The Number of Entangled Clusters

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In bond percolation on the simple cubic lattice, each bond is independently "open" with probability p. Suppose we view each open bond as a solid but flexible bar, with all bars that share an endpoint being joined at that point. Then it is possible for two disjoint connected components to be topologically entangled.

Is it possible for all connected components to be finite, and yet for an infinite number of them to form a single entangled cluster? G. Grimmett and A. Holroyd showed that this happens (almost surely) for some values of p, but not when p is close to 0. They then asked whether the number of entangled clusters (modulo translation) with exactly N edges is bounded exponentially in N. We prove that the answer is yes. Among our corollaries we obtain

(1) an alternative lower bound on the critical value for this "entanglement percolation", and

(2) exponential decay of the tail probabilities for the size of the entangled cluster containing the origin, when p is small.

This is joint work with Mahshid Atapour.