

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Elizabeth Gross Email/Phone: egross7@uic.edu

Speaker's Name: Aldo Conca

Talk Title: Regularity for powers of ideals of maximal minors

Date: 12/6/12 Time: 11:30 (am/pm (circle one))

List 6-12 key words for the talk: Castelnuovo-Mumford regularity, powers of ideals, linear powers, maximal minor, rational normal scroll, linear resolutions
Please summarize the lecture in 5 or fewer sentences: _____

Explores the Castelnuovo-Mumford regularity of powers of homogeneous ideals. Shows that every rational normal scroll has linear powers.

CHECK LIST

(This is NOT optional, we will not pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Regularity for powers of ideals of maximal minors

①

Joint work w/ Bruns-Varbaro

See: arXiv: 1203.1176.

$$S = K[x_1, \dots, x_n]$$

I homogeneous $m = \max$ degree of generator of I

$$\begin{aligned} \text{reg } I &= \max \{ j-i : \beta_{ij}(I) \neq 0 \} \\ &= \max \{ j+i : H_m^i(I)_j \neq 0 \} \end{aligned}$$

$\text{reg } I$ bounds the degrees in Gröbner basis of I in generic coordinates w/ revlex order.

$$m \leq \text{reg } I$$

Def I has a m -linear resolution if $I = (I_m)$ $m = \text{reg } I$

$$\begin{array}{ccccccc} \rightarrow & F_2 & \rightarrow & F_1 & \rightarrow & F_0 & \rightarrow 0 \\ & & & & & S^{*}(-m-1) & \parallel \\ & & & & & & S(-m)^* \end{array}$$

Example

$$0 \rightarrow S(-3)^2 \rightarrow S(-2)^3 \rightarrow (x^2, xy, y^2) \rightarrow 0$$

linear

②

$$0 \rightarrow S(-n) \rightarrow S(-2)^2 \rightarrow 0 \quad \text{ideal } (x^2, y^2)$$

(non-linear)

Brodmann 1979

$$I \quad \{I^k\} \quad k \in \mathbb{N}$$

$$\text{Ass}(S/I^k) \text{ for } k \gg 0$$

constant

$$\text{Ass}(S/I^k) = \{P_1, \dots, P_s\} \quad \forall k \gg k_0$$

$$I^k = Q_{1k} \cap Q_{2k} \cap \dots \cap Q_{sk} \quad \text{primary decomposition}$$

$$\overline{Q_{ik}} = P_i$$

Swanson 1997

$\exists \alpha = \alpha(I)$ such that

$$\exists Q_{ik} \text{ s.t. } P_i^{k\alpha} \subseteq Q_{ik}$$

Corollary

$$\exists \beta = \beta(I) \text{ s.t. } \text{reg } I^k \leq \beta_k$$

Thm

(Herzog, Cutkosky-Trung, Kodiyalam)

$\exists a(I), b(I), c(I)$ such that

$$\text{reg } I^k = a(I)k + b(I) \quad \forall k \geq c(I)$$

Assume

$$I = (I_m)$$

Then $a(I) = m$

what about $b(I)??$ $c(I)??$

$$\mathcal{R}(I) = \bigoplus_{k=0}^{\infty} I^k = \left\{ \sum a_i t^i : a_i \in I^i \right\} \quad (3)$$

$\subseteq S[t]$

$$\mathcal{R}(I) = K[x_1, \dots, x_m, f_1 t, \dots, f_s t]$$

$$I = (f_1, \dots, f_s)$$

$$K[x, y] \xrightarrow{\phi} \mathcal{R}(I)$$

give degree $(1,0)$ to the x 's $\rightarrow x_i \mapsto x_i$
and $(0,1)$ to the y 's $\rightarrow y_i \mapsto f_i t$

$$\text{reg}_{(1,0)} \mathcal{R}(I) = \max \{ j-i : \beta_{i,(j,*)}(\mathcal{R}) \neq 0 \}$$

$$\text{reg}_{(0,1)} \mathcal{R}(I) = \max \{ j-i : \beta_{i,(*,j)}(\mathcal{R}) \neq 0 \}$$

[Römer] $\text{reg } I^k \leq km + \text{reg}_{(1,0)} \mathcal{R}(I)$

$\forall k$

$$b(I) \leq \text{reg}_{(1,0)} \mathcal{R}(I)$$

$$c(I) \leq \text{reg}_{(0,1)} \mathcal{R}(I)$$

If $\overline{TI} = m$ [Eisenbud - Ulrich] (2010)

Best possible behavior:

$$\text{reg } I^k = km \quad \forall k$$

$$b(I) = 0, c(I) = 1$$

linear powers

i.e. all the powers have a linear resolution

Thm TFAE

(4)

$$1) \operatorname{reg} I^k = km \quad \forall k$$

$$2) \operatorname{reg}_{(1,0)} \mathcal{R}(I) = 0 \quad \left. \begin{array}{l} \uparrow \text{Römer} \\ \downarrow \end{array} \right\} \text{B-C-V}$$

$$1) \dim S/I \leq 1$$

If $\operatorname{reg} I = m \Rightarrow$ linear powers

ex: 20 pts in \mathbb{P}^3

2) Strongly stable ideals

Polymatroidal ideals

Product of ideals of linear form

3) Herzog - Hibi - Zheng

I monomial of deg 2

$\operatorname{reg} I = 2 \Rightarrow$ linear powers

If $\operatorname{reg} I = m \not\Rightarrow \operatorname{reg} I^2 = 2m$

Can find I such that

$$\operatorname{reg} I^k = km \quad \forall k < d$$

$$\operatorname{reg} I^d > dm$$

Rational Normal Scrolls

I prime

$$\operatorname{reg} I = 2$$