Title: Symmetrization
Speaker: Almut Burchard

Abstract: These lectures give an introduction to rearrangement or symmetrization methods in geometric and functional analysis. Rearrangements are geometric manipulations that improve the "shape" of a body while preserving its "size."

We will mainly consider Steiner symmetrization (which introduces a hyperplane of symmetry) and polarization (which pushes mass across a hyperplane towards the origin); these can be iterated to produce full rotational symmetry.

Starting from Steiner's 1830's "simple proof" of the isoperimetric inequality, we will discuss the role of symmetrization in geometric optimization problems (e.g., minimizing the electrostatic capacity and Santalo's inequality in convex geometry), functional inequalities (the Sobolev and Hardy-Littlewood-Sobolev inequalities), differential equations (eigenvalue problems and Talenti's elliptic estimate), and probability (inequalities for Brownian motion and related processes). The goal is to acquaint students with a useful tool that can greatly simplify problems (if it applies).