Title for my talk in the workshop "Combinatorial Commutative Algebra and Applications" December 3 7, 2012

On the stable set of associated prime ideals of a monomial ideal

Abstract: By a classical result of Brodmann it is known that in any Noetherian ring, the set of associated prime ideals $\mathrm{Ass}(I^s)$ for the powers of an ideal I stabilizes for $s \gg 0$. In other words, there exists an integer s_0 such that $\mathrm{Ass}(I^s) = \mathrm{Ass}(I^{s+1})$ for all $s \geq s_0$. This *stable set of associated prime ideals* is denoted by $\mathrm{Ass}^\infty(I)$. The smallest integer s_0 such that $\mathrm{Ass}(I^s) = \mathrm{Ass}(I^{s+1})$ for all $s \geq s_0$ is called the *index of stability*.

In this lecture we discuss the following questions:

(i) Which finite sets of monomial prime ideals are of the form $\mathrm{Ass}^\infty(I)$ for a suitable (squarefree) monomial ideal I?

(ii) Is there a global bound of the index of stability?

It can be shown that for any finite set $\mathcal{P}$ of non-zero monomial prime ideals there exists a monomial ideal I such that $\mathcal{P} = \mathrm{Ass}^\infty(I)$. However, an answer to question (i) in the squarefree case is widely open. We give explicit descriptions of $\mathrm{Ass}^\infty(I)$ for certain classes of matroidal and polymatroidal ideals.

There is no example known of a monomial ideal in the polynomial ring in n variables whose index of stability is $\geq n$. Thus we expect that this index is always $< n$. We show that this is indeed the case for any polymatroidal ideal.

The subjects of the lecture summarize results in joint papers with Bandari, Bayati, Hibi, Rauf, Rinaldo, Vladoiu and Qureshi.