Conformal blocks in 2D CFT, the Calogero-Sutherland model and the AGT conjecture

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Notes taken by Samuel S Watson

We will make some connections between the integrable structure of the Calogero-Sutherland model and SLE and other topics discussed this week. The relationship between the coupling constant g and κ from SLE is $g = \kappa/4$. When g is positive, we obtain a fractional quantum Hall effect with non-abelian statistics, and when g is negative we get AGT conjecture. Recall that the central charge is related via

$$c = 1 - \frac{(g-1)^2}{g}.$$

We will describe a link with supersymmetric 4D theory and a 2D CFT, called the AGT conjecture. The correspondence is between the n-point correlation function of the Liouville theory and Nekrasov's instanton partition function of a gauge theory with gauge group $\mathfrak{su}(2)_1 \otimes \cdots \otimes \mathfrak{su}(2)_{n-3}$.

One may give a full dictionary relating quantities in Liouville theory to corresponding ones in gauge theory (see slides).

The proof of AGT in the mathematical literature consists of a general proof (yet unpublished), as well as special cases in 2008 and 2012.

The eigenfunctions in the Calogero-Sutherland model can be taken to be Jack polynomials. There is a correspondence between holomorphic correlators and wave functions for the FQHE (fractional quantum hall effect). One can take advantage of a certain property of Jack polynomial related to conjugates of Young diagrams.

Two second-level degenerate fields

$$(L_{-1}^2 - gL_{-2})\Phi_{(1|2)} = 0 \qquad (L_{-1}^2 - \frac{1}{g}L_{-2})\Phi_{(2|1)} = 0$$

("duality" in the SLE literature). How can we characterize the excited states above the ground states? One consequence of duality is

$$\mathcal{F}^{a,b}_{M,N}(w;z) = \sum_{\lambda} P^{\tilde{\alpha},a}_{\lambda}(w) P^{\alpha,b}_{\lambda}(z)$$

The correlators of second-order degenerate fields are ground states of the CS model.

We can rotate the boson basis to obtain

$$c_m = (a_m + b_m)/\sqrt{2} \qquad \tilde{c}_m = \frac{1}{\sqrt{2}}(a_m - b_m)$$

and introduce the one-component bosonised CS Hamiltonians. This is related to the Benjamin-Ono heirarchy.

It is possible to extend these ideas to a large number of cases (coset and superconformal theories; parafermionic theories).

Open problems:

Is it possible to do non-polynomial eigenfunctions?