Homology of Littlewood complexes

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Given the action of a classical group G on several vectors (and covectors) V, the ring of polynomial invariants is the coordinate ring of a determinantal variety, and its minimal free resolution has a lot of rich combinatorial structure that was worked out by Lascoux and Jozefiak-Pragacz-Weyman. There is an even richer structure if we consider the minimal free resolution (over an ambient polynomial ring) of the whole ring of functions on V considered as a module over this ring of invariants.

I will review the known situation and explain how it generalizes. This gives several families of ideals where the Koszul homology can be calculated explicitly with (simple!) combinatorial formulas which are formally similar to the formulas in the Borel-Weil-Bott theorem. I will also explain how this relates to instability phenomena for the representation theory of the classical groups. This is based on joint work with Andrew Snowden and Jerzy Weyman.