Navigating Standards: Teacher and Student Learning through Different Standards Paths

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Standards = Instructional Roadmap

- Standards provide a "roadmap" of student learning of mathematics.
- Textbooks are written according to the roadmap.
- Teachers teach using the roadmap.
- Student learning is framed by the roadmap.
- Teachers understand their student learning of mathematics as guided by the roadmap.

Today's Presentation

Compare and contrast California Content Standards, Common Core Standards, and Japanese Standards (Course of Study)

- Highlight different ways they frame student learning
  - Focus topic: Whole number addition and subtraction
- Examples from textbooks for different content treatments

Consider Standards as Instructional Roadmap

- Ways to support teachers as they navigate student learning on the path

Whole-Number Addition and Subtraction: Standards Categories

California Content Standards	Common Core Standards	Japanese Course of Study
Grades K – 4th	Grades K – 4th	Grades 1 - 3
• Number Sense • Algebra and Functions	• Operations and Algebraic thinking • Number and operations in base ten • Measurement and Data	• Numbers and operations • Quantitative Reasoning

Standards across Years

	California Content Standards		Common Core Standards		Japanese Course of Study	
	Add/Sub	*Total*	Add/Sub	*Total*	Add/Sub	*Total*
K		*18*		*22*		*--*
1st		*29*		*20*		*17*
2nd		*35*		*26*		*21*
3rd		*49*		*25*		*32*
4th		*54*		*28*		*31*

Whole-Number Addition and Subtraction: Emphasis in Early Years

	California Content Standards		Common Core Standards		Japanese Course of Study	
	Add/Sub	*Total*	Add/Sub	*Total*	Add/Sub	*Total*
K	1	*18*	5	*22*	--	--
1st	10	*29*	11	*20*	5	*17*
2nd	6	*35*	9	*26*	4	*21*
3rd	1	*49*	1	*25*	3	*32*
4th	1	*54*	1	*28*	0	*31*

Whole-Number Addition and Subtraction: Goals

	California Content Standards		Common Core Standards		Japanese Course of Study	
	Add/Sub	*Total*	Add/Sub	*Total*	Add/Sub	*Total*
K	1	*18*	5	*22*	--	--
1st	10	*29*	11	*20*	5	*17*
2nd	6	*35*	9	*26*	4	*21*
3rd	1	*49*	1	*25*	3	*32*
4th	1	*54*	1	*28*	0	*31*
Last Standard	Demonstrate an understanding of, and the ability to use, **standard algorithms** for the addition and subtraction of multi-digit numbers.		Fluently add and subtract multi-digit whole numbers using the **standard algorithm.**		Understand the procedures of **vertical algorithm**.	

Examples of Standards (Grade 1): Whole Number Addition and Subtraction

California Content Standards	Common Core Standards	Japanese Course of Study
Know the addition **facts** (sums to 20) and the corresponding subtraction **facts** and commit them to **memory**	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10, **use strategies** such as counting on, making ten, decomposing a number leading to a ten; **using the relationship** between addition and subtraction; and creating equivalent but easier known sums	**Consider/invent the methods** for addition and subtraction using two single-digit numbers and perform the operations successfully.
Grade 1; Number Sense 2.1	Grade 1; Operations and Algebraic Thinking 6	Grade 1; Number and Operations 2a

California Content Standards:
Focus on End Product and Fluency

End Product and Fluency (Knowing the destination)

- K: Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10).
- G1: Solve addition and subtraction problems with one and two digit numbers (e.g., 5+58=___).
- G2: Find the sum or difference of two whole numbers up to three digits long.
- G3: Find the sum or difference of two whole numbers between 0 and 10,000.
- G4: Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers.

Common Core Standards:
Focus on Process and Strategies

Process and Strategies (Multiple alternate options)

- K: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equations (e.g., 5=2+3 and 5=4+1).
- G1: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10, use strategies such as counting on, making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.
- G2: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- G3: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- G4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Japanese Course of Study: *Invent, Consider, and Extend*

Invent, Consider and Extend (Flexibility in rerouting paths)

- G1: Consider/invent the methods for addition and subtraction using two single-digit numbers and perform the operations successfully.
- G2: Consider/invent methods for addition and subtraction as the reverse process of addition, understand that they could use the single-digit addition and subtraction methods as foundation for two-digit operations, accurately follow the procedures, and understand the vertical algorithm.
- G3; Consider/invent ways to add and subtract three-digit and four-digit numbers and understand that they could use the aspects of two-digit number operations as foundations for it, and understand the procedures of vertical algorithm.

General Themes for Instructional Trajectories

California Content Standards	Common Core Standards	Japanese Course of Study
End Product and Fluency	Process and Strategies	Invent, Consider, and Extend

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Product and Fluency	Process and Strategies	Invent, Consider, and Extend
Knowing the Destination	Multiple Alternate Road Options	Flexibility in Rerouting the Paths

General Themes for Instructional Trajectories

California Content Standards	Common Core Standards	Japanese Course of Study
Product and Fluency	Process and Strategies	Invent, Consider, and Extend
Knowing the Destinations	Multiple Alternate Road Options	Flexibility in Rerouting the Paths
• **Destination address** • General knowledge of the driver is crucial	• **Online map program** • Multiple possibilities are given, and the driver is informed in advance	• **GPS** • As the trip unfolds, the driver decides and navigates the road, depending on the road/traffic challenges

Structure of Japanese Course of Study

For each grade level section:

- Overall goals for the grade level
- Content goals
- Mathematical activities
  - Sample activities to support student learning
- Vocabulary and terms

Examples in Textbooks: Grade 2, Whole Number Addition and Subtraction

California Content Standards	Common Core Standards	Japanese Course of Study
Understand and use the **inverse relationship between addition and subtraction** (e.g., an opposite number sentence for 8+6=14 is 14-6=8) to solve problems and check solutions.	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the **relationship between addition and subtraction.**	Think/invent methods for addition and **subtraction as the reverse process of addition.** They will understand that they could use the single-digit addition and subtraction methods as a foundation for two-digit operations ...
Grade 2: Number Sense 2.1	Grade 2: Number and Operations in Base Ten 5	Grade 2: Number and Operations 2a

Instructional Approach in Japanese Textbook: Inverse Relationships between Addition and Subtraction (G2)

"There are 17 blue ribbons and 24 red ribbons, whose total is 41. Make a math problem using the situation."

[Unit **"Addition and Subtraction"** (Study with Friends Mathematics, Volume 1)]

Instructional Approach in Japanese Textbook: Inverse Relationships between Addition and Subtraction (G2)

"There are 17 blue ribbons and 24 red ribbons, whose total is 41. Make a math problem using the situation."

[Unit **"Addition and Subtraction"** (Study with Friends Mathematics, Volume 1)]

There are 17 blue ribbons and 24 red ribbons. How many are there together?	There are 41 ribbons. 17 of them are blue. How many red ribbons are there?
Total: ☐ ribbons	Total 41 ribbons
	
17 blue ribbons 24 red ribbons	17 blue ribbons ☐ red ribbons
Number sentence: 17 + 24	Number sentence: 41 - 17

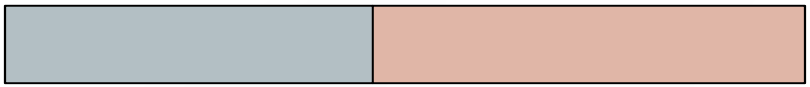
Instructional Approach in Japanese Textbook: Inverse Relationships between Addition and Subtraction (G2)

Instructional Highlights

- Sharing of problems and solution methods
- Discussion of similarity and differences among problems
- Discussion of the operations used (addition and subtraction) and their relationships in the problems and solutions
- Focus on representations of situations and problem solutions

There are 17 blue ribbons and 24 red ribbons. How many are there together?

Total: ☐ ribbons



17 blue ribbons 24 red ribbons

Number sentence: 17 + 24

There are 41 ribbons. 17 of them are blue. How many red ribbons are there?

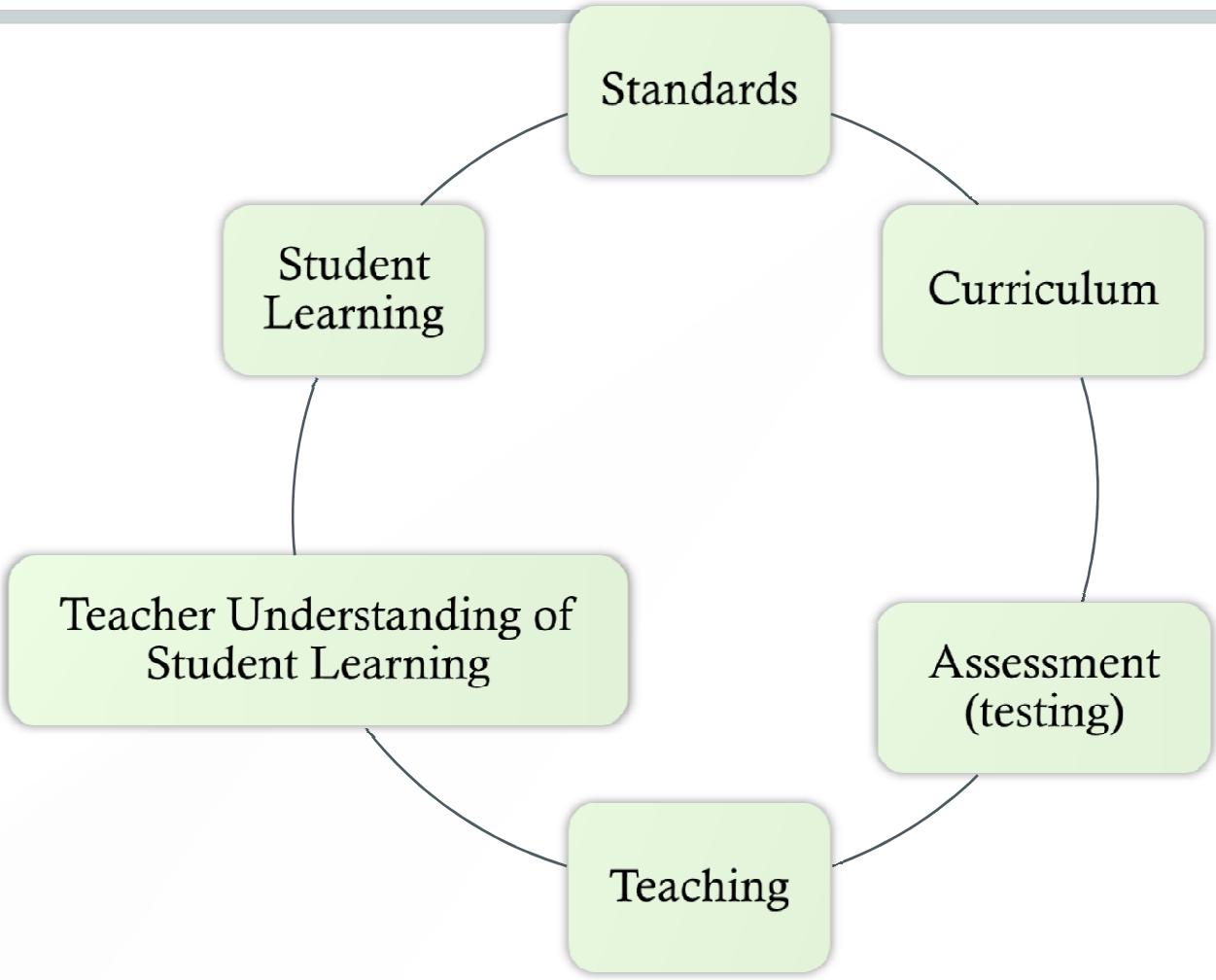
Total 41 ribbons

17 blue ribbons ☐ red ribbons

Number sentence: 41 - 17

Instructional Approach in U.S. Textbook: Inverse Relationships between Addition and Subtraction (G2)

Mathematics Education as a System



Standards = Instructional Roadmap

How do we make the instructional roadmap more flexible and interactive with student differences (like GPS)?

Japanese teaching context

- Textbooks/manuals provide detailed guidelines
  - Research, assessment, and instruction sections/volumes
  - Focus on coherence, integrity, and general patterns of student learning and thinking with flexibility
- Lesson study
  - Support for teacher learning in the professional community
  - Teachers as important part of decision making system
- Multi-grade teaching
  - Knowledge of and interest in student learning over time

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