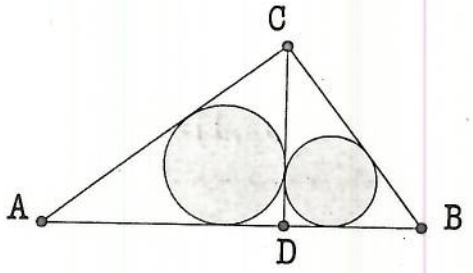
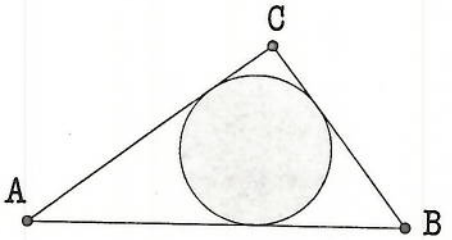
3. In each triangle below, BC = 3, AC = 4, and AB = 5. On the left, $\overline{CD} \perp \overline{AB}$.

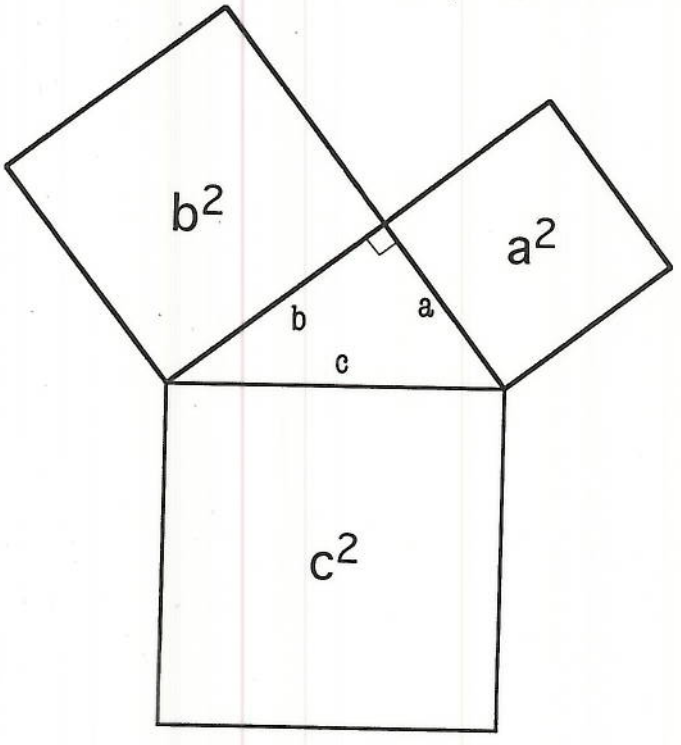
What is the total area of the 2 incircles on the left?

What is the area of the incircle on the right?

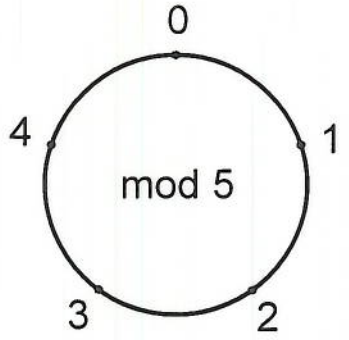




4. Explain how the previous three exercises helped you gain insight into the Pythagorean Theorem.



$$a^2 + b^2 = c^2$$



$u^2 = 2u + 2$

3u										
1										4

$$\frac{4u + 3}{3u + 1} =$$

$$\frac{2u + 2}{3u + 2} =$$

$$\frac{u}{2u + 1} =$$

$$\frac{4u + 2}{4u + 4} =$$

$$\frac{2u + 1}{u + 1} =$$

$$\frac{u + 2}{3u + 3} =$$

$$\frac{3u + 4}{2u + 4} =$$

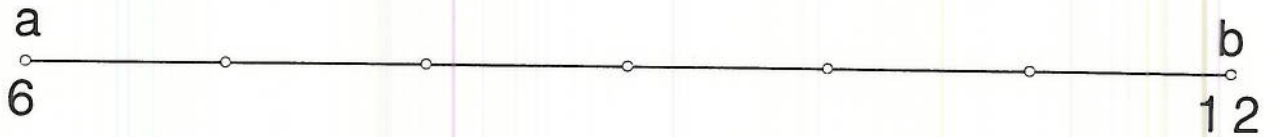
$$\frac{u + 1}{3u + 4} =$$

Bargaining

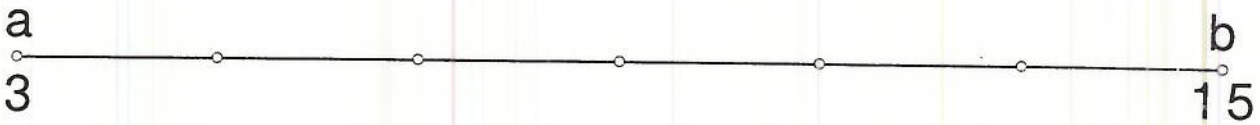
-50% 33 1/3% -16 2/3%

50% 100% 150%

$$\frac{2ab}{a+b} = 3 \qquad \frac{b-a}{b+a} = 50\%$$



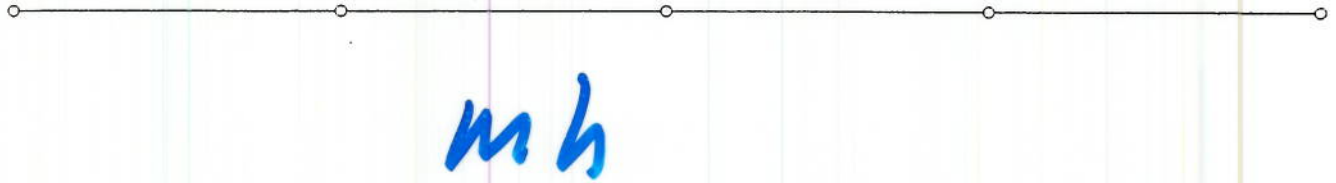
$$\frac{2ab}{a+b} = \qquad \frac{b-a}{b+a} = \_\_\ \%$$



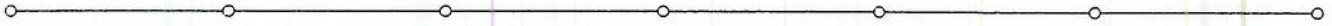
$$\frac{2ab}{a+b} = \qquad \frac{b-a}{b+a} = \_\_\ \%$$

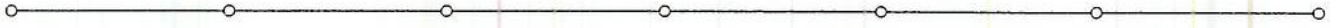
$$\frac{2ab}{a+b} = \qquad \frac{b-a}{b+a} = \_\_\ \%$$

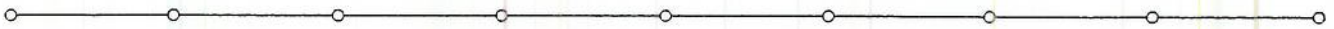
The Harmonic Mean

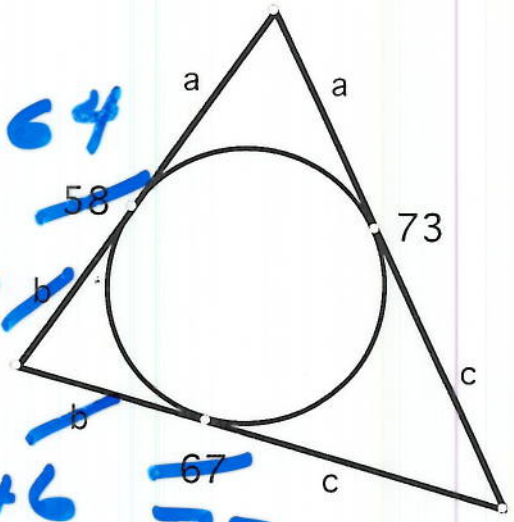


m_h









64

6+6

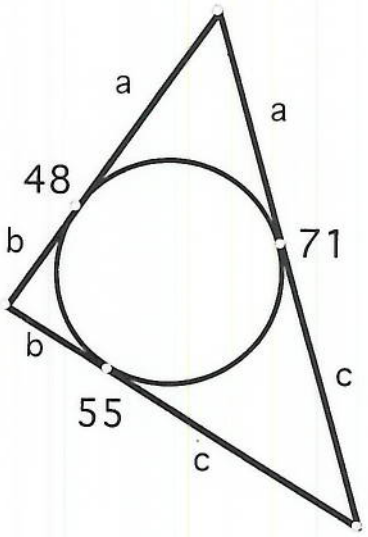
6+6

73

a =

b =

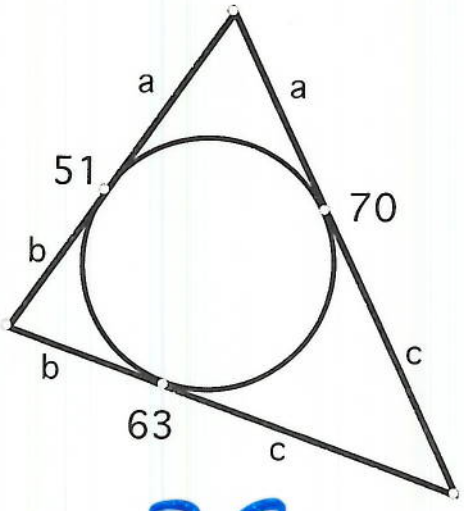
c =



a =

b =

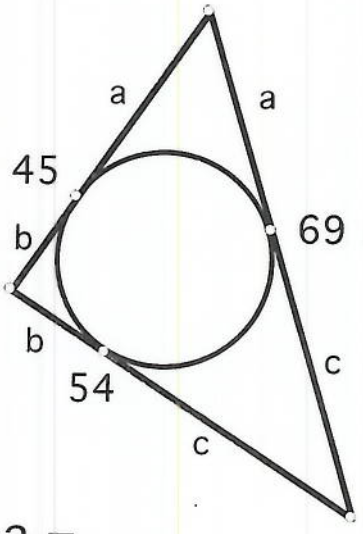
c =



a = 29

b = 23

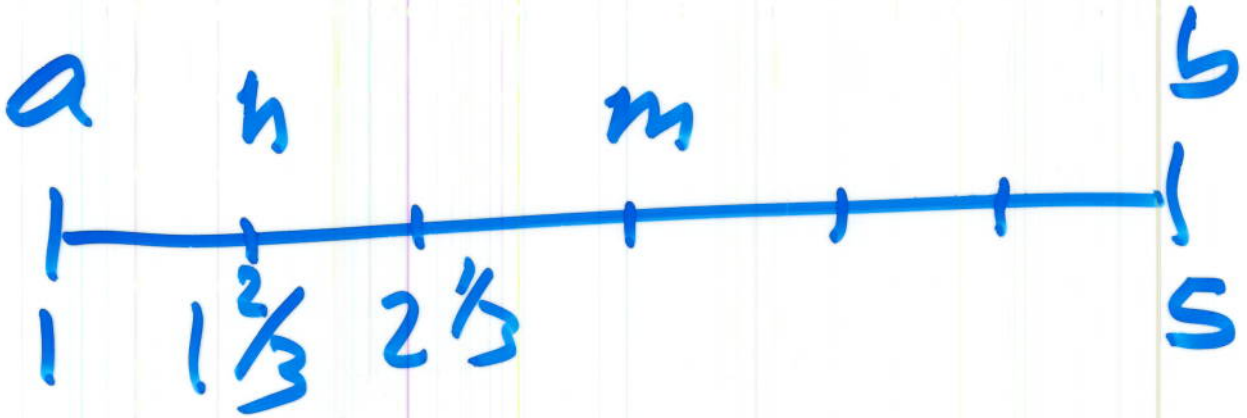
c = 44



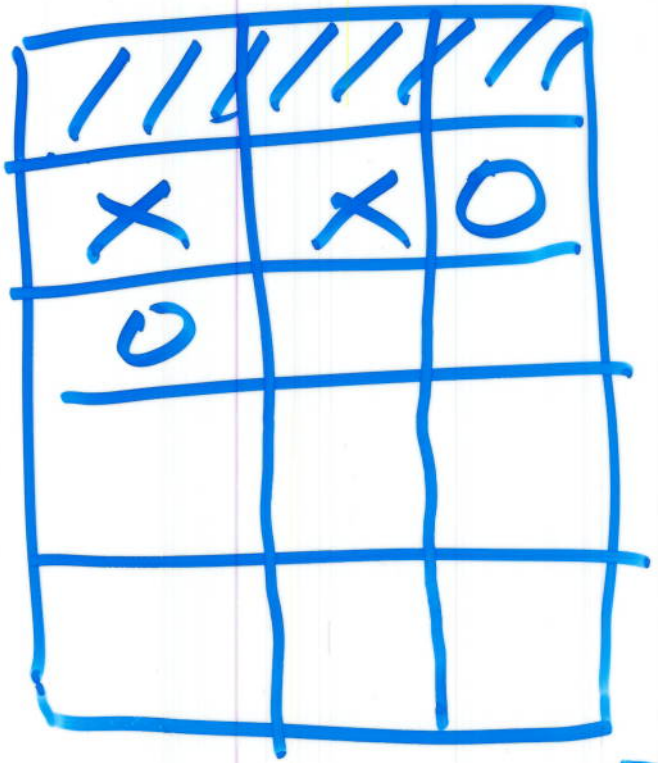
a =

b =

c =



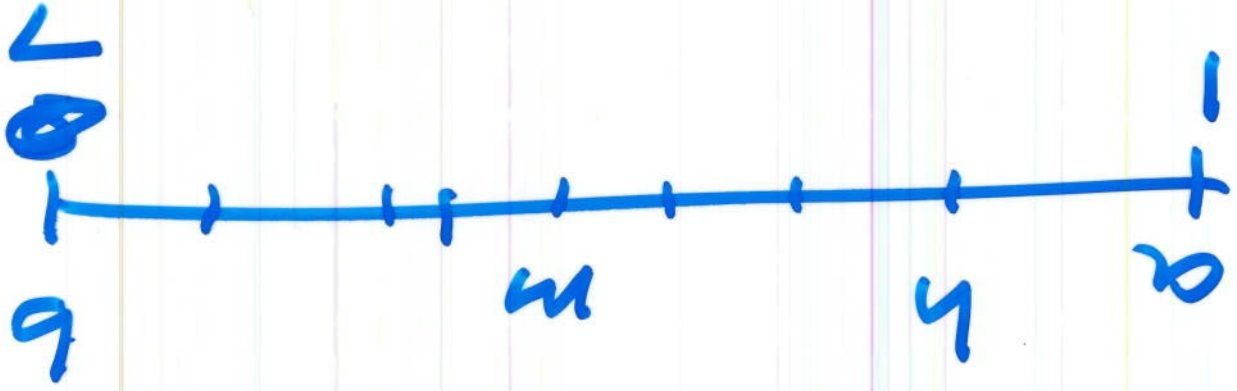
$$\frac{b-a}{b+a} = \frac{5-1}{5+1} = \frac{2}{3}$$



$$\frac{9+b}{90c}$$

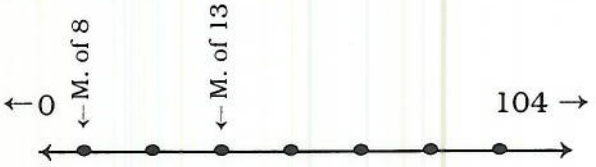
$$\left(\frac{b+9}{b-9}\right)b+b$$

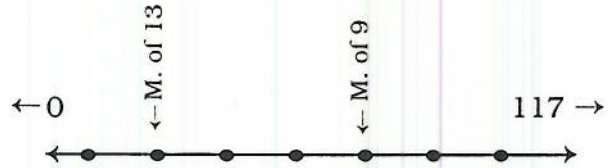
$$\frac{b+9}{b-9}=\frac{1+L}{1-L}$$

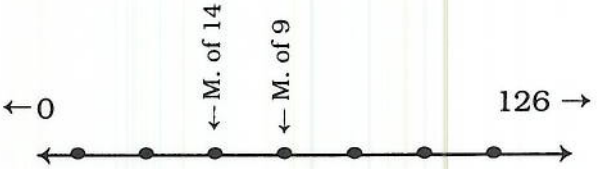


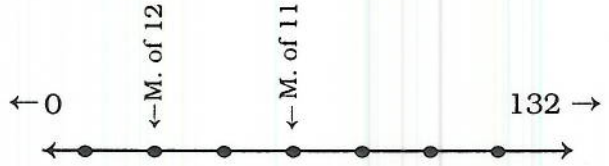

2
4
8

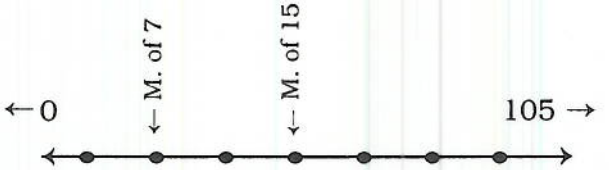
Lost on the Number Line

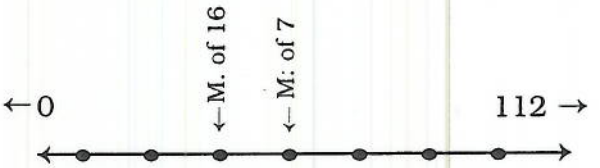


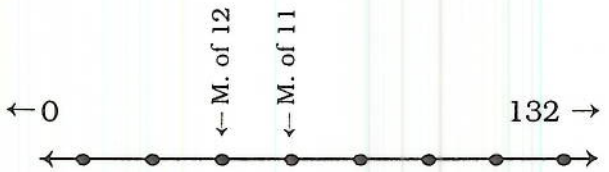










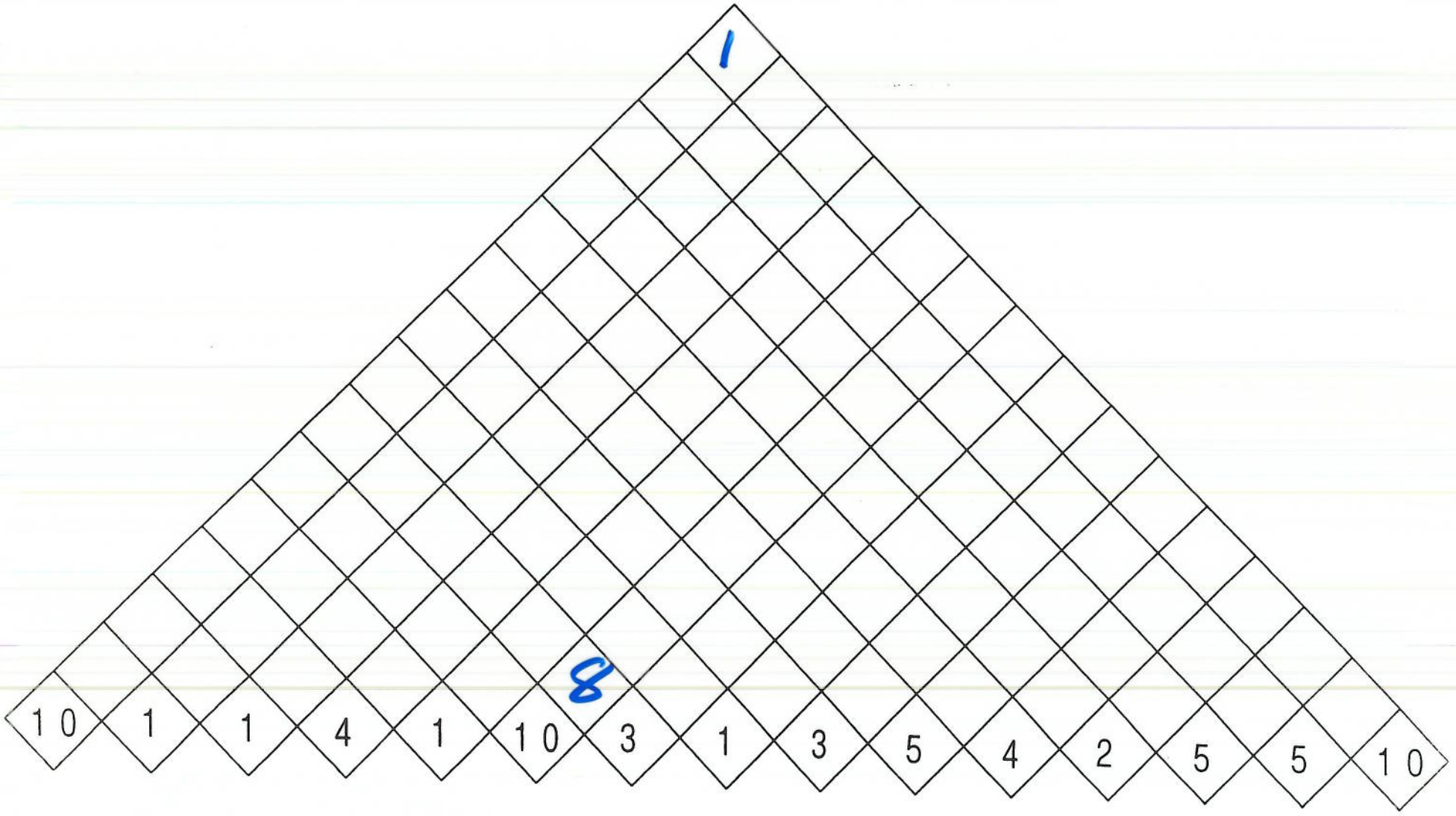


M. = multiple

489 x 7 = 3423	298 x 7 = 2086	590 x 7 = 4130	385 x 7 = 2695	723 x 7 = 5061
308 x 7 = 2156	659 x 7 = 4613	902 x 7 = 6314	739 x 7 = 5173	388 x 7 = 2716
330 x 7 = 2310	671 x 7 = 4697	367 x 7 = 2569	893 x 7 = 6251	550 x 7 = 3850
781 x 7 = 5467	924 x 7 = 6468	836 x 7 = 5852	572 x 7 = 4004	396 x 7 = 2772
616 x 7 = 4312	979 x 7 = 6853	436 x 7 = 3052	693 x 7 = 4851	707 x 7 = 4949
935 x 7 = 6545	286 x 7 = 2002	994 x 7 = 6958	218 x 7 = 1526	234 x 7 = 1638
776 x 7 = 5432	198 x 7 = 1386	675 x 7 = 4725	845 x 7 = 5915	715 x 7 = 5005

The Christmas Tree

0, 11, 22, 33, 44, 55, 66, 77, 88, 99



In each problem, try to complete the equation by putting a whole number into each parenthesis. But fourteen of the problems are impossible. If (and only if) a problem is impossible, add 40 to the right side of the equation. After doing this, the equation can be completed. Three problems, including one of the impossible ones, are done as examples.

5( ) + 8( ) = 1	5( ) + 8( ) = 21
5( ) + 8( ) = 2	5( ) + 8( ) = 22
5( ) + 8( ) = 3	5( ) + 8( ) = 23
5(4) + 8(3) = 4 + 40	5( ) + 8( ) = 24
5( ) + 8( ) = 5	5( ) + 8( ) = 25
5( ) + 8( ) = 6	5( ) + 8( ) = 26
5( ) + 8( ) = 7	5( ) + 8( ) = 27
5( ) + 8( ) = 8	5( ) + 8( ) = 28
5( ) + 8( ) = 9	5( ) + 8( ) = 29
5( ) + 8( ) = 10	5( ) + 8( ) = 30
5( ) + 8( ) = 11	5( ) + 8( ) = 31
5( ) + 8( ) = 12	5( ) + 8( ) = 32
5( ) + 8( ) = 13	5( ) + 8( ) = 33
5( ) + 8( ) = 14	5( ) + 8( ) = 34
5( ) + 8( ) = 15	5( ) + 8( ) = 35
5( ) + 8( ) = 16	5( ) + 8( ) = 36
5( ) + 8( ) = 17	5(1) + 8(4) = 37
5(2) + 8(1) = 18	5( ) + 8( ) = 38
5( ) + 8( ) = 19	5( ) + 8( ) = 39
5( ) + 8( ) = 20	

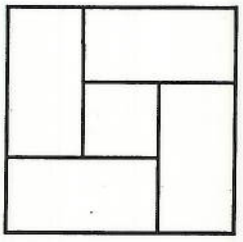
Twentieth Century Arithmetic

20 x 99 = 1980
20 x 98 =
20 x 97 = 1940
20 x 96 =
20 x 95 = 1900
21 x 95 =
21 x 94 =1974
21 x 93 =
21 x 92 = 1932
21 x 91 =
22 x 90 = 1980
22 x 89 =
22 x 88 = 1936
22 x 87 =
23 x 86 = 1978
23 x 85 =
23 x 84 = 1932
23 x 83 =
24 x 83 = 1992
24 x 82 =
24 x 81 = 1944
24 x 80 =
25 x 79 = 1975
25 x 78 =
25 x 77 = 1925
25 x 76 =

26 x 76 = 1976
26 x 75 =
26 x 74 = 1924
27 x 74 =
27 x 73 = 1971
27 x 72 =
27 x 71 = 1917
28 x 71 =
28 x 70 = 1960
28 x 69 =
28 x 68 = 1904
29 x 68 =
29 x 67 = 1943
29 x 66 =
30 x 66 = 1980
30 x 65 =
30 x 64 = 1920
31 x 64 =
31 x 63 = 1953
31 x 62 =
32 x 62 = 1984
32 x 61 =
32 x 60 = 1920
33 x 60 =
33 x 59 = 1947
33 x 58 =

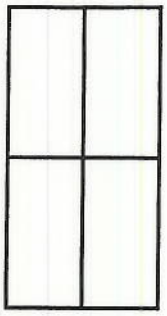
34 x 57 = 1938
34 x 56 =
35 x 56 = 1960
35 x 57 =
35 x 55 = 1925
36 x 55 =
36 x 54 = 1944
37 x 54 =
37 x 53 = 1961
37 x 52 =
38 x 52 =
38 x 51 = 1938
38 x 50 =
39 x 50 =
39 x 51 =
39 x 49 = 1911
40 x 49 =
40 x 48 =
41 x 48 =
41 x 47 =
42 x 47 =
42 x 46 = 1932
43 x 46 =
43 x 45 =
44 x 45 =
44 x 44 =

Write the area in square centimeters in each subregion of the four figures below.



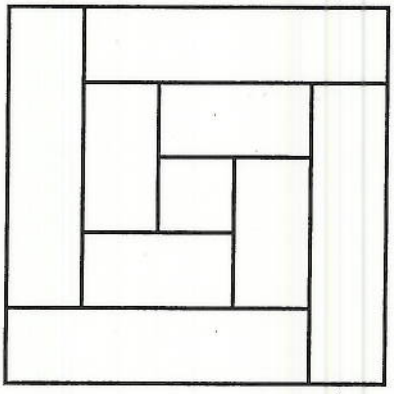
The total area is ____ cm^2

The perimeter is ____ cm



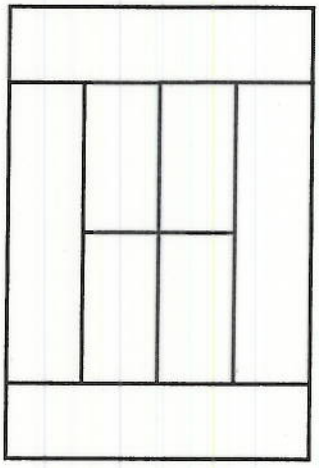
The total area is ____ cm^2

The perimeter is ____ cm



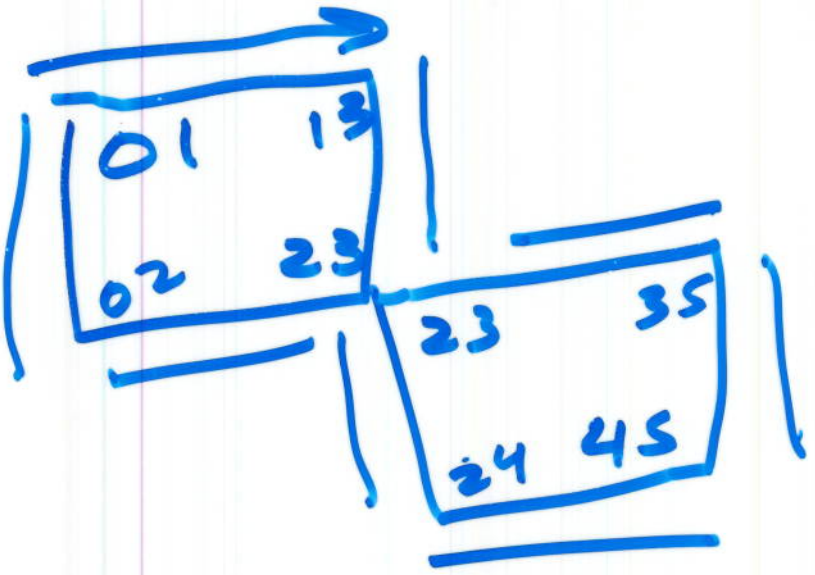
The total area is ____ cm^2

The perimeter is ____ cm



The total area is ____ cm^2

The perimeter is ____ cm



0 1 3

2 2

3 5 4

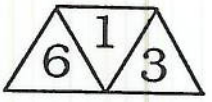
~~B B B~~

B 1 B D

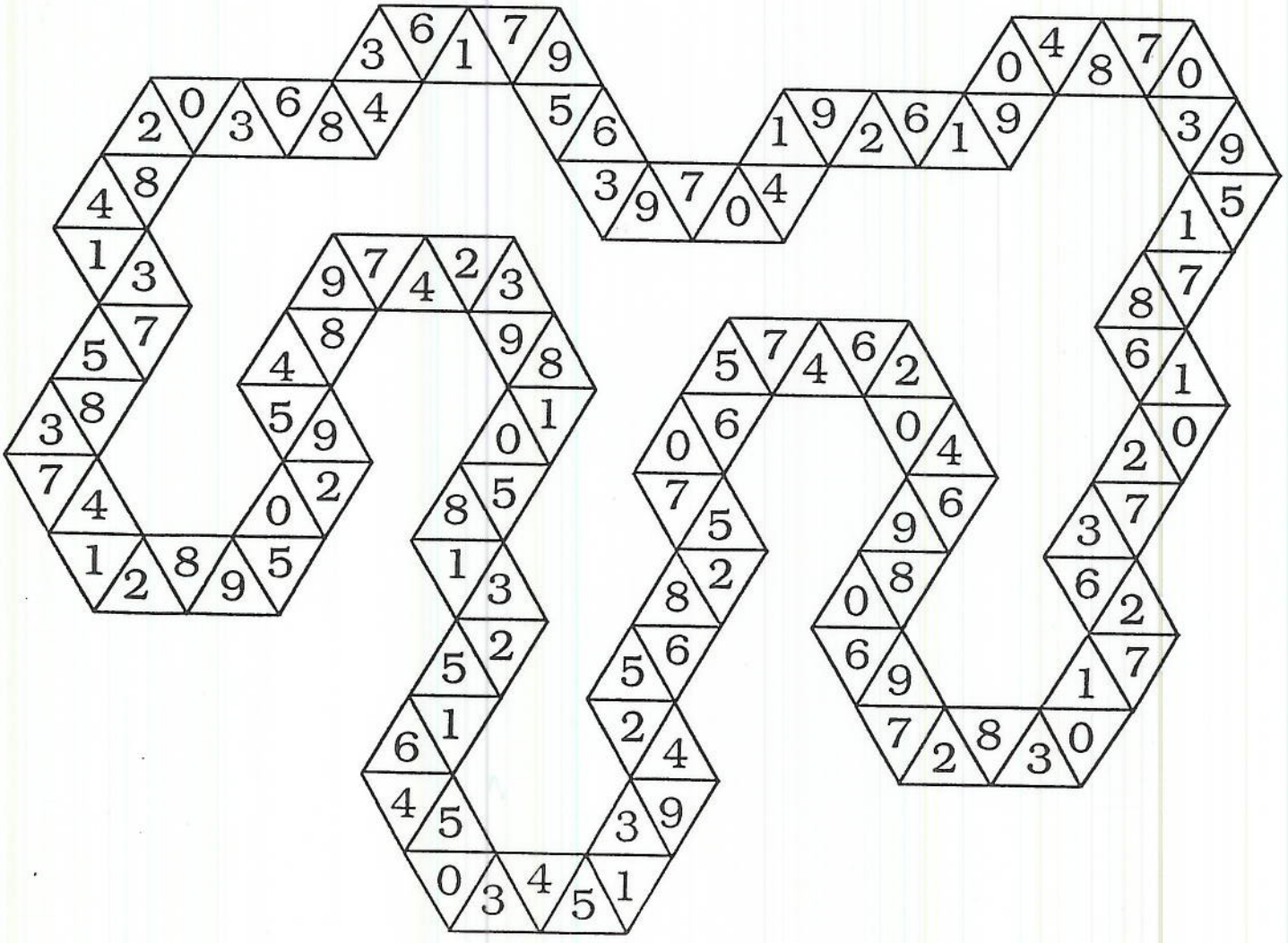
Directions: Choose one of the boxes below.
Draw a line around it (cover the dots).

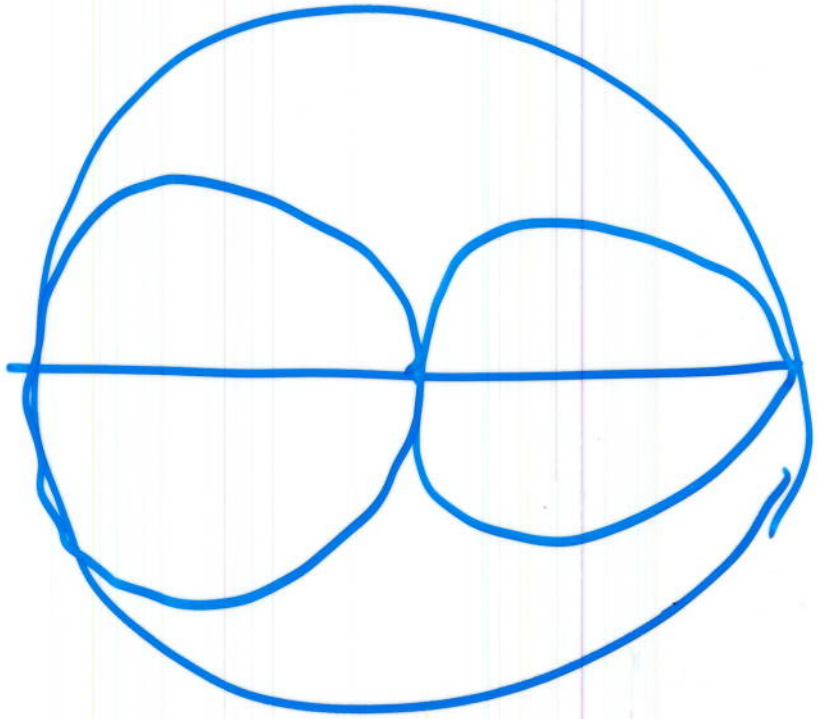
10 or 20	11 or 21	12 or 22	3, 13, or 23	4, 14, or 24
5 or 15	6 or 16	7 or 17	8 or 18	9 or 19

The shape on the right is a trapezoid that has 3 triangles in it. The numbers in it add up to 10.



In the loop below, find a trapezoid that has 3 numbers in it that add up to one of the numbers in the box you chose. Find a total of 12 such trapezoids and lightly shade them in.

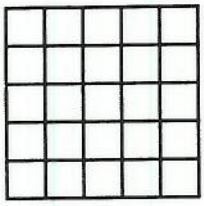


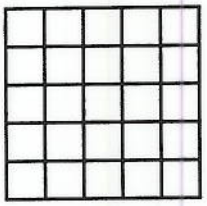


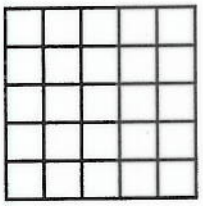
All of the polynomials are elements of $F_5[x]$

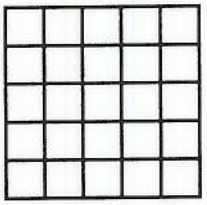
polynomial	factorization	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
x^2						
$x^2 + 1$						
$x^2 + 2$						
$x^2 + 3$						
$x^2 + 4$						
$x^2 + x$						
$x^2 + x + 1$						
$x^2 + x + 2$						
$x^2 + x + 3$						
$x^2 + x + 4$						
$x^2 + 2x$						
$x^2 + 2x + 1$						
$x^2 + 2x + 2$						
$x^2 + 2x + 3$						
$x^2 + 2x + 4$						
$x^2 + 3x$						
$x^2 + 3x + 1$						
$x^2 + 3x + 2$						
$x^2 + 3x + 3$						
$x^2 + 3x + 4$						
$x^2 + 4x$						
$x^2 + 4x + 1$						
$x^2 + 4x + 2$						
$x^2 + 4x + 3$						
$x^2 + 4x + 4$						

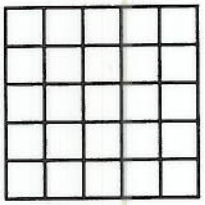
1	2	3	2	1
2	4	5	4	2
3	5	6	5	3
2	4	5	4	2
1	2	3	2	1

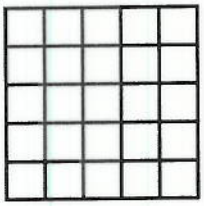
1,1 

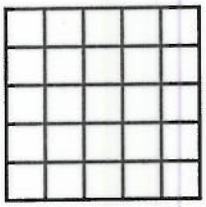
1,1 

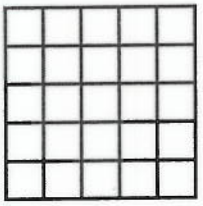
1,2 

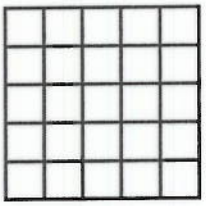
1,2 

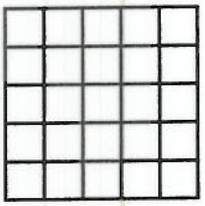
1,2 

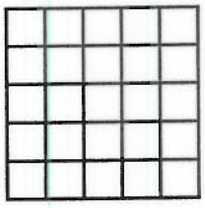
1,2 

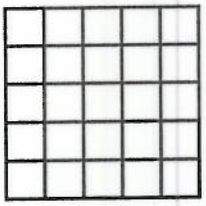
1,3 

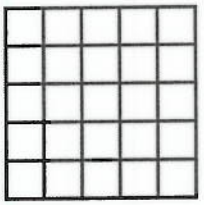
1,3 

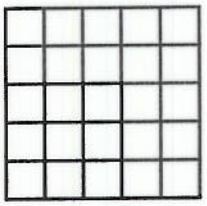
1,4 

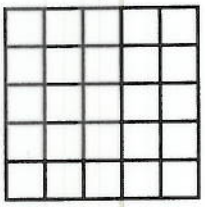
1,4 

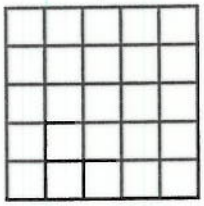
1,4 

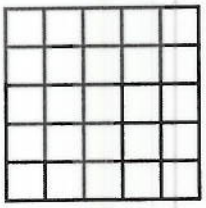
1,5 

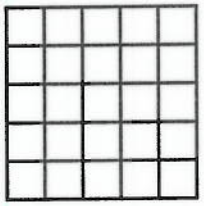
1,5 

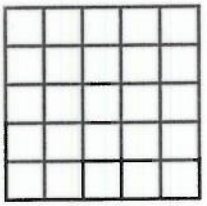
1,6 

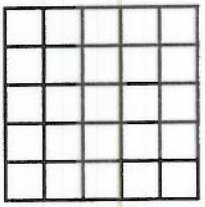
2,2 

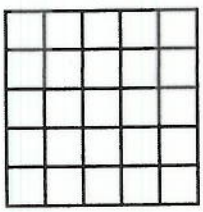
2,2 

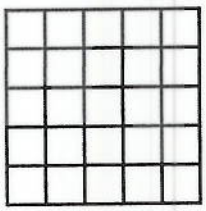
2,2 

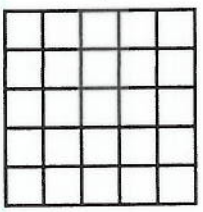
2,2 

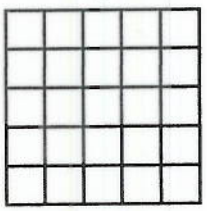
2,2 

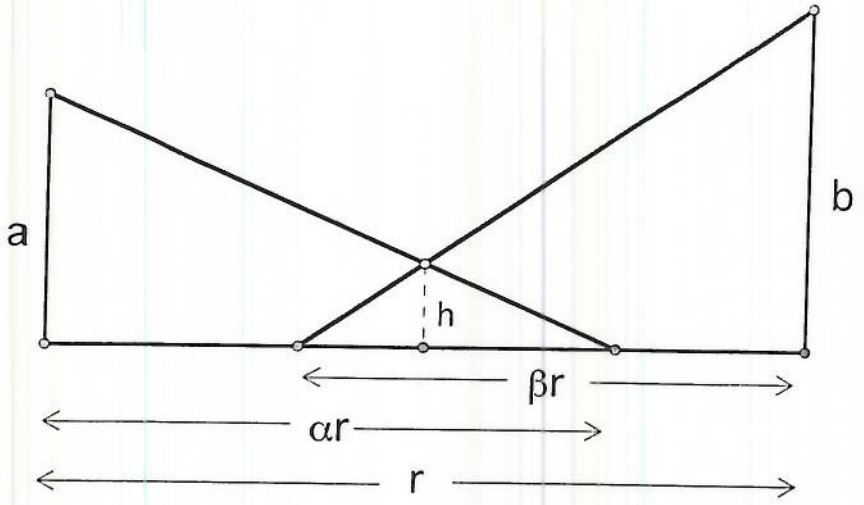
2,2 

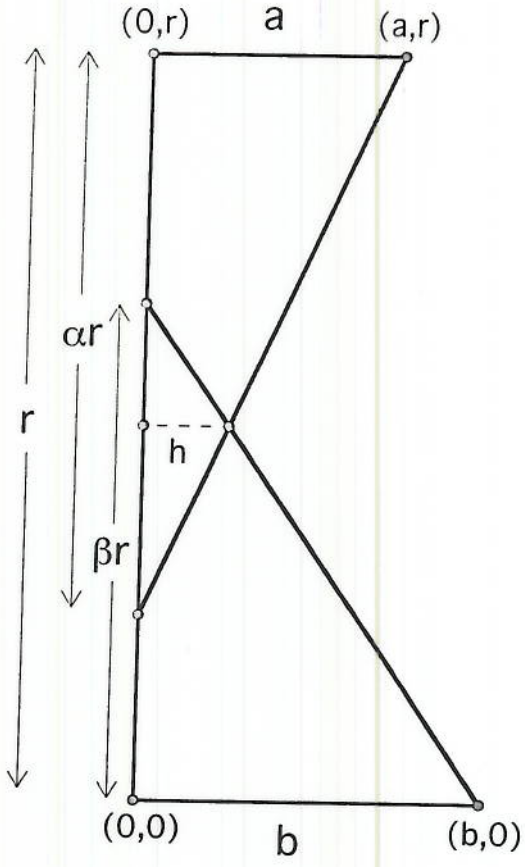
2,3 

2,3 

2,3 

2,3 





To simplify the the derivation of a formula for h, the original diagram on the left has been turned on its side.

$$y = \frac{-\beta r}{b}x + \beta r$$

$$y = \frac{\alpha r}{a}x + r - \alpha r$$

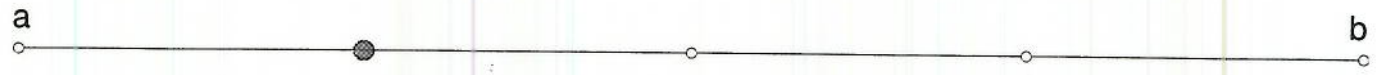
$$\frac{-\beta r}{b}x + \beta r = \frac{\alpha r}{a}x + r(1 - \alpha)$$

$$\frac{b\alpha r}{ab}x + \frac{a\beta r}{ab}x = \beta r - r(1 - \alpha)$$

$$\frac{\beta a + \alpha b}{ab}x = \alpha + \beta - 1$$

$$x = \frac{ab(\alpha + \beta - 1)}{\beta a + \alpha b} = h$$

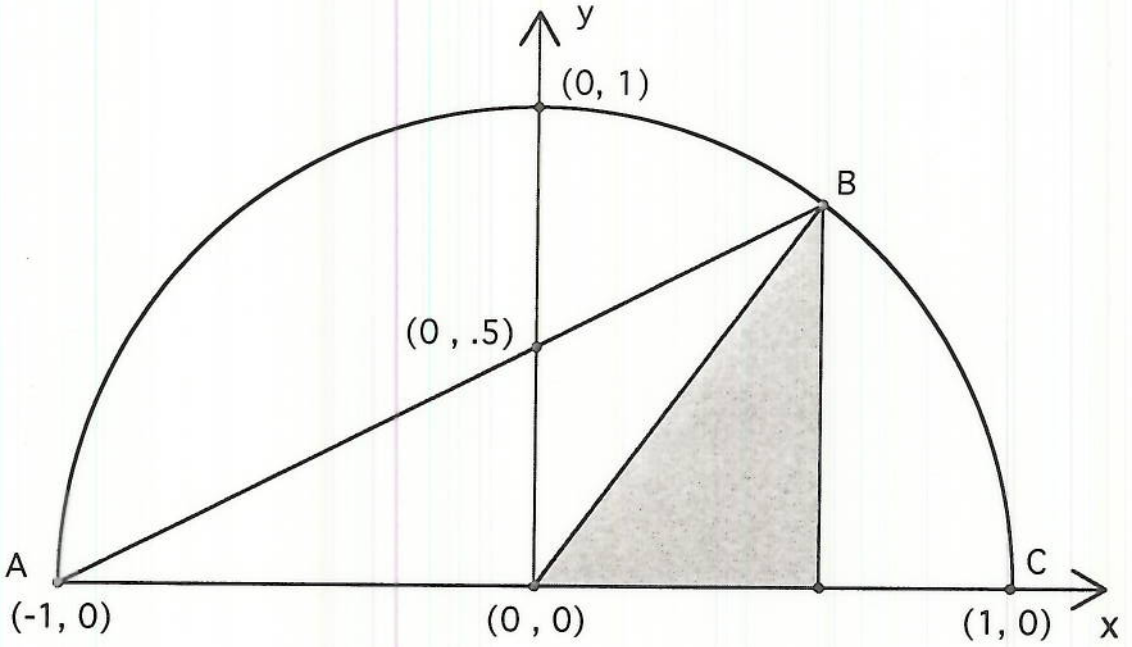
Label each enlarged point with an algebraic expression in two different ways.

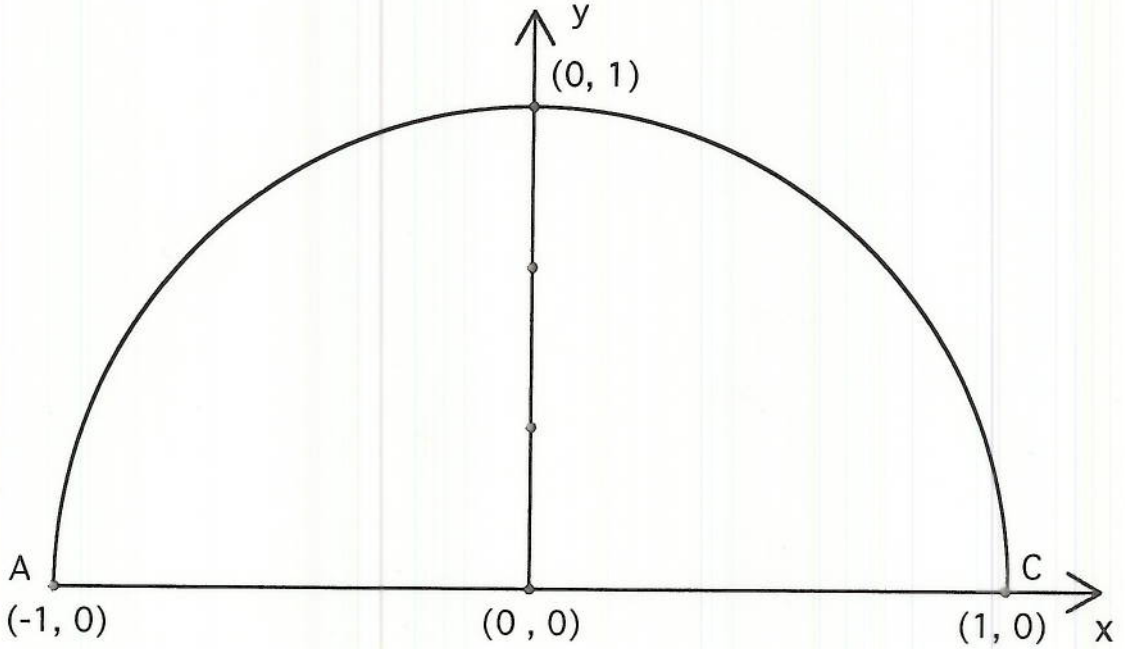


$$\frac{2\left(\frac{2}{5}\right)}{1+\left(\frac{2}{5}\right)^2} =$$

$$\frac{2\left(\frac{3}{5}\right)}{1+\left(\frac{3}{5}\right)^2} =$$

$$\frac{2\left(\frac{4}{5}\right)}{1+\left(\frac{4}{5}\right)^2} =$$


y
(0, 1)
B
(0 , .5)
A
(-1, 0)
(0 , 0)
C
(1, 0)
x


y
(0, 1)
A
(-1, 0)
(0 , 0)
C
(1, 0)
x

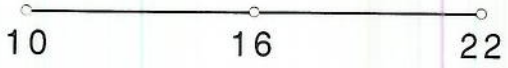
$$m = \frac{b + a}{2}$$

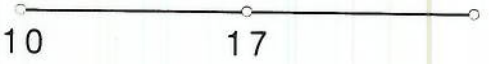
$$d = \frac{b - a}{2}$$

b and a are positive integers, both even or both odd, b > a, and ba = n

difference of squares of positive integers

n	*ba*	$(m + d)(m - d)$	$m^2 - d^2$	
5	5 X 1	$(3 + 2)(3 - 2)$	$3^2 - 2^2$	9 - 4
8				
11				
13				
16				
19				
21				
21				
24				
24				
27				
27				
32				
32				
36				
37				
40				
40				

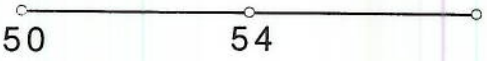


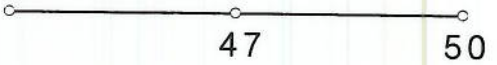


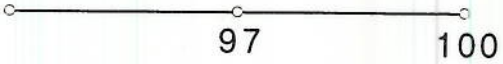
$$16^2 = (10)(22) + 6^2$$

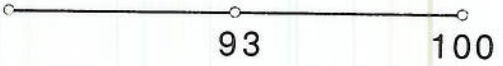
$$16^2 = 220 + 36$$

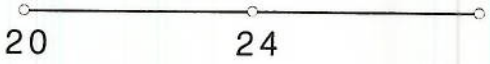
$$16^2 = 256$$

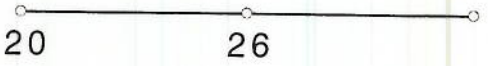












= 1	= 2	= 3	= 4	= 4	= 5
= 6	= 6	= 7	= 8	= 8	= 9
= 9	= 10	= 10	= 11	= 12	= 12
= 12	= 13	= 14	= 14	= 15	= 15
= 16	= 16	= 16	= 17	= 18	= 18
= 18	= 19	= 20	= 20	= 20	= 21
= 21	= 22	= 22	= 23	= 24	= 24
= 24	= 24	= 25	= 25	= 26	= 26
= 27	= 27	= 28	= 28	= 28	= 29
= 30	= 30	= 30	= 30	= 31	= 32
= 32	= 32	= 33	= 33	= 34	= 34
= 35	= 35	= 36	= 36	= 36	= 36
= 36	= 37	= 38	= 38	= 39	= 39
= 40	= 40	= 40	= 40	= 41	= 42
= 42	= 42	= 42	= 43	= 44	= 44
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= 47	= 48	= 48	= 48	= 48	= 48
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= 82	= 83	= 84	= 84	= 84	= 84
= 84	= 84	= 85	= 85	= 86	= 86
= 87	= 87	= 88	= 88	= 88	= 88
= 89	= 90	= 90	= 90	= 90	= 90
= 90	= 91	= 91	= 92	= 92	= 92
= 93	= 93	= 94	= 94	= 95	= 95
= 96	= 96	= 96	= 96	= 96	= 96
= 97	= 98	= 98	= 98	= 99	= 99
= 99	= 100	= 100	= 100	= 100	= 100

Multiples Chart

-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81
-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72
-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63
-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54
-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45
-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36
-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27
-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18
27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36
45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45
54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54
63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63
72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72
81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81

1. Find the length and width of a rectangle that has a perimeter of 132 units and an area of 1025 square units?

2. Find the length and width of a rectangle that has a perimeter of 112 units and an area of 703 square units.

3. Find the length and width of a rectangle that has a perimeter of 140 units and an area of 1176 square units.

4. Find the length and width of a rectangle that has a perimeter of 152 units and an area of 1323 square units.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
f(x)														
g(x)														
h(x)														

$f(x) = 2x + 10$

$g(x) = 3x + 3$

$h(x) = f(x) \cdot g(x)$

C N T

1( ) + 0( ) + 0( ) =

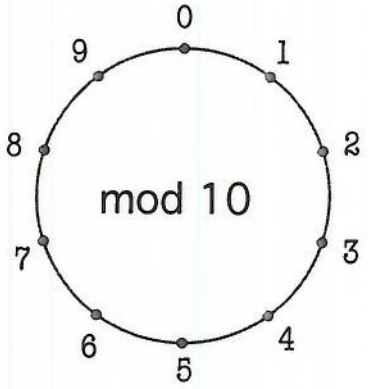
1( ) + 1( ) + 0( ) =

1( ) + 2( ) + 1( ) =

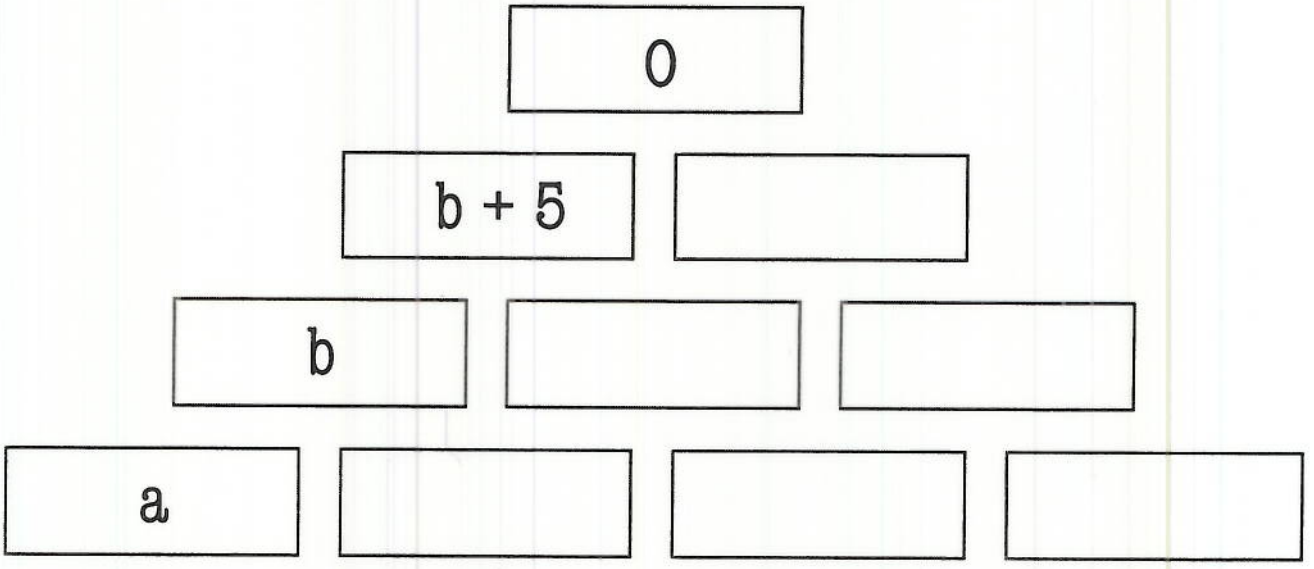
$A = \frac{T}{2}, \quad B = N - A$

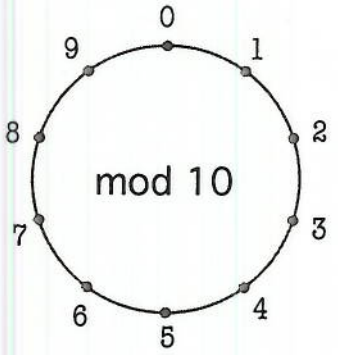
x	h(x)	Δ	Δ^2
0			
1			
2			
3			
4			

$h(x) = (2x + 3)(3x + 3) = Ax^2 + Bx + C =$



$$a - a + a - a + b - b + b + b - 2b + 3b + c - c + c - 2c + c - 3c =$$

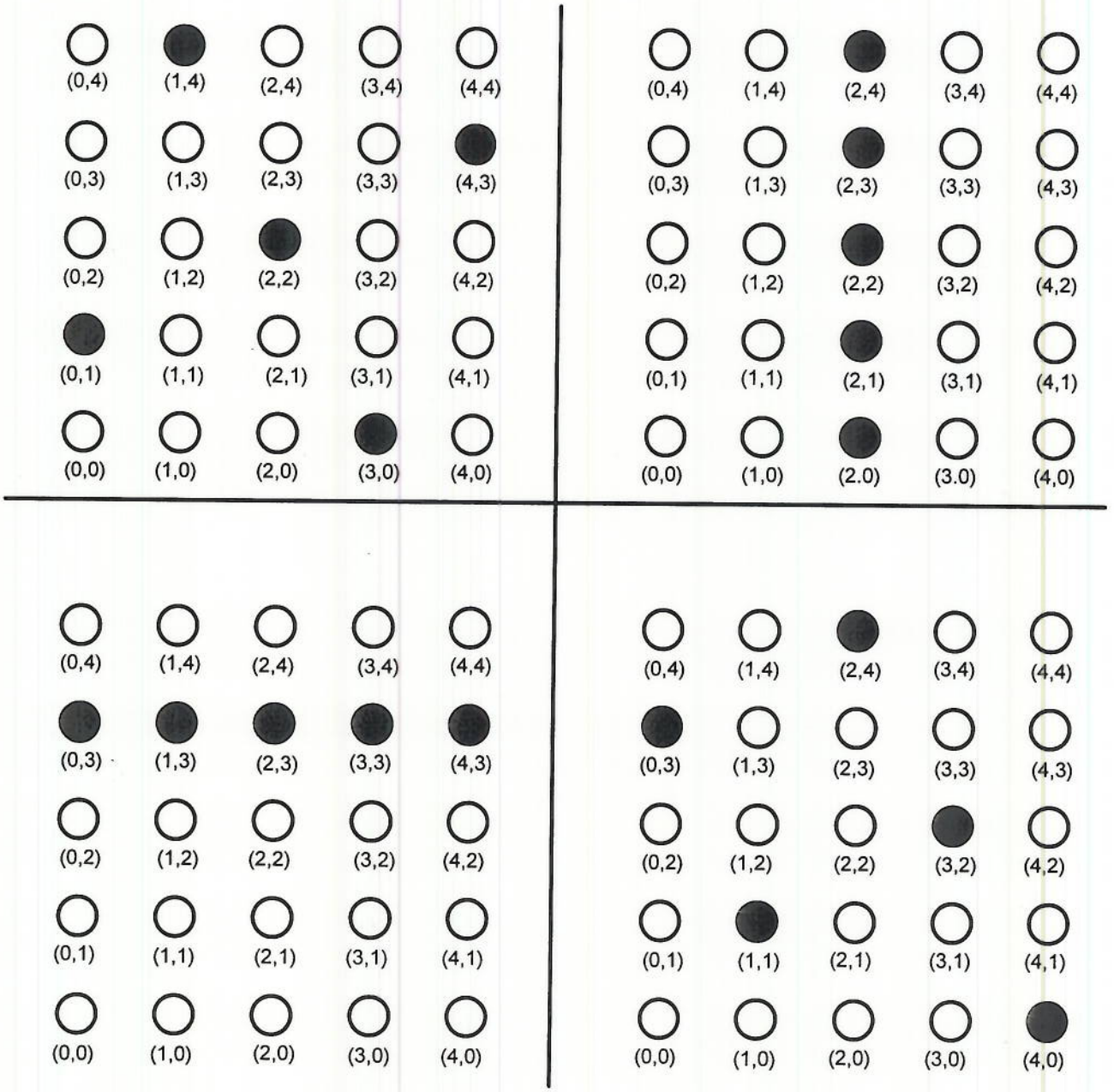


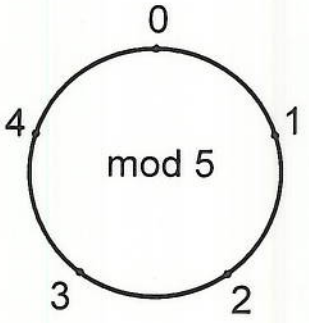


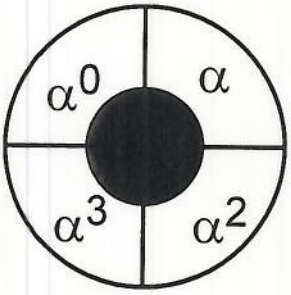
		0		
	2a		-2a	
2a + 5		5		- (2a + 5)
a	a + 5		-a	-a + 5

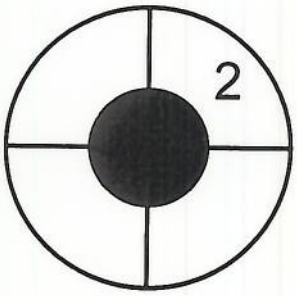
Substitute 3 for a in the triangle above to complete the triangle below.

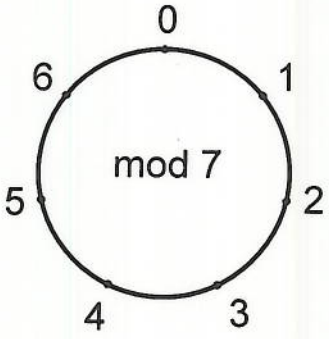
		0		
		5		
3				

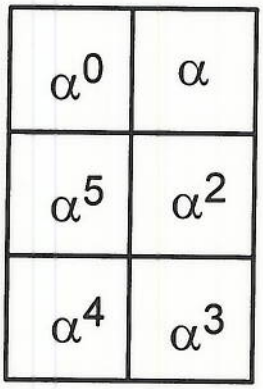

(0,4) (1,4) (2,4) (3,4) (4,4)
(0,3) (1,3) (2,3) (3,3) (4,3)
(0,2) (1,2) (2,2) (3,2) (4,2)
(0,1) (1,1) (2,1) (3,1) (4,1)
(0,0) (1,0) (2,0) (3,0) (4,0)
(0,4) (1,4) (2,4) (3,4) (4,4)
(0,3) (1,3) (2,3) (3,3) (4,3)
(0,2) (1,2) (2,2) (3,2) (4,2)
(0,1) (1,1) (2,1) (3,1) (4,1)
(0,0) (1,0) (2.0) (3.0) (4,0)
(0,4) (1,4) (2,4) (3,4) (4,4)
(0,3) (1,3) (2,3) (3,3) (4,3)
(0,2) (1,2) (2,2) (3,2) (4,2)
(0,1) (1,1) (2,1) (3,1) (4,1)
(0,0) (1,0) (2,0) (3,0) (4,0)
(0,4) (1,4) (2,4) (3,4) (4,4)
(0,3) (1,3) (2,3) (3,3) (4,3)
(0,2) (1,2) (2,2) (3,2) (4,2)
(0,1) (1,1) (2,1) (3,1) (4,1)
(0,0) (1,0) (2,0) (3,0) (4,0)

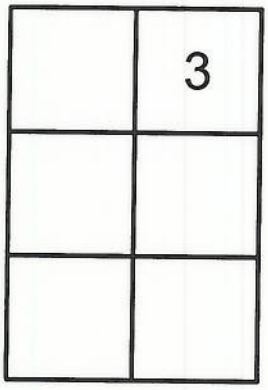

0
1
2
3
4
mod 5

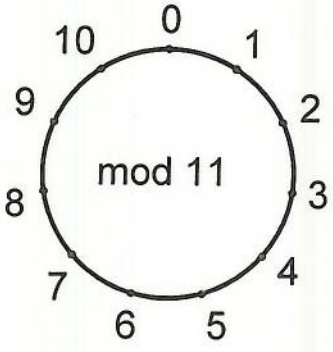

α^0
α
α^3
α^2

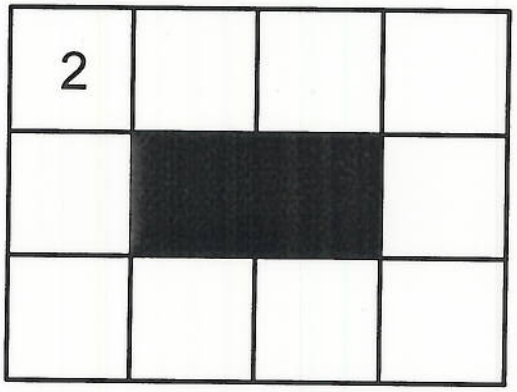

2

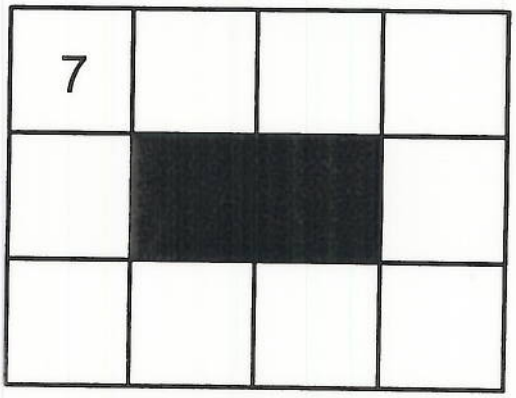

0
1
2
3
4
5
6
mod 7


α^0
α
α^5
α^2
α^4
α^3

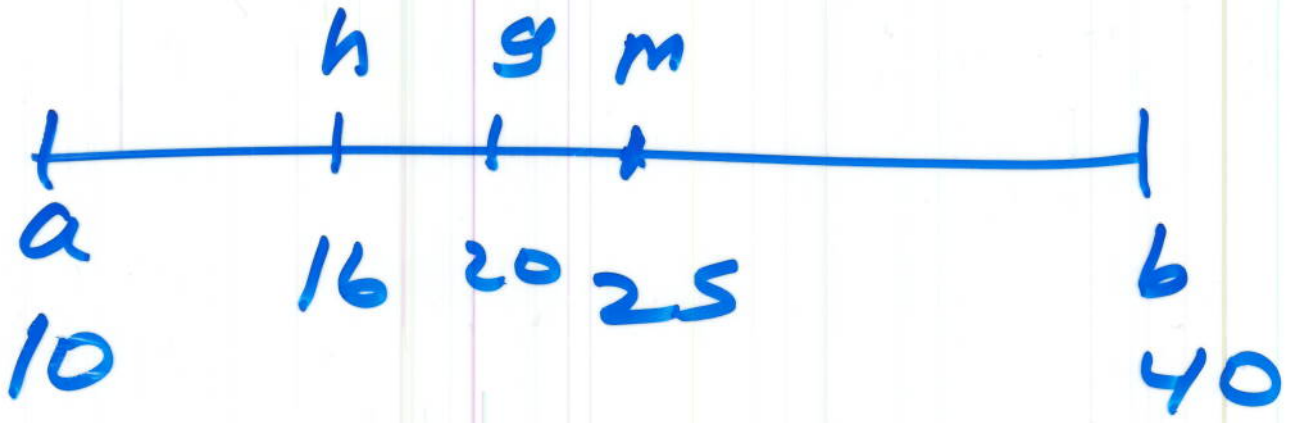

3

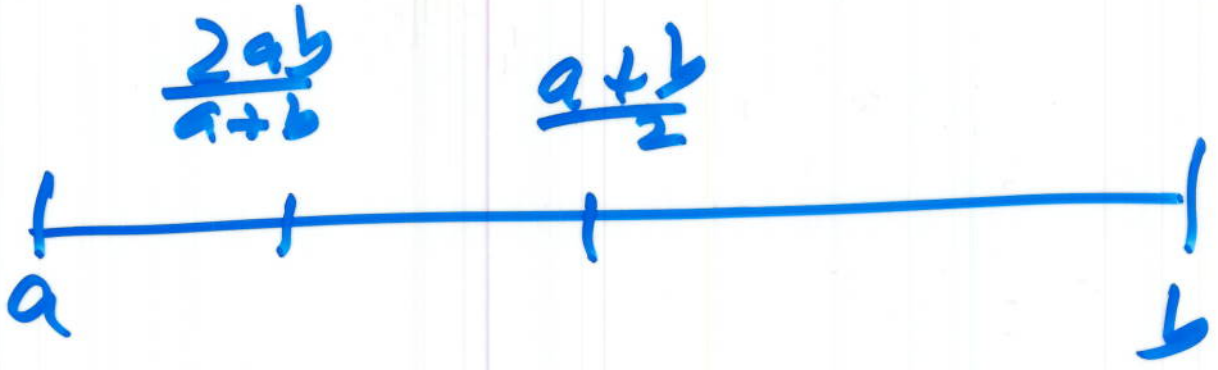

0
1
2
3
4
5
6
7
8
9
10
mod 11


2


7

$$mh = ab$$





$$\frac{ab + ba}{a+b}$$

Fourth of July Timelines

$\frac{a+b}{2}$

9:28 a.m. — 7:42 p.m.

$a + \frac{b-a}{2}$

11:14 a.m. — 7:36 p.m.

6:57 a.m. — 9:29 p.m.

$\frac{2a+b}{3}$

4:12 a.m. — 9:18 p.m.

$a + \frac{b-a}{3}$

4:35 a.m. — 9:23 p.m.

11:15 a.m. — 10:21 p.m.

Ivan Niven

Bob Wirtz

Eugene Novak – Margo Nanny

Dave Logothetti

B.C.

The Wizard of Id

Anneli Lax

Zalmen Usiskin

Dick Stanley

Martin Gardner

John Fletcher

Joe Buhler

George Polya

Jean Pedersen & Peter Hilton

Uri Treisman

Alfred Posamentier