Heat flow in non-equilibrium CFT

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Notes taken by Samuel S Watson

We will describe what happens when we put two isomorphic one-dimensional gapless systems, prepared at different temperatures (in some Gibbs state) , into contact with one another. We want to know what happens close to the contact point. We will study very large systems, where our observation domain is much smaller than the system size. The steady state is obtained by letting the system evolve over $[-t_0, 0]$ and letting $t_0 \to \infty$.

Define $\Delta_t Q$ be the heat transferred during the time duration t in the stationary regime. The mean heat current is given by $J = -\partial_t(\Delta_t Q)$, and in the steady state we have

$$\langle J \rangle = \frac{c\pi}{12\hbar} k_B^2 (T_l^2 - T_r^2),$$

where c is the central charge. Let

$$F(\lambda) = \lim_{t\to\infty} t^{-1} \log(e^{i\lambda\Delta_t Q})$$

be the large deviation function. It is universal, depending only on the central charge. It satisfies the fluctuation relation

$$F(i(\beta_l - \beta_r) - \lambda) = F(\lambda).$$

The fluctuation relation deals with the probabilities of opposite heat transfers across the interface.

How do we prepare the stationary state? We have two dynamics: $H_0 = H_l + H_r$ before contact and H after contact. If the limit exists,

$$\lim_{R \gg t_0 \to \infty} e^{-it_0 H} \rho_0 e^{it_0 H}$$

is stationary. The far left and right parts of the two subsystems serve as effective reservoirs (for R large), which are different temperatures.

In CFT, the energy and momentum (chiral) densities are expressed in terms of their chiral component whose modes are the Virasoro generators.

The moment of contact is modeled by making h obey reflection before and transmission after (thus there is a wall between the two systems which is removed).

Define Q to be half the energy difference in the two subsystems. The time evolution is that of the coupled system. Then

$$Q(t) = Q + \int_0^t dx(h_-(x) - h_+(-x)),$$

We measure the charge/energy Q at time 0, and let the system evolve during time t. The wavefunction collapses when we make an observation, so we then observe at time t the transition probability

$$P_t(q, q_0) = \mathrm{Tr}(P_q e^{-itH} P_{q_0} \rho_{\mathrm{stat}} P_{q_0} e^{itH} P_q).$$

The energy/heat transferred is $q - q_0$. The generating function is defined by

$$\langle e^{i\lambda \Delta_t Q} \rangle = \sum_{q,q_0} e^{i\lambda(q-q_0) P_t(q,q_0)}.$$

(a standard definition for generating functions in QFT).

It turns out that the large deviations function $F(\lambda)$ admits a decomposition as a sum of Poisson processes (recall the elementary Levy-Kintchin representation of an infinitely divisible process as a sum of Gaussian, drift, and jump components).

To sum up, the probability distribution of heat transfer is universal (meaning that it's independent of microscopic data and of ν). Also, the large deviation admits a Poissonian representation with a nice representation. Is this universal?