Restriction Properties of annulus SLE

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We will define annulus SLE as an intermediate step in the proof of the reversal of whole-plane SLE. It is defined using the annulus Loewner equation. The driving term is $\sqrt{\kappa}B_t$ plus a drift function, which is determined by a function Λ_κ.

Theorem 1. Define a measure by

$$\frac{d\mu_L}{d\mu} = Z^{-1}\mathbf{1}_{\beta\cap L=\emptyset}\exp(c\mu_{\text{loop}}(\mathcal{L}_{L,p})).$$

Then ν_L is the distribution of a time-change of an annulus SLE$(\kappa;\Lambda_{\kappa;\langle s\rangle})$ trace in $A_p \setminus L$ started from z_0 with marked point w_0.

When $\kappa = 8/3$, the central charge $c = 0$ so we obtain the restriction property. For other values of κ this is a "weak" restriction property.

To define annulus SLE, we need to choose a way to uniformize $g_t : A_p \to A$. Properties of the resulting trace include continuity. The annulus Loewner equation is

$$\partial_t\tilde{g}_t(z) = \mathbf{H}(p-t, \tilde{g}_t - \xi(t)),$$

where $\mathbf{H}$ relates to the covering map from the strip to the annulus.

The covering trace is defined to be the lift $\tilde{\beta}$ of the annulus trace under the covering map.

We can also define SLE(κ) with an additional force point on the outer boundary. If the force Λ satisfies a certain PDE, the annulus $\mathsf{SLE}(\kappa;\Lambda)$ commutes with an annulus $\mathsf{SLE}(\kappa;\Lambda^-)$ process growing in the same domain with the force point and initial point switched. For each κ, there is a special drift function Λ_κ which solves this PDE. This function is defined in terms of Jacobi's theta function.

We decompose the lift function of the covering trace. We can decompose the resulting sets according to which preimage of the endpoint at which the lifted trace ends.

To prove the restriction property (Theorem 1), we use a family of polygonal crosscuts.

We can also prove that annulus SLE defined in this way is equivalent to annulus SLE as defined by Lawler.