Edge reinforced random walks, Vertex reinforced jump process, and the SuSy hyperbolic sigma model.

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Edge-reinforced random walk (ERRW), introduced by Coppersmith and Diaconis in 1986, is a random process which takes values in the vertex set of a graph G, and is more likely to cross edges it has visited before. We show that it can be represented in terms of a Vertex-reinforced jump process (VRJP) with independent gamma conductances: the VRJP was conceived by Werner and first studied by Davis and Volkov (2002,2004), and is a continuous-time process favouring sites with more local time. We show that the VRJP is a mixture of time-changed Markov jump processes and calculate the mixing measure. The mixing measure is interpreted as a marginal of the supersymmetric hyperbolic sigma model introduced by Disertori, Spencer and Zirnbauer.

This enables us to deduce that VRJP and ERRW are strongly recurrent in any dimension for large reinforcement (in fact, on graphs of bounded degree), using a localisation result of Disertori and Spencer (2010).

(Joint work with Pierre Tarrès.)