MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Elizabeth Gross **Email/Phone:** egross7@uic.edu

Speaker's Name: Anne Shiu

Talk Title: Extensions of Birch's Theorem, with applications to dynamical systems

Date: 12 / 3 / 12 **Time:** 3 : 30 am / pm (circle one)

List 6-12 key words for the talk: chemical reaction networks, endotactic networks, Birch's Theorem, persistence, global attractor conjecture, polytopes

Please summarize the lecture in 5 or fewer sentances: Introduces & explores the problem of persistence for chemical reaction systems. Using a generalization of Birch's theorem, shows that strongly endotactic (inward-"pointing") networks are persistent.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Extensions of Birch's Thm,
with applications to dynamical systems

[Dec. 3, 2012
MSRI -
Combl. Comm. Alg.]

Anne Shiu
(jt. w/ M. Gopalkrishnan + E. Miller)

- SPEAK SLOWLY! -

Outline: (I) Persistence (main result)
(II) Algc./combl. ideas that go into the proof

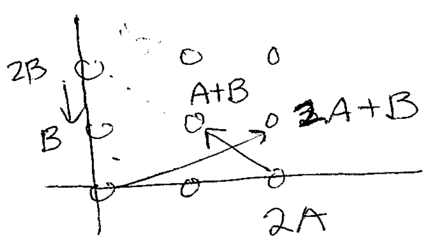
(I) ex: (chemical reaction)

$A+B \longrightarrow C$

(reactant) (product)

species: A, B, C

ex: (chemical reaction network)



(2-d: will return)

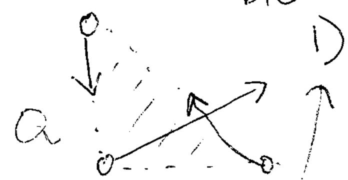
Question (studied since 1970s: Horn, Jackson, Feinberg)

Which networks are:
1) bistable?
2) convergent to a ! st. ste?
* 3) persistent?

(def) given any species i & any trajectory $(x_1(t), \dots, x_s(t))$ of the netwk.,
lim inf $x_i(t) > 0$

Main Thm - Strongly endotactic ("inward-pointing") networks are persistent.

ex/defn: the following net. is str. end.



b/c
1) for each face F* of cvx hull of reactants (i.e. Q), there is a rxn. ~~from F~~ that points ~~from F into Q~~ out of F (into Q)
2) no rxn pts. out of Q

*: as long as F maxs. some $\langle w, - \rangle$ where w is not $\perp$ to all rxns.

ex: this net. is endotactic but not str. end. (stsfs. only 2)

Conj. (Craciun, Nazarov, Pantea) - Endotactic networks are persistent.

Rmk: This gens. the long-standing Global attractor conjecture (Horn 1972). (rel. to Q2.)

(Remainder: dyns. + ideas of pf.)

(II) Three main ideas in the proof: (of our thm)

(1)

$\boxed{\text{Lemma A}}$ (usual Ly. fnc. is Ly.-like)

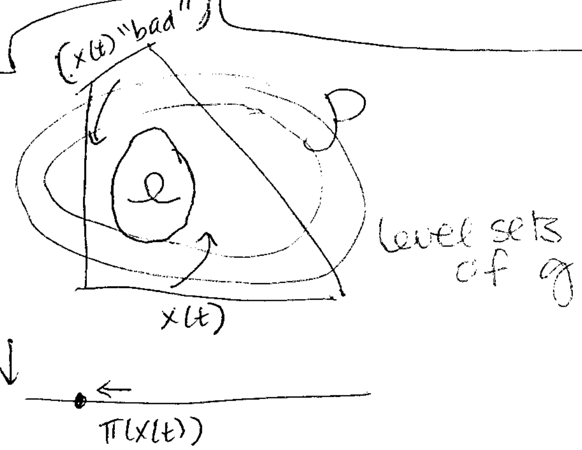
Fix a strongly endo. net., fix rate constants, fix $\mathcal{P}$. Then

$$g(x_1, \dots x_s) := \sum_{i=1}^{s} x_i(\log x_i - 1)$$

decrs. along trajs. outside a cpc. set $\Omega \subseteq \mathrm{int}(\mathcal{P})$,

i.e. $\frac{d}{dt}\, g(x(t)) < 0$ whenever $x(t) \in \mathrm{int}(P) \setminus \Omega$.

Pic:



Pf ~~idea~~ outline: (strengthens technique of Anderson '11.)

- Take seq. $x(i) \in \mathcal{P}$ going to b'dy (or ∞)
- Write (compt.-wise) $\log x(i)$ in polar coords.:

$$\log x(i) = \underbrace{r(i)}_{\to\infty}\ \underbrace{\underline{w}(i)}_{\in S^{s-1}}$$

$$= (\log \theta(i))\ \underline{w}(i)$$

i.e. $x(i) = \theta(i)^{\underline{w}(i)} := (\theta(i)^{w_1(i)}, \dots, \theta(i)^{w_s(i)})$

- Take subseq. so; $\underline{w}(i)$ convgs. to some $\underline{w}^*$

Al. stat?! → "at ∞"

tropical

- Check: the rxns. that dom. the ODEs along $x(i)$ are those in the face F of $Q := \mathrm{cvx}(\text{rcts.})$ that maximize $\langle w^*, - \rangle$
- Check: By defn. of str. end.,
  1) no rxn. pts. out of Q (no dominant pos. term in $\frac{d}{dt} g(x)$)
  2) $\exists$ rxn. from F into Q (dom. neg. term in $\frac{d}{dt} g(x)$) (none in endo case!)
- Thus $\frac{d}{dt} g < 0$ eventually.
- Check: This can be done uniformly to get Ω. (w^*: 1st order dir., $w^{*\prime}$: 2nd...) ☒

(2)

(def. of str. en.)

Potential pblm: what if ω^* is $\perp$ to all rxns. (i.e. $\omega^* \in S^{\perp}$)?

("I think this is B's Thm")

Birch's Thm: Let $S \subseteq \mathbb{R}^s$ be a subsp., let $x_0 \in \mathbb{R}^s_{>0}$, let $\mathcal{P} := (x_0 + S) \cap \mathbb{R}^s_{\geq 0}$. Then:

$$(*)\ \underbrace{\left\{ \theta^{\underline{w}} \;\middle|\; \begin{array}{l} \underline{w} \in S^{\perp} \\ \theta > 0 \end{array} \right\}}_{\text{lim of pts. here}} \cap \mathcal{P} = \left\{ \begin{array}{l} \text{the !} \\ \text{Birch} \\ \text{point} \end{array} \right\}.$$

note: 1) This gives $\exists$-ce/!-ness of c-bal. st. stes. (Def. Zero Thm.)

2) This gives $\exists$-ce/!-ness of MLE for log-linear models (Birch 1963).

(Translation: matrix A that defns. model: $S = \ker A$.)

common to SPS talk

[extns. of B's Thm]

$\boxed{\text{Lem. B}}$ - (*) still holds if closures of two sets of LHS taken in $[0,\infty]^s$,

i.e. Pot. pblm. never occurs.

③ → explain "bad" pblm ("historical") via picture.

$\boxed{\text{Lem C}}$ – (informally) – generalizes $\boxed{\text{Pantea 2012}}$

[Trajectories near the b'dy can be projected to lower-dim'l. systs.,
so persistence can be proven by induction.]

Main msg: – tropical / geo.
– two p'topes } (language!)
– complement to $\exists$-g. dyn. systs. approach

1616 Walnut St

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