> Please describe in detail what you think each child did in response to this problem. **(Alexis)**

Participant A

"Drew tally marks for each bag of 43. I'm not sure how she counted the total. My guess would be either by 5's then the extras <u>OR</u> the fact that she has the groups of 40 circled may mean she counted by 40's then went back to count the extra threes."

Participant B

"I believe Alexis separated the 6 bags visually and counted 43 M&M's in each bag. She then added all of her M&M's and came out with the answer. Her technique was simple and understandable."

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Interpretation of Children's Understandings (Participant A)

__Prompt:__ Please explain what you learned about these children's understandings.

"Alexis has a clear understanding of the problem and a very basic way to figure it out. Without knowing how she counted all of those tally marks, it's hard to say more about her number sense.

Cassandra has a clear understanding of place value (at least tens and ones). She also is flexible with numbers. Her strategy of taking the 20 off of 172 to make a 100 shows facility with friendly numbers. I'd like to ask her if she checked her work.

Josie understands place value – tens and ones. She knows how to skip count and count on."

Interpretation of Children's Understandings (Participant B)

<u>Prompt:</u> Please explain what you learned about these children's understandings.

"I learned how children use rounding and grouping in order to solve problems that involve more than counting on your fingers. I learned that one problem can be solved in infinite ways when it is in the hands of a learning and imaginative child."

Three component skills of *noticing children's mathematical thinking*

1. **Attending** to children's strategies

   Please describe in detail what you think each child did in response to this problem.

2. **Interpreting** children's understandings

   Please explain what you learned about these children's understandings.

3. **Deciding how to respond** on the basis of children's understandings

   Pretend that you are the teacher of these children. What problem or problems might you pose next?

Deciding How to Respond on the Basis of Children's Understandings (Participant A)

Pretend that you are the teacher of these children. What problem or problems might you pose next?

"More of this same problem type.

For Alexis I would choose some smaller numbers with clear friendly numbers within to try to move her past direct modeling. Maybe 6 bags with 25 M&M's.

Cassandra needs more problems with similar numbers to try and streamline her process. Can she begin to group all the 40's at once, or at least more of them at once?

Josie also needs more of the same. Can she begin to combine large groups without needing to count up one group at a time? Maybe some work with numbers she has more facility with – maybe 50's or 20's."

Deciding How to Respond on the Basis of Children's Understandings (Participant B)

Pretend that you are the teacher of these children. What problem or problems might you pose next?

"I might pose a question about needing 4 groups in the class and having 29 students. Then I would ask if we had any extra.

The students would then have to use a process of counting and distributing children into groups. They would have to learn how many kids needed to be in each group. They would learn later that one student was extra."

So what?

Participant A consistently focused on children's thinking whereas Participant B focused on other things.

Why do we care?

→ Instruction will look different

In what ways?

Consider how Participant A's and Participant B's classrooms might look differently.

Studying Teachers' Evolving Perspective (STEP) Research Project

Special thanks to…

National Science Foundation (NSF)

Collaborators at San Diego State University

Vicki Jacobs, co-PI
Lisa Lamb
Jessica Pierson Bishop
Bonnie Schappelle
Candy Cabral
John Siegfried

STEP Participant Groups (*N*=131, 30+ per group)

K–3 Teachers

Emerging Teacher Leaders	At least 4 years of sustained professional development and some leadership activities
Advancing Participants	2 years of sustained professional development
Initial Participants	0 years of sustained professional development
Prospective Teachers	Undergraduates enrolled in a first mathematics-for-teachers content course

*Average of 14–16 years of teaching per group; range 4–33 years

CGI Professional Development

- Focus on what we've learned from research and practice about children's mathematical thinking and how to use this knowledge to inform instruction

- 5 full-day meetings per year

- Discussion of classroom artifacts (video and written student work)

- Problems to try in teachers' own classrooms between meetings

Categorized the written responses to the M & M task

- Read the written responses without knowing which participant group they were in
- Decided whether there was evidence of a focus on children's thinking in each of the responses

  Evidence / No Evidence

  Robust Evidence / Limited Evidence / No Evidence

- Looked to see which individuals show the highest level of expertise for at least one of the noticing skills of attending, interpreting or deciding how to respond

Participant Groups Means (Standard Deviations) for Overall Scores of the Component Noticing Skill

Component Skill	Scale	Prospective Teachers	Initial Participants	Advancing Participants	Emerging Teacher Leaders
Attending to CMTg	0–1	0.42 (0.50)	0.65 (0.49)	0.90 (0.30)	0.97 (0.17)
Interpreting CMTg	0-2	0.47 (0.51)	0.94 (0.63)	1.19 (0.54)	1.76 (0.44)
Deciding how to respond on the basis of CMTg	0-2	0.14 (0.35)	0.29 (0.53)	0.84 (0.73(	1.45 (0.79)

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Statistically Significant Differences

Focusing on Children's Mathematical Thinking…

- …supports teachers' developing more sophisticated means of professional noticing of children's mathematical thinking.
- …supports teachers learning mathematics.
- …supports teachers developing more sophisticated beliefs.
- …supports teachers in developing a stance of inquiry toward their practice.

Discussion

- Questions?
- Comments?