MSRI
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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Elizabeth Gross **Email/Phone:** egross7@uic.edu

Speaker's Name: Hidefumi Ohsugi

Talk Title: Cut ideals & their application to regular designs in statistics

Date: 12 / 4 / 12 **Time:** 3 : 30 am / (pm) (circle one)

List 6-12 key words for the talk: cut ideals, toric ideals, normality, regular design, algebraic statistics, cut polytope

Please summarize the lecture in 5 or fewer sentances:
Survey results on cut ideals and shows that if G has no K_5-minor then $K[G]$ is normal. Applies the study of cut ideals to the problem of regular designs in statistics.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

$G = (V, E)$: finite graph

$(V = \{1, 2, \ldots, d\})$

$\mathcal{P}(V) := \{ A \mid B \mid A \cup B,\ A \cap B = \emptyset \}$

$(|\mathcal{P}(V)| = 2^{d-1})$

$K[q] = K[\, q_{A|B} \mid A|B \in \mathcal{P}(V)]$

$K[s,t] = K[\, s_{ij}, t_{ij} \mid \{i,j\} \in E]$

For each $A|B \in \mathcal{P}(V)$

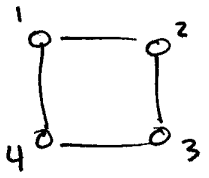
$\mathrm{Cut}(A|B) = \{ \{i,j\} \in E \mid \begin{matrix} i \in A,\ j \in B \\ \text{or}\ \ j \in A,\ i \in B \end{matrix} \}$

$\phi_G : K[q] \to K[s,t]$

$$q_{A|B} \longmapsto \prod_{\{i,j\} \in \mathrm{Cut}(A|B)} s_{ij} \prod_{\{i,j\} \notin \mathrm{Cut}(A|B)} t_{ij}$$

Example

$G =$



ϕ_G :

$q_{\emptyset|1234} \mapsto t_{12} t_{23} t_{34} t_{14}$

$q_{1|234} \mapsto s_{12} t_{23} t_{34} s_{14}$

$q_{21|34} \mapsto s_{12} s_{23} t_{34} t_{14}$

$q_{31|24} \mapsto t_{12} s_{23} s_{34} t_{14}$

$q_{41|23} \mapsto t_{12} t_{23} s_{34} t_{14}$

$q_{12|34} \mapsto t_{12} s_{23} t_{34} s_{14}$

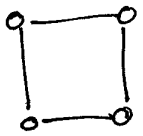
$q_{13|24} \mapsto s_{12} s_{23} s_{34} s_{14}$

$q_{14|23} \mapsto s_{12} t_{23} s_{34} t_{14}$

$I_G := \ker(\phi_G)$ cut ideal of G

$K[G] := \mathrm{Im}(\phi_G) \cong K[q]/I_G$

Example

$G =$ 

$I_G = \langle a-b, a-c, a-d \rangle$

$a = q_{\emptyset|1234} \cdot q_{13|24}$

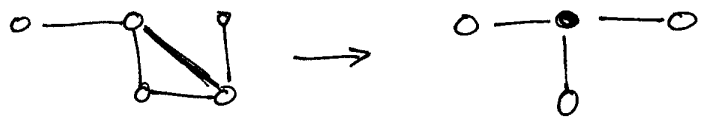
$b = q_{1|234} q_{3|124}$

$c = q_{2|134} q_{4|123}$

$d = q_{12|34} q_{14|23}$

Minors

- Edge contraction

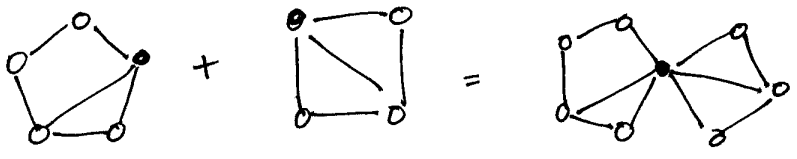


- Edge deletion

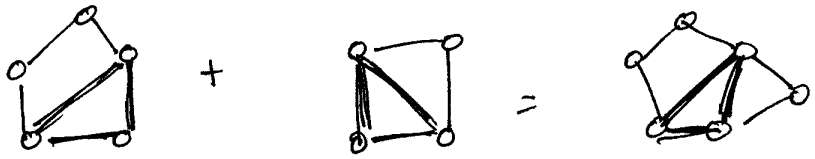
A graph H is a minor of a graph G
$\iff$ H can be obtained from G by deleting and/or contracting

Clique Sum

0-sum



2-sum



Sturmfels-Sullivant

① $G \rightsquigarrow G'$ contracting an edge

Then, $\mathrm{Cut}^{\square}(G')$ is a face of $\mathrm{Cut}^{\square}(G)$

↑ Cut polytope of G'

G satisfies (◎), then G' satisfies (◎)

↑ here can put:
$K[G]$ is normal
$K[G]$ is Kozul
I_G is generated by binomials of degree $\leq m$ $(m \geq 2)$

② $G = G_1 \# G_2$ $(0, 1, 2\text{-sum})$

Then, I_G is the "toric fiber product" of I_{G_1} & I_{G_2}.

G_1 & G_2 satisfy ——— $\Rightarrow$ G satisfies ———

↑ normal
I_G is generated by degree $\leq m$

Fact

If graph G has K_n-minor (← complete graph), then K_n is obtained from G by only contraction.

- I_G is generated by quadratic binomials $\Rightarrow$ G has no K_4-minor
  $\Leftarrow$ (Engström)
- I_G is generated by binomials of degree ≤ 4 $\Rightarrow$ G has no K_5-minor
- $K[G]$ is normal $\Rightarrow$ G has no K_5-minor