

Maximal stream, minimal cutset and maximal flow in d-dimensional first passage percolation

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We consider the standard first passage percolation model in the rescaled graph $\mathbb{Z}^d/n$ for $d \geq 2$. We interpret it as a model of porous medium: the edges of the graph are small tubes, to which we associate a family of i.i.d. random capacities. Let Ω be a domain of $\mathbb{R}^d$ and denote by Γ its boundary. Let Γ_1 and Γ_2 be two disjoint open subsets of Γ , representing the parts of Γ through which some water can enter and escape from Ω . A law of large numbers for the maximal flow from Γ_1 to Γ_2 in Ω in $\mathbb{Z}^d/n$, when n goes to infinity, is already known. We investigate here the asymptotic behaviour of a maximal stream and a minimal cutset. A maximal stream is a vector measure $\vec{\mu}_{\max}$ that describes how the maximal amount of fluid can circulate through Ω . Under conditions on the regularity of the domain and on the law of the capacities of the edges, we prove that a.s. the sequence $(\vec{\mu}_{\max})$ converges weakly, when n goes to infinity, to the set of the solutions of a continuous non-random problem of maximal stream in an anisotropic network. A minimal cutset is a set of edges whose capacities restrict the flow. It can be seen as the boundary of a set $E_{\min}$ that separates Γ_1 from Γ_2 in Ω and whose random capacity is minimal. Under the same conditions, we prove that a.s. the sequence $(E_{\min})$ converges for the topology L^1 , when n goes to infinity, towards the set of the solutions of a continuous non-random problem of minimal cutset. We deduce from this a continuous non-random max-flow min-cut theorem, and a new proof of the law of large numbers for the maximal flow.