Toric algebra of hypergraphs

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The edges of any uniform hypergraph parametrize a monomial algebra called the edge subring of the hypergraph. We study presentation ideals of these edge subrings, and describe their generators in terms of balanced walks on hypergraphs. Our results generalize those for the defining ideals of edge subrings of graphs, which are well-known in the commutative algebra community, and popular in the algebraic statistics community. One of the motivations for studying toric ideals of hypergraphs comes from algebraic statistics, where generators of the toric ideal give a basis for random walks on fibers of the statistical model specified by the hypergraph. Further, understanding the structure of the generators gives insight into the model geometry.