Critical Issues in Mathematics Education

Math as Arbitrary Rules

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"Teachers are coming to believe that pupils can and should understand their arithmetic operations."

- Textbooks
- Not choice of content
- Not math errors
- Making sense
- Using mathematical reasoning
- If no sense, we teach by rote.

- "Teachers are coming to believe that pupils can and should understand their arithmetic operations."
- Grade 5, Teacher's Edition, Growth in Arithmetic
- 1956, Small Town Kansas
- No one alive should remember rote learning
- Second grade teacher (and Third grade)

Fractions

- Fractions start as parts of Pizzas or cookies
- But always as "fraction of"
- (to mathematicians: element of dual of reals)
- Fractions are difficult sophisticated mathematics
- Even at the 4th and 5th grade level
- Ultimately, fractions are numbers, not pizzas

The Transition to numbers

- One page it's ½ pizza + ¼ pizza = ¾ pizza
- The next page, it's ½ + ¼ = ¾
- Suddenly the pizzas are gone
- But that doesn't mean we have defined fractions as numbers
- They just hang there without sense
- You can't multiply pizzas!

Adding Fractions

- Common denominators
- Least common multiple of denominators
- 1/3 + 1/5
- 3,6,9,12,15,18,21
- 5,10,15,20
- Aha! 15, 1/3 + 1/5 = 5/15 + 3/15 = 8/15.
- Try 1/17 – 1/19, makes no sense.

Dividing fractions

- Typical 6th grade textbook:
- 3 examples: 1 divided by 1/5,
- ¾ divided by 3/8, and 3 divided by ½
- Then, presto, the rule:
- To divide by a fraction, multiply by its reciprocal.
- Not enough sense!

Understanding, sense, reasoning

- All missing!
- Rules stated without reasoning to make sense of them is rote learning.

Ratios

"A ratio is a special type of comparison of two numbers or measures."

Ratios are senseless!

"A rate compares two quantities that have different units of measure."

Linear equations and graphs

- No one shows that the graph of Ax+By=C is a line.
- No one shows that the equation for a line in the plane is of the form Ax+By=C.
- No one shows slope is well-defined.
- Few show m in y=mx+b is the slope.
- Everything is just stated as facts, i.e. no explanations.
- I found one book that says these things should be proven.

Slope

- I found 2 books that showed slope is well-defined.

- Japan and Singapore didn't do it.

- One book that did it was the old "new math" book (SMSG).

- The other was the book my kid used.

quadratics

- Second differences with no explanation
- Parabolas are defined as graphs of quadratics
- Some texts love symmetry
- "Parabolas are symmetric about a central line called the axis of symmetry."
- End of story.
- Some show y=x squared has symmetric graph.

Proof of symmetry and max/min

$$y = ax^2 + bx + c$$

- Complete the square

$$y = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a^2}$$

- Easy to see symmetry and max/min
- I found 2 books that just did this, no more.

My 1962 Algebra II book

- Stuck –b/2a in $\quad y = ax^2 + bx + c$
- Then –b/2a + k
- The difference is $\quad ak^2$
- This shows both symmetry and max/min
- It is more algebraic manipulation than most texts today.
- Important for STEM fields.
- Too late to start in pre-calculus.

Area of parallelogram

- I checked 12 high school geometry books
- Including Singapore and Japan
- Singapore and Japan didn't do it completely.
- 3 did it, including
- The ancient "new math" book

Conclusion

- Too much fundamental mathematics is stated as fact without explanation.

- They are just arbitrary rules.

- There is no mathematical reasoning to make sense of them.

- If textbooks today are to be believed, contrary to what my 1950s textbooks say, rote learning is alive and well.