MSRI
Mathematical Sciences Research Institute

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Elizabeth Gross **Email/Phone:** egross7@uic.edu

Speaker's Name: David Eisenbud

Talk Title: Twenty Points in $\mathbb{P}^3$

Date: 12 / 6 / 12 **Time:** 3 : 30 **am / pm (circle one)**

List 6-12 key words for the talk: Gorenstein linkage, glicci, complete intersection, Hilbert scheme, Rao invariant, computational algebra

Please summarize the lecture in 5 or fewer sentances:

Explores the problem of determining whether the ideal of a set of 20 general points in $\mathbb{P}^3$ is Gorenstein-linked to a complete intersection. Gives a positive result & outlines the ideas of the proof.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☐ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☐ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - **Computer Presentations**: Obtain a copy of their presentation
  - **Overhead**: Obtain a copy or use the originals and scan them
  - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - **Handouts**: Obtain copies of and scan all handouts
- ☐ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☐ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☐ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

Twenty Points in $\mathbb{P}^3$

work with Hartshorne & Schreyer.

Peskini-Szpiro 1974. $I \supset_K C J \subset S = K[x]$

K complete intersection

$(K : I) = J$, $(K : J) = I$

Schenzel 82 - Take K Gorenstein.

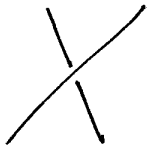
Rao 1978 $S = k[x_0, \dots, x_3]$

$\cup$

I codim 2

Rao invariant $Ext^3(S/I, S)$

Example



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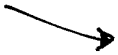

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Rao invariant = k.

Let S/I be Cohen-Macaulay.

$$0 \to F_c \to \cdots \to F_1 \to S \to S/I$$

$$0 \to S \to \cdots \wedge^2 S^c \to S^c \to S \to S/K$$

$$S/J \leftarrow S^* \leftarrow F_c^* \leftarrow \bigoplus S^c$$

Huneke - Ulrich - generic links of ideals

If I is Cohen - Macaulay & the minimum degree of $F_c \leq (c-1)d$ where $d = \max F_1$

$\Rightarrow$ I is not licci

Conjecture If $I, J \subset k[x_0, \ldots, x_n]$ are CM of same codim $\Rightarrow$ linked by Gorenstein ideals.

Survey 2001, Kleppe - Migliore - Miro-Roig - Nagel - Peterson.

Hartshorne's Question 1: Is a set of 20 general points in $\mathbb{P}^3$ glicci? (2001)

thm (E - Hartshorne - Schreyer) If $1 \leq d \leq 33$ or $d = 37, 38$ $\Rightarrow$ general set of d points is glicci.

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k. V is a projective variety

Problem find $P \in V(k)$

If $k = \mathbb{Q}$ - then no.

If $k = \mathbb{F}_q$ $C \subset \mathbb{P}^2$

<u>Proposition</u> At least 1/2 the lines in $\mathbb{P}^2$ have a pt of $C(k)$ on them.

$\# C(k) = q + 1 - a \qquad |a| \leq 2g\sqrt{q}$

$(p, L \mid p \in L) \quad q \sim q^2$

$\swarrow \qquad \searrow$

$p = C(k) \qquad \mathbb{P}^2$

$q \qquad q^2$

~~Theorem (Deligne, Katz)~~

Aside: $\Gamma \subset L = \mathbb{P}^1$ $\quad \Gamma \supset$ subset of d' pts def $/k$

d) reduced $\qquad$ if $P \supset$ pts ...

<u>Theorem</u> (Deligne, Katz)

Problem of finding a partition D of d as the part of pts $L \cap C$ $\sim$ prob of D as conjucacy classes of a permutation in Σ_d.

Gr_n $\qquad$ n-vector = Hilb function of artinian reduction

$H_{d_{21}}$ $\qquad$ $H_{e\ a}$

$\uparrow$

Hilbert scheme of d points

$h = 1\ 3\ 6\ 10\ 6\ 3\ 1$ – Gorenstein

$1\ 3\ 6\ 10\ 1$ ← h-vector of 11 pts

$5\ 3\ 1$

Now

$$\mathrm{Gor}_h \times H_d \times H_e$$

$$\overset{\cup}{\mu} = \{ G, P, Q \mid G = P \cup Q \text{ reduced} \}$$

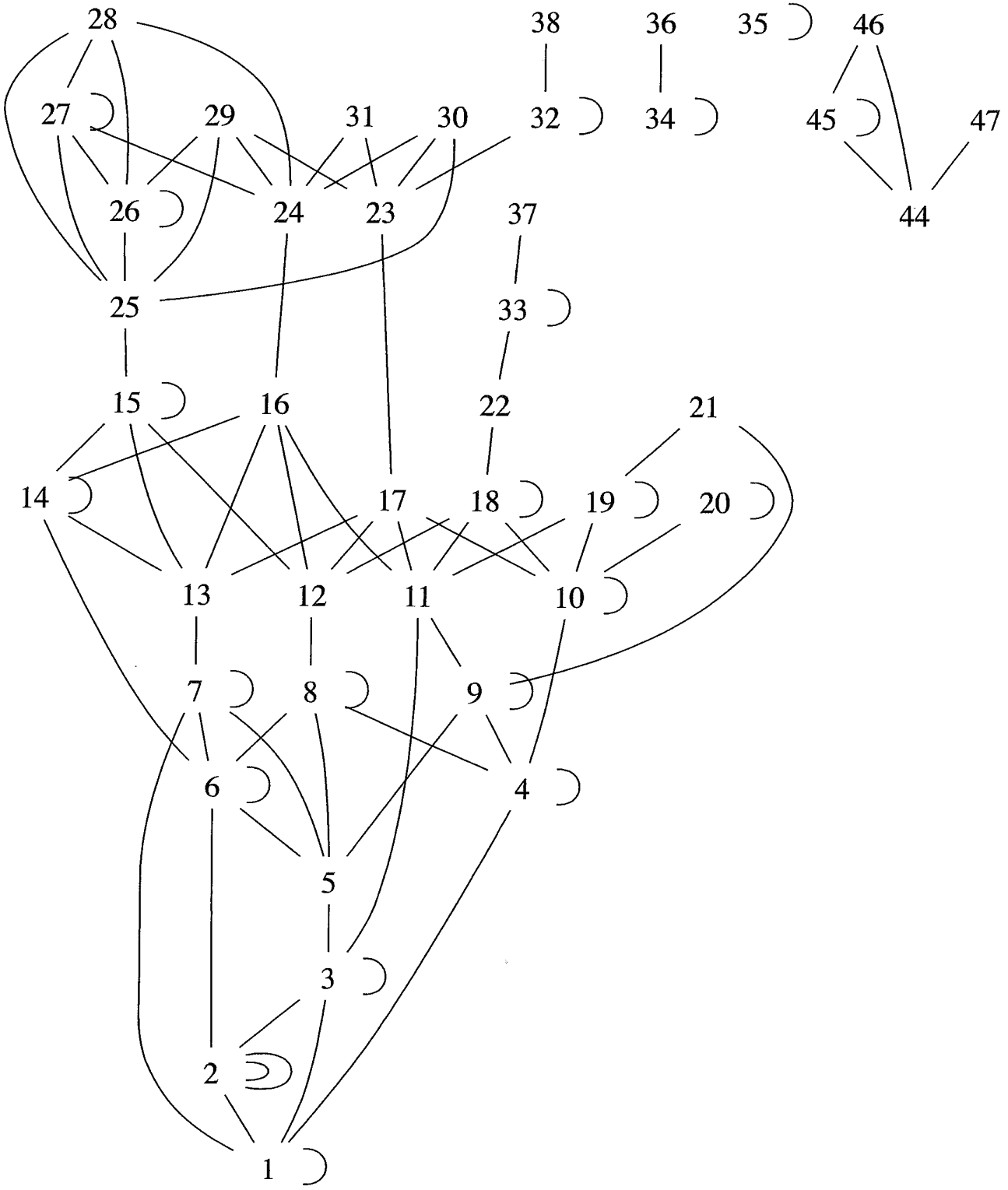
H_d H_e

21 9

Thm $(-, H, s)$: The d, e with bidcon direct Gor link are

... see handout ...

(this is over $\mathbb{C}$)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