Monday, April 30, 2012

Speaker: Dmitry Dolgopyat

Title: Dynamical point of view on some random walk models.

Note taker: Xiaoqin Guo

I. RW in Markovian environment

Model:
- P_{ij} – transition prob. of an irred. aperiodic MC on S (finite set)
- At each $z \in \mathbb{Z}^d$, $(x(z,n))_{n\in\mathbb{N}}$ a chain w/ transition prob. P_{ij}. $x(z,\cdot)$ at different sites z are independent.
- $\forall s \in S$, have a prob. meas. $p(s,\cdot)$ on $\Lambda \subset \mathbb{Z}^d$ (finite)
- RW S_n:

$$P(S_{n+1}-S_n = e \mid \mathcal{F}_n) = p(x(S_n,n), e)$$

Thm 1 (Liverani, D.) $\exists$ velocity v, matrix $\mathcal{D}$ s.t. with prob. 1,

$$\frac{S_n - nv}{\sqrt{n}} \Rightarrow \mathcal{N}(0, \mathcal{D}^2)$$

proof. Use environment seen from particle process:

let $\omega_n := (x(S_n,n),\ S_{n+1}-S_n) \in A = S\times\Lambda$.

$\xi,\ \xi\omega_0,\ \xi\omega_0\omega_1,\ \xi\omega_0\omega_1\omega_2, \dots,\ \underbrace{\xi\omega_0\cdots\omega_n}_{=\xi_n}$ is a Markov chain

with transition kernel $P_n(\xi\to\eta)$

By mixing, $P_n \to$ invariant measure P_∞ exponentially fast.

Prop. $\forall a$, $\xi \mapsto P_\infty(a\mid\xi)$ is Hölder continuous

Thm (Ruelle ~ '60)

$\xi \mapsto P_\infty(a\mid\xi)$ Hölder $\Rightarrow \exists!$ invariant meas. ν_p for the Markov chain (ξ_n) and if $\varphi(\xi)$ is also Hölder then

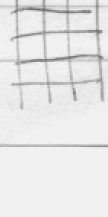
$$\frac{\sum_{i=0}^{n-1}\varphi(\xi_n) - n\nu_p(\varphi)}{\sqrt{n}}$$

satisfies CLT.

II Finitely reinforced walks on a ladder

ladder: $\mathbb{Z}\times\{1,\dots,d\}$

$\mathbb{Z}$



Model: Each edge has weight 1 initially. During the first k visits to an edge, the weight of the edge increased by δ, thereafter stay at $1+k\delta$:

$$\omega(0,e) = 1$$

random walks:

$$P(S_{n+1}=u \mid S_n = v) = \frac{\omega(n, \overset{\text{edge}}{uv})}{\sum_{\bar{u}\sim v} \omega(n, \bar{u}v)}$$

<u>Thm</u> (Sellke) $\forall\, l, k$, $\exists\, \delta_0 > 0$ such that if $|\delta| < \delta_0$

$\Rightarrow$ reinforced RW recurrent.

<u>Thm</u> $\forall\, l, k$, $\exists\, \delta_1$ such that if $|\delta| < \delta_1$, then $\exists\, \alpha$,

$$\frac{X_{nt}}{\sqrt{n}} \Rightarrow W(t)$$

where $W(t)$ is an "$(-\alpha)$ perturbed Brownian motion":

$$W(t) = \underbrace{B(t)}_{\text{Brownian motion.}} - \alpha \left(\max_{s\le t} W(s) + \min_{s \le t} W(s) \right)$$

Question 1. $\delta > 0 \Rightarrow \alpha > 0$?

Question 2. $\delta > 0 \Rightarrow X_n$ recurrent ?